

Letters to the Editor

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Note on the Scattering of Radiation in an Inhomogeneous Medium†

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WE consider the scattering of a plane sound wave when passing through a medium in which the sound velocity c fluctuates slightly and irregularly around the mean value \bar{c} . The corresponding problem of the scattering of electromagnetic radiation in an optically inhomogeneous medium can be treated along similar lines. We write

$$c(x, y, z) = \bar{c} + \Delta c(x, y, z), \quad (1)$$

and compute the intensity of sound scattered by a volume V of the medium, which is large enough to contain several oscillations in sign of Δc . We choose a coordinate system centered at V and assume an incident plane wave traveling in the negative z direction. If φ denotes the sound potential then we have

$$\begin{aligned} \varphi &= e^{ik(ct+z)} + \varphi_1, \\ \varphi_1 &= -\frac{k^2}{2\pi} \int \left(\frac{\Delta c}{c}\right)' \frac{\exp[ik(ct+z'-R)]}{R} dv', \end{aligned} \quad (2)$$

where

$$dv = dx'dy'dz', \quad R = [(x-x')^2 + (y-y')^2 + (z-z')^2]^{\frac{1}{2}}, \quad (3)$$

and c has been written for \bar{c} . From an analysis of the scattering by a sphere of radius a of slightly different sound velocity it is found that expression (2) is valid if $(ak)(\Delta c/c) \ll 1$. We should, therefore, expect the perturbation method used in deriving (2) to be valid if $(a'k)(\Delta c/c) \ll 1$, where a' denotes the dominant "wave-length" of Δc .

If the distance r of the point of observation is large in comparison with the wave-length, we have

$$\begin{aligned} R &\cong r - \frac{xx'}{r} - \frac{yy'}{r} - \frac{zz'}{r}, \quad f' \cong z' + \frac{xx'}{r} + \frac{yy'}{r} + \frac{zz'}{r}, \\ \varphi_1 &= -\frac{k^2}{2\pi r} \int \left(\frac{\Delta c}{c}\right)' \cos k(ct+f'-r) dv', \end{aligned} \quad (4)$$

where we have taken the real part of (2). The energy E scattered in an elementary solid angle $d\Omega$ is given by

$$\begin{aligned} E &= -\rho \frac{\partial \varphi}{\partial t} \cdot \frac{\partial \varphi}{\partial r} r^2 d\Omega = c\rho k^2 \left(\frac{k^2}{2\pi}\right)^2 d\Omega \int \left(\frac{\Delta c}{c}\right)' \\ &\quad \times \cos k(ct+f'-r) dv' \int \left(\frac{\Delta c}{c}\right)'' \cos k(ct+f''-r) dv''. \end{aligned} \quad (5)$$

We now average over time, and writing $E_0 = \frac{1}{2}\rho ck^2$, obtain

$$\begin{aligned} E &= E_0 \left(\frac{k^2}{2\pi}\right)^2 d\Omega \int \left(\frac{\Delta c}{c}\right)' dv' \\ &\quad \times \int \left(\frac{\Delta c}{c}\right)'' \cos k \left[\left(1 + \frac{z}{r}\right) \gamma + \frac{\alpha x}{r} + \frac{\beta y}{r} \right] \\ &= E_0 \left(\frac{k^2}{2\pi}\right)^2 d\Omega \int \left(\frac{\Delta c}{c}\right)' dv' \\ &\quad \times \int \frac{\Delta c}{c}(x'+\alpha, y'+\beta, z'+\gamma) \\ &\quad \times \cos k \left[\left(1 + \frac{z}{r}\right) \alpha + \frac{x\beta}{r} + \frac{\gamma\gamma}{r} \right] d\alpha d\beta d\gamma, \end{aligned} \quad (6)$$

with

$$\alpha = x'' - x', \quad \beta = y'' - y', \quad \gamma = z'' - z'.$$

The axes of α, β, γ are parallel to the axes of the original system, and the limits of α , say, are $-(x_1+x_2)$ and $+(x_1+x_2)$ if the limits of x are $-x_1$ and $+x_2$. In particular we have

$$\begin{aligned} \left(1 + \frac{z}{r}\right) \alpha + \frac{x\beta}{r} + \frac{\gamma\gamma}{r} &= \cos\theta_0(1 + \cos\theta) \\ &\quad + \sin\theta_0 \sin\theta \sin(\varphi - \varphi_0), \end{aligned} \quad (7)$$

where θ_0, φ_0 refer to the direction of r , and θ, φ are the spherical coordinates in the (α, β, γ) system. We now define the autocorrelation $R(\alpha, \beta, \gamma)$ of the values of $(\Delta c/c)$ at two points inside V whose x', y', z' coordinates differ, respectively, by α, β, γ by:

$$\begin{aligned} \int \int \int \frac{\Delta c}{c}(x', y', z') \frac{\Delta c}{c}(x'+\alpha, y'+\beta, z'+\gamma) dx' dy' dz' \\ \equiv V \left\langle \left(\frac{\Delta c}{c}\right)^2 \right\rangle_{Av} R(\alpha, \beta, \gamma). \end{aligned} \quad (8)$$

Substituting in (6) we get

$$\begin{aligned} E &= E_0 \left(\frac{k^2}{2\pi}\right)^2 d\Omega V \left\langle \left(\frac{\Delta c}{c}\right)^2 \right\rangle_{Av} \int \int \int R(\alpha, \beta, \gamma) \\ &\quad \times \cos k \left[\left(1 + \frac{z}{r}\right) \gamma + \frac{x\alpha}{r} + \frac{y\beta}{r} \right] d\alpha d\beta d\gamma. \end{aligned} \quad (9)$$

In the case of spherical isotropy we have

$$R(\alpha, \beta, \gamma) = R(\rho), \quad \rho = [\alpha^2 + \beta^2 + \gamma^2]^{\frac{1}{2}}, \quad (10)$$

and using the relation

$$\begin{aligned} \int_0^\infty \rho^2 R(\rho) d\rho \int_0^\pi \sin\theta d\theta \int_0^{2\pi} \cos k[\cos\theta_0(1 + \cos\theta) \\ + \sin\theta_0 \sin\theta \cos(\varphi - \varphi_0)] d\varphi \\ = 4\pi \int_0^\infty \rho^2 R(\rho) \left(\frac{\sin b k \rho}{b k \rho}\right) d\rho, \quad b = 2 \cos(\theta_0/2), \end{aligned} \quad (11)$$

we obtain

$$E = E_0 \frac{k^4}{\pi} d\Omega V \left\langle \left(\frac{\Delta c}{c}\right)^2 \right\rangle_{Av} \int_0^\infty \rho^2 R(\rho) \left(\frac{\sin b k \rho}{b k \rho}\right) d\rho. \quad (12)$$

For a Markoffian autocorrelation function²

$$R(\rho) = e^{-\rho/a}, \quad (13)$$

where a represents a measure of the "size" of the scattering elements, and Eq. (12) yields

$$E = E_0 \frac{2a^3 k^4}{\pi} d\Omega V \left\langle \left(\frac{\Delta c}{c}\right)^2 \right\rangle_{Av} \left[1 + 4a^2 k^2 \cos^2\left(\frac{\theta_0}{2}\right) \right]^{-2}. \quad (14)$$

It should be noted that θ_0 is measured from the negative z

axis. If the angle is measured from the forward direction of the incident wave, $\cos(\theta_0/2)$ in (11) and (14) should be changed to $\sin(\theta_0/2)$.

In the interpretation of light scattering in solutions, it is customary to assume a model for the scattering elements, say a sphere of radius a , and apply the theory of scattering by a cloud of randomly distributed spheres.³ Experiments indicate, however, that a has a considerable statistical scatter. The model of randomly distributed spheres of variable radii is perhaps not simpler to visualize than the autocorrelation function R itself, which is the only statistical property of the scatterers' population determining the angular distribution of scattered light, as shown in Eqs. (9) and (12).

This note is taken from a study of scattering problems made by the writer in May, 1942 for O.S.R.D., of which only the results were published. The manuscript was made available to members of Division 6 and was loaned to several members of Division 17 of O.S.R.D. Among the problems treated in this study, either exactly or approximately, are the scattering of sound by (1) a vortex sheet, (2) a cylindrical vortex, (3) a spherical vortex, (4) a convective cloud in which turbulence is isotropic, and (5) a corrugated surface.

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¹ See G. I. Taylor, Proc. Lond. Math. Soc. 20, 196 (1920); Proc. Roy. Soc. A164, 478 (1938). N. Wiener, Acta Mathematica 55, 118 (1930); A. L. Patterson, Phys. Rev. 46, 372 (1934).

² C. L. Pekeris, Phil. Mag. [7] 33, 541 (1942).

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The Half-Life of Be¹⁰

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THE long-lived nucleus Be¹⁰ has been reported as formed only in the Be⁹(d, p) reaction.^{1,2} The half-life could only be estimated to be in the range 10^3 to 10^7 years¹⁻³ because the yield of the reaction producing the Be¹⁰ is not known. Bretscher⁴ tried to produce Be¹⁰ from the Be⁹(n, γ) reaction but found no activity. Levinger and Meiners² also found no Be¹⁰ activity in Be irradiated in the Clinton pile.

Because the extremely long half-life of Be¹⁰ is of interest theoretically, it was decided to attempt its measurement by isolation of the activity produced by the (n, γ) reaction in two BeO samples which had been irradiated for different periods of time in the Hanford pile (kindly supplied by O. C. Simpson of the Argonne Laboratory). Although the BeO was of high purity, it had a gross counting rate of about 5000 counts per min. per mg of Be. As the expected Be¹⁰ activity would be only a few counts per min. per mg, it was necessary to purify the samples carefully. The purification was carried out by group separation and the addition of suspected impurities of the third and fourth groups as carriers, the final specific separation for Be being the basic acetate process.⁵ The main impurities were found to be carried with titanium in excess NaOH and with zinc in the H₂S-NH₄CH₃COO process.

The final samples, which did not decrease in activity with further purification, had activities of 3.3 counts per min. per mg of Be for one sample and 6.7 for another sample that had been irradiated twice as long. Absorption curves of the activities gave an energy of 0.58 ± 0.03 Mev which is in close agreement with the findings of Levinger and Meiners² and McMillan and Ruben¹ and the mass difference of Be¹⁰ and B¹⁰ (0.58 Mev¹).

In order to obtain the half-life from the nvt and observed activity it is necessary to know the cross section of the (n, γ) reaction. The cross section for Be⁹(n, γ) was measured by comparing the effect on the reactivity of the Argonne graphite pile of several kilograms of BeO with that of graphite. The value obtained was 0.01 barn (barn = 10^{-24} cm²) in agreement with the measurements of S. K. Allison,⁶ of A. Langsdorf,⁶ and of R. Nobles.⁶ The half-life of Be¹⁰ calculated from the activity, nvt , and the cross section is almost the same for the two samples, the average value being

$$T_{1/2} = 2.9 \times 10^6 \text{ years.}$$

The amount of activity to be expected in the Hanford graphite from the C¹³(n, α)Be¹⁰ (fast neutron) reaction was then calculated, using the measured half-life and an (n, α) cross section estimated by comparison with measured pile cross sections for other endothermic reactions.⁷ A small amount of graphite which had been in the Hanford pile for about a year was ignited, several mg of Be carrier added, and the Be purification carried out. An activity was obtained of magnitude within a factor of two of that predicted, which did not decay, and had the correct beta-energy for Be¹⁰. It seems certain then that the activity was Be¹⁰ produced by the C¹³(n, α) reaction and the finding constitutes additional support for the correctness of the half-life determination.

Because of the importance of obtaining Be¹⁰ in large amounts and of higher specific activity than can be done by the (n, γ) reaction, arrangements are now being made to irradiate boron in the Hanford pile for the production by the (n, p) reaction and subsequent chemical separation of the Be¹⁰.

* Declassified January 6, 1947.

¹ E. M. McMillan and S. Ruben, Phys. Rev. 70, 123 (1946).

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³ G. T. Seaborg, Rev. Mod. Phys. 16, 1 (1944).

⁴ E. Bretscher, Nature 146, 94 (1940).

⁵ Noyes and Bray, A System of Qualitative Analysis for the Rare Elements (The Macmillan Company, New York, 1943).

⁶ Unpublished Manhattan Project work (1943-46).

⁷ D. J. Hughes, N. Goldstein, and W. D. B. Spatz (unpublished).

Photoelectric Emission from a Rough Surface*

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EXPERIMENTS on the normal energy distribution of photoelectrons from alkali metals ordinarily make use of evaporated metals which have a matte surface rather than the smooth surface considered in the Fowler-DuBridge¹ theory. The following is a modification of the theory for the case of a rough surface at zero degrees Kelvin.