The Free Fall of Atoms and the Measurement of the Velocity Distribution in a Molecular Beam of Cesium Atoms

I. ESTERMANN, O. C. SIMPSON,* AND O. STERN**
Research Laboratory of Molecular Physics, Carnegie Institute of Technology, Pittsburgh, Pennsylvania
(Received November 29, 1946)

The free fall of atoms is observed in long molecular beams of potassium and cesium atoms. The measurement of the intensity distribution in a beam deflected by gravity represents the velocity distribution of the beam atoms and permits an accurate determination of this distribution. The results show that the measured values agree in general with those calculated for a modified Maxwellian velocity distribution in the beam. At larger deflections, i.e., for slow atoms, a deficiency of intensity was observed, which increased both with increasing deflection and with increasing pressure in the oven where the beam originates. This deficiency is explained by collisions in the immediate vicinity of the oven slit.

1. INTRODUCTION

A METHOD for the exact measurement of the Bohr magneton devised recently by one of us[1] is based on the compensation of the acceleration of gravity acting on a molecular beam of cesium atoms by the magnetic acceleration produced by an inhomogeneous magnetic field. The effect of gravity on atoms, that is their free fall, is easily observable in a long molecular beam. In a cesium beam 2 meters long, the fall distance of atoms having the most probable velocity corresponding to an oven temperature of about 450°K is approximately 0.2 mm. The magnetic inhomogenity required to compensate the acceleration of gravity is in this case only about 25 gauss/cm. It is, therefore, possible to produce the magnetic field by a current in a straight conductor of circular cross section; thus the magnetic inhomogeneity does not need to be measured, but can be calculated from the geometrical dimensions and the electric current. Hence, it is only necessary to measure the current I_0 through the conductor which produces a magnetic field of the inhomogeneity $(\partial H/\partial r)_0$ exactly compensating the acceleration of gravity for the Cs atoms with the largest magnetic moment μ_0 equal to one Bohr magneton. Since the compensation of the acceleration is independent of the velocity, this method is in principle independent of the velocity distribution in the beam. For the actual measurement of I_0, however, the exact knowledge of the velocity distribution in the beam is necessary.

FIG. 1. Principle of the method.

If the current I through the conductor is less than I_0, all the atoms in the beam are deflected downwards. If $I>I_0$, those with the largest magnetic moment are deflected upwards. The compensating current I_0 can be determined by placing a detector slightly above the plane of the beam (position D' in Fig. 1), and by increasing I gradually until atoms deflected upwards strike the detector. In practice, the beam intensity i striking the detector must be measured as a function of the current I, and I_0 is found by an extrapolation, which requires the knowledge of the actual velocity distribution in the beam.

From published measurements by Cohen and Ellett[2] of the velocity distribution in beams of alkali-metal atoms undergoing magnetic deflection, it appeared that in such beams the modified Maxwellian distribution was correct to within a few percent down to velocities of one-half of the most probable velocity. Experiments for the measurement of the Bohr magneton carried out by us from 1938 to 1942 showed, however, that considerable deficiencies of slow molecules (up to 50 percent of the number expected on the basis of Maxwell's law) are present even at oven pressures lower than those used in

* At present on leave to the Argonne National Laboratory, Chicago, Illinois.
** Now in Berkeley, California.
[1] O. Stern, Phys. Rev. 51, 852 (1937).
[2] V. W. Cohen and A. Ellett, Phys. Rev. 52, 502 (1937).

the magnetic deflection experiments of Cohen and Ellett.* It was, therefore, necessary to determine these deviations from the modified Maxwellian distribution as a preliminary work. Since the apparatus designed for the measurement of the Bohr magneton allowed such measurements by means of deflection by gravity ("free fall"), this method was used.

2. PRINCIPLE OF THE METHOD

To observe the free fall of molecules, the following arrangement was used (Fig. 1). A beam of Cs atoms produced by the oven slit O and the collimating slit S was detected by a hot tungsten wire D. Both slits as well as the detector wire were horizontal. The Cs atoms striking the detector wire became ionized and the ions were collected on a negatively charged cylinder surrounding the wire (Langmuir-Taylor method).[3] The ion current between the detector wire and the collecting cylinder gave directly the number per second of Cs atoms impinging upon the wire. The dotted lines show the path of Cs atoms with different velocities.

We assume $OS=SD=l$. Then for an atom with the velocity v, the fall distance is $S=gl^2/v^2$ (not $gl^2/2v^2$). For Cs atoms of the most probable velocity α the fall distance S_α is 0.174 mm† for an oven temperature $T=450°$K and for $l=100$ cm. Since the width of the slits defining the beam and the thickness of the detector wire in our experiments was only 0.02 mm, this fall distance is easily observable. For potassium atoms, S_α is approximately 0.04 mm, which is still well observable under our experimental conditions. The intensity distribution in the beam deflected by gravity measured in the vertical direction pictures the velocity distribution of the atoms in the beam.

With slits of the width b, the intensity distribution in the undeflected beam is given by a trapezoid (Fig. 2). Assuming Maxwellian distribution in the oven, the fraction of atoms in the beam in the velocity interval between v and

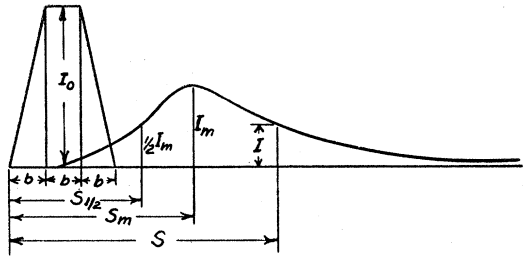

FIG. 2. Intensity distribution in a molecular beam deflected by gravity.

$v+dv$ is

$$\frac{dn}{n_0}=\frac{2v^3}{\alpha^3}\exp\left[-\frac{v^2}{\alpha^2}\right]d\left(\frac{v}{\alpha}\right)=x\cdot e^{-x}dx;\qquad x=\left(\frac{v}{\alpha}\right)^2 \tag{1}$$

The intensity at the displacement S is given by:

$$\frac{i}{i_0}=\frac{S_\alpha}{b}\left[\frac{S}{S_\alpha}\exp\left[-\frac{S_\alpha}{S}\right]-\frac{S-b}{S_\alpha}\exp\left[-\frac{S_\alpha}{S-b}\right]-\frac{S-2b}{S_\alpha}\exp\left[-\frac{S_\alpha}{S-2b}\right]+\frac{S-3b}{S_\alpha}\exp\left[-\frac{S_\alpha}{S-3b}\right]\right]. \tag{2}$$

This is the intensity distribution as function of the displacements which would be measured with an infinitely thin detector wire. The finite width of the detector can easily be taken into account by integration; in our Cs experiments, this correction amounted to less than $1\frac{1}{2}$ percent near the maximum of the intensity distribution and was negligible elsewhere.

To compensate for the force of gravity by a magnetic force, one must produce a magnetic force on the Cs atoms of the same magnitude, but of direction opposite to the gravitational force. This can be accomplished by an electric current I underneath and parallel to the beam. At a distance d from the current, the magnetic field strength is $H=2I/d$, the inhomogeneity $\partial H/\partial r=2I/d^2$, and the force F on an atom whose component of magnetic moment in the direction of the field is μ_H is $F=\mu_H\cdot 2I/d^2$. Compensation takes place if

$$\mu_H\cdot 2I_0/d^2=mg, \tag{3}$$

* Because of a different slit form, their "effective" pressure at the oven slit might have been much lower than their measured pressure. A comparison of their results with our results is, therefore, not possible.

[3] J. B. Taylor, Zeits. f. Physik. **57**, 242 (1929).

† The value 0.177 mm given in reference 1 is incorrect.

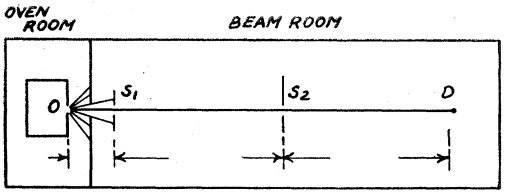

FIG. 3. General arrangement of the apparatus.

where m is the mass of the atom and g the acceleration of gravity. Because of the nuclear spin (7/2) of the Cs atom and the use of "weak" fields (not strong enough to uncouple the nuclear and electronic moments) the cesium beam is split by space quantization into 16 beams, each beam consisting of atoms with different values of μ_H. One-half of these values of μ_H result in a magnetic force in the same direction as gravity; for the other half, compensation is possible and will take place for those with the largest value of μ_H at the smallest value of I. (One-sixteenth of all the atoms will have the largest value of $\mu_H=\mu_B-\mu_N$, where μ_B is the Bohr magneton and μ_N the magnetic moment of the nucleus, which is of the order of magnitude of 1/1000 of the Bohr magneton.)

To measure I_0, the detector is placed a small distance above the position of the undeflected beam (position D'). As long as $I<I_0$, all atoms are deflected downwards and no atoms strike the detector. As soon as $I>I_0$, all the atoms with the largest magnetic moment ($\frac{1}{16}$ of the total number) will be deflected upwards and those in the proper velocity interval will strike the detector. As stated before, this method of measuring of I_0 is in principle independent of the velocity distribution in the beam. However, for practical reasons (small intensity of deflected atoms and background of scattered atoms) the intensity of the deflected atoms has to be measured at several detector positions as a function of I, and I_0 must be found by extrapolation. The extrapolation requires the knowledge of the actual velocity distribution in the beam. This is the reason for the experiments described in this paper.

To find the value of μ_B, the quantities m, g, and d in addition to I_0 have to be known. The mass* m is known to an accuracy of about one part in 10^4, and g, of course, at least to one part in 10^6. However, d is not only difficult to measure, but it is not simple to design an arrangement in which d is sufficiently defined and constant.

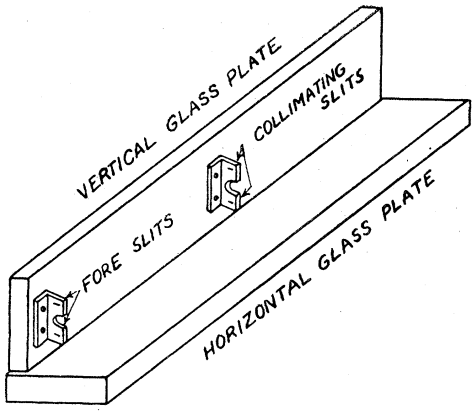

FIG. 4. Glass plates carrying slit system.

If we want the result to be exact to one part in 1000, the distance d between the center line of the current and the center of the beam has to be exact to one part in 2000. In our apparatus, this distance d was 2 cm. It is not possible to make this distance much larger, because then I_0 would be too large. Even with $d=2$ cm, I_0 is about 500 amp. Therefore, d has to be exact to 1/100 mm. Even with a solid rod as conductor it is not easy to have the radius exact to 1/100 mm over a length of more than 2 meters, nor is it easy to hold this rod in such a fashion that the center line is straight and parallel to the beam within this accuracy. Moreover, because of the heat developed by the current, it is necessary to provide cooling. This requires the use of a tube instead of a rod. Even in the best precision tubes obtainable, the wall thickness was not uniform to 1/100 mm.

To overcome this difficulty, we used the following arrangement: In addition to the beam above the conductor, a second beam was arranged at the same distance below. If we determine the I_0 for both beams and take the average, this value will be the I_0 for d equal to one-half of the distance between the two beams within the following accuracy: If δ is the deviation from the 2-cm distance between the beam and the center line of the conductor, then the error in the average I_0 for both beams is of the order of mag-

* If Eq. (3) is multiplied by Avogadro's number N, it becomes $N\mu_B 2I_0/d^2=M_B\cdot 2I_0/d^2=Mg$, where M is the atomic weight. Therefore, we measure actually the Bohr magneton per mole, M_B, with the accuracy discussed here.

nitude $(\delta/d)^2$. The reason for this is that if the distance in one place is too small for one of the beams by the amount δ, it is too large by the same amount for the other beam. The distance between the two beams can be defined and measured very accurately by using a slit system in which both the source slits and the collimating slits are so designed that the edges of a single piece of metal form the lower jaw of the upper slit and the upper jaw of the lower slit. The distance between these edges was measured with a micrometer slide to a few thousandths of a mm. Even if the center line of the conductor should be misaligned by as much as 0.1 mm, the error produced hereby is only of the order of magnitude 10^{-4}. In our experiments, the error in alignment was less than 0.05 mm.

3. APPARATUS

(a) General Arrangement (Fig. 3)

The molecular beam was produced by Cs vapor streaming out from an oven slit and was defined by a foreslit S_1 and a collimating slit S_2. The introduction of a foreslit was necessary for two main reasons: First, in order to get a good vacuum in the "beam room," it was necessary to separate the latter from the "oven room." By this separation, it was possible to maintain a very good vacuum (10^{-7} mm Hg or better) in the beam room, where, because the beam was 2 m long, a long mean free path was required. In the oven room, the pressure was usually around 10^{-6} mm, but the beam length there was only 10 cm. Secondly, it was easier to maintain the correct mechanical alignment between foreslit, collimating slit, and detector than between the oven slit and the other slits, since the oven position is not exactly fixed and changes during the heating. The alignment was the most important and difficult part of the experiments. For the magnetic deflection experiments, the detector was placed only a few hundredths of a mm above the "undeflected" beam, since the intensity i of deflected atoms decreases rapidly with increasing distance. In order to obtain the compensating current I_0, this intensity i was measured as a function of the current I producing the magnetic field. During these measurements, the relative position of the slits and the detector had to remain constant to about one-thousandth of a mm. Since the distance between the two slits and between the collimating slit and the detector was one meter in each case, the demands on the rigidity of the apparatus were considerable. In the first apparatus, the detector and the slits were mounted on a self-contained metal framework which was supported inside the evacuated beam room, but this construction did not produce the necessary rigidity. We finally chose the following arrangement.

The slits and detectors were mounted on a slab of plate glass ("vertical glass plate") 204 cm long, 9.5 cm high, and 1.9 cm thick, as shown in Fig. 4. The slits were mounted horizontally. The high degree of rigidity is, therefore, required only in the vertical direction, where the glass plate was very rigid with respect to bending because of its height of 10 cm. A slight displacement of the slits in the horizontal direction (up to 1/10 mm) does not affect the accuracy. The vertical glass plate was resting on another glass plate 210 cm long, 3 cm high, and 11.5 cm wide, which was mounted horizontally. This glass plate carried a

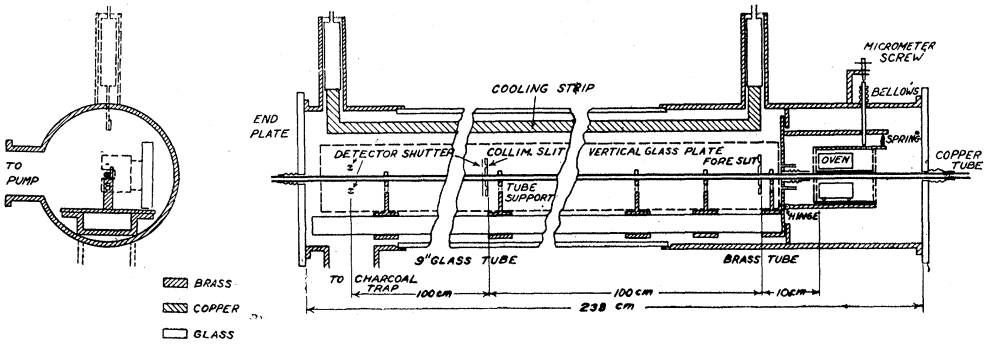

FIG. 5. (a) End view, (b) side view of apparatus.

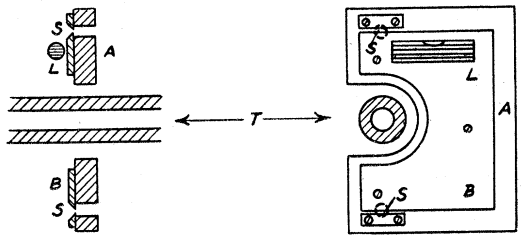

FIG. 6. (a) End view, (b) front view of slit system.

copper tube (0.947 cm O.D. and 0.47 cm I.D.) which served as conductor for the current producing the magnetic field. The horizontal glass plate was supported by two brass rings inside the vacuum envelope.

(b) Vacuum Envelope

The envelope consisted of a glass tube approximately 9″ in diameter with two brass end pieces as shown in Fig. 5. The glass tube had two side arms in the center for the shutter and for gas inlets, while the brass end pieces carried pump connections, traps, etc. It was divided by a brass plate into an oven room about 20 cm long, located inside one of the brass end pieces, and a beam room about 210 cm long. The oven room was evacuated by one large octoil diffusion pump,[4] and the beam room by two similar pumps. Large liquid-air cooled traps were arranged between pumps and apparatus. The dividing plate had a central hole for the copper tube conductor and two narrow channels for the two beams. The central hole was equipped with a bushing mounted on a sylphon bellows which permitted a slight motion of the tube but furnished a seal of high flow resistance. The channels for the beams were 20 mm long, 5 mm wide, and 1 mm high. The total flow between the two parts was about 1 liter/sec. as compared with a pumping speed of more than 50 liters/sec. for the beam room; that means that a pressure of 10^{-6} mm Hg in the oven room produced only 2×10^{-8} mm in the beam room. The ends were closed with plate glass disks of 26 cm and 1.9 cm ($\frac{3}{4}$″) thickness which were equipped with central holes of 2 cm and with bushings carried by sylphon bellows allowing a slight motion of the copper tube. All vacuum seals were made with Apiezon sealing compound *Q*.

In order to obtain the required vacuum of the order of magnitude of 10^{-8} mm, it was necessary to pump for several days. For the efficient condensation of vapors, a copper strip 4.5 cm high, 0.5 cm thick, and 185 cm long was suspended parallel to the beam from two metal Dewar vessels filled with liquid nitrogen. For the mag-

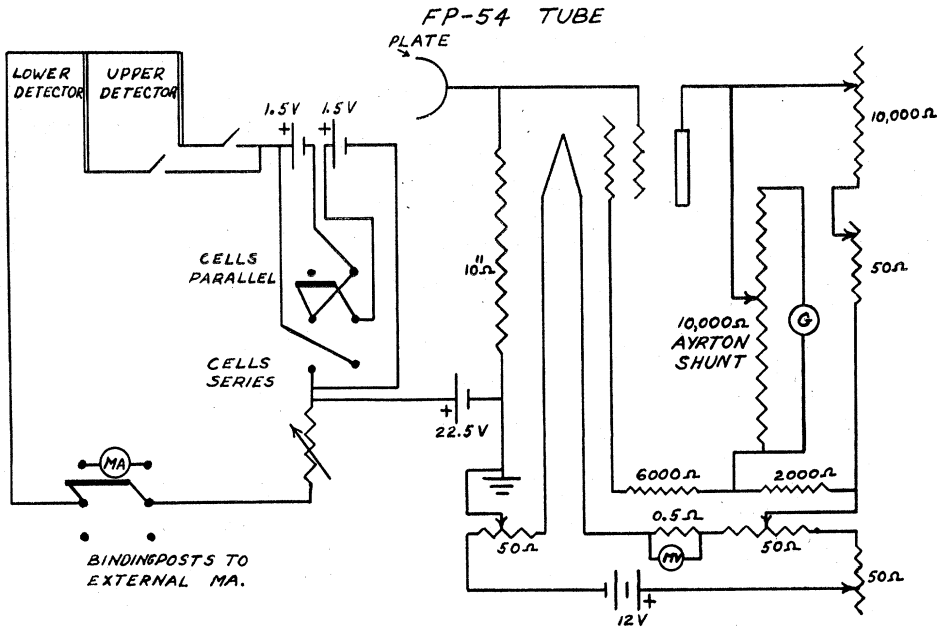

FIG. 7. Detector and amplifier circuit.

[4] L. Malter and N. Marcuvitz, Rev. Sci. Inst. 9, 92 (1938).

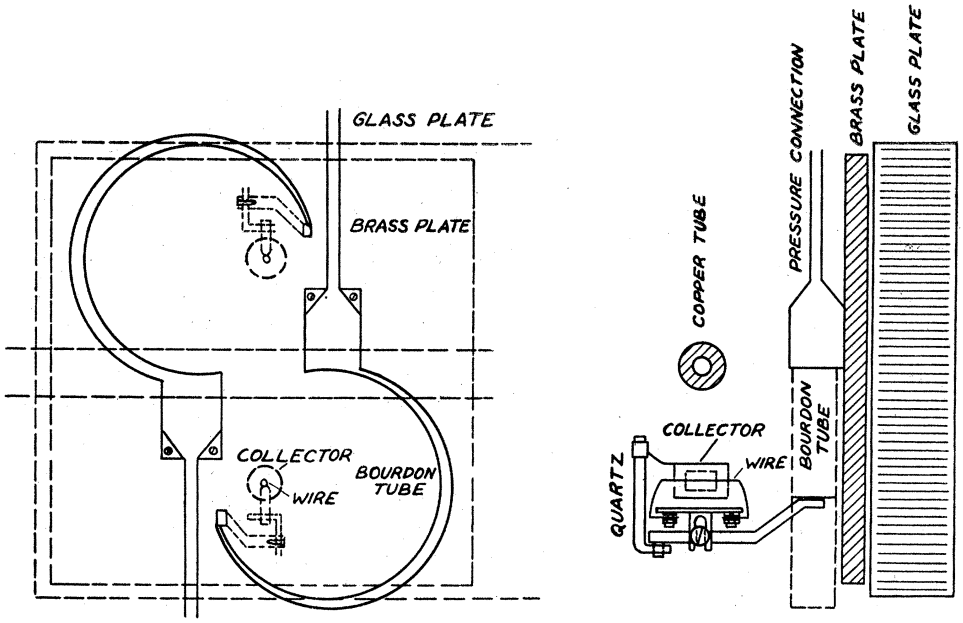

FIG. 8. (a) Side view, (b) end view of detector assembly and mounting.

netic deflection measurements, this copper strip could not be cooled, since it did not reach temperature equilibrium for several hours and changed the position of the beam during this period, apparently by changing the temperature of the slit system unevenly.

(c) Slit and Receiver System

The two foreslits and two collimating slits were each 3 mm long and the distance between the upper and lower slits was 4 cm. The upper slits were 0.022 mm and the lower slits 0.026 mm wide. They consisted of a base plate A and a slit plate B (Fig. 6). Plate A was made of brass and was screwed to a brass angle which in turn was fastened to the vertical glass plate by means of screws inserted through holes in the glass plate. The plates A and B had cut-outs for the copper tube T and plate A had two 3-mm holes underneath the slits, defining the "length" of the beams. The centers of the holes were 4 cm apart and in the same plane as the center of the copper tube. The upper jaw of the lower slit and the lower jaw of the upper slit were formed by the sharp edges of the center piece B. The distances between these edges were 3.994 cm for the foreslits and 3.991 cm for the collimating slits. They were measured with a microscope and a Gaertner micrometer slide to a few thousandths of a mm. The upper jaw of the upper slit and the lower jaw of the lower slit were formed by two small brass pieces screwed to the base plate A. The width of the slits was determined by two methods: First, by a direct measurement with microscope and eyepiece micrometer, and secondly, by measuring the distance of diffraction fringes with sodium light.

(d) Detector

The beam atoms were detected by the Langmuir-Taylor method, in which each Cs atom striking a hot tungsten wire is re-evaporated as an ion and collected on a negatively charged plate. The plate current gives directly the number per second of atoms striking the wire surface, or the intensity of the beam. This current was measured with a FP-54 DC amplifier circuit and a galvanometer (Fig. 7). The FP-54 tube and the grid leak resistor of $10^{11}\Omega$ were mounted inside the vacuum in a side tube (see Fig. 5) and the connecting wire between the collector plates and the grid cap of the FP-54 tube was self-supporting. Each detector consisted of a tungsten wire of 0.02-mm diameter and 15-mm length

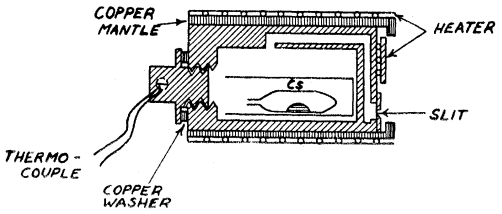

FIG. 9. Oven.

which was mounted between springs. The collector plates consisted of nickel cylinders with windows for letting the beam through and were insulated with quartz rods. The detectors had to be movable in the vertical direction by about 1.5 mm in order to measure the beam intensity as a function of the displacement. A movement which was insensitive against vibration, free from friction, and reliably reproducible was obtained in the following way (Fig. 8). Each detector assembly was attached to the tip of a Bourdon tube from a pressure gauge. This Bourdon tube was connected by a very flexible metal tube, wound in the form of a spiral, to a thin copper tube which was soldered through the vacuum mantle and led to a small glass bulb in which the pressure could be changed from vacuum to about two atmospheres. Calibration with a micrometer showed that a pressure change of 1 cm Hg produced a displacement of the detector wire by almost exactly 1/100 of a mm. The pressure was measured with a mercury manometer and adjusted with a mercury leveling bulb to about 1/10 mm. The position of the detector wire was therefore reproducible with an accuracy of 1/1000 mm, the actual displacements were measured to better than one percent.

(e) Alignment

The parallelism of foreslit, collimating slit, and detector wire was insured by making all of them horizontal. Each slit plate B was provided with a small level bulb L. Both edges of the slit plates were machined to be accurately parallel. Then a surface plate was made horizontal by means of a sensitive level. The slit plate was put on the surface plate so that the edges were horizontal, and finally the level bulbs were so adjusted that the bubble was in the center position. After mounting the slit plates on the vertical glass plate, it was only necessary to bring the bubbles back to this position in order to guarantee parallelism of the slits with an accuracy of one part in 1000. The detector wires were made horizontal with the aid of a microscope with a cross hair, which was adjusted horizontally with respect to the same level.

(f) Conductor Tube

The copper tube conducting the current for the magnetic field was mounted on the horizontal glass plate. It rested on ten "chairs" spaced 20 cm apart, which were fastened to the glass plate by brass clamps. The chairs were lined up by means of stretched tungsten wires. The tube was loosely tied to the chairs with thread so that it could move parallel to its axis to allow for thermal expansion.

The position of the slits with respect to the tube, that is of the two molecular beams, had to be invariable to a few thousands of a mm, but did not have to be known to better than one-tenth of a mm because of the two-beam arrangement. For the slit alignment, the vertical glass plate rested on two supports on the horizontal glass plate, and its position was secured with stops and springs. The slits were then adjusted so that the center lines of the beams were directly above and below and equally distant from the center line of the copper tube. This adjustment was made with the aid of a microscope and adjustable parallel blocks and had to be accurate only to one-tenth of a mm. The whole alignment was made outside the vacuum envelope. The two glass plates could be separated and reassembled without destroying the alignment.

TABLE I. Ratio of measured to calculated intensities of the cesium beam.

T °K	p mm Hg	I_{meas}/I_{calc} for $S\alpha$	$2S\alpha$	$3S\alpha$	Beam
439	1.9×10^{-2}	1.01	0.87		Lower
443	2.27	0.90	0.79	0.66	Lower
449	2.95	0.93	0.75	0.67	Upper
453	3.52	0.91	0.75		Lower
453	3.52	0.93	0.75	0.56	Lower
456	3.99	0.93	0.77	0.58	Lower
457	4.17	0.94	0.70		Lower
476	8.87	0.79	0.51	0.33	Upper
477	9.23	0.86	0.52	0.35	Lower
478	9.59	0.82	0.52	0.39	Upper

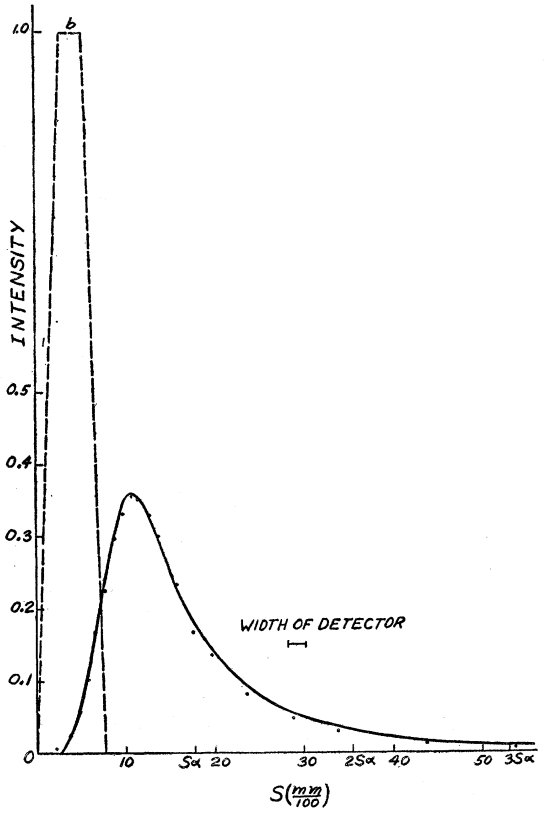

FIG. 10. Gravity deflection of a cesium beam.

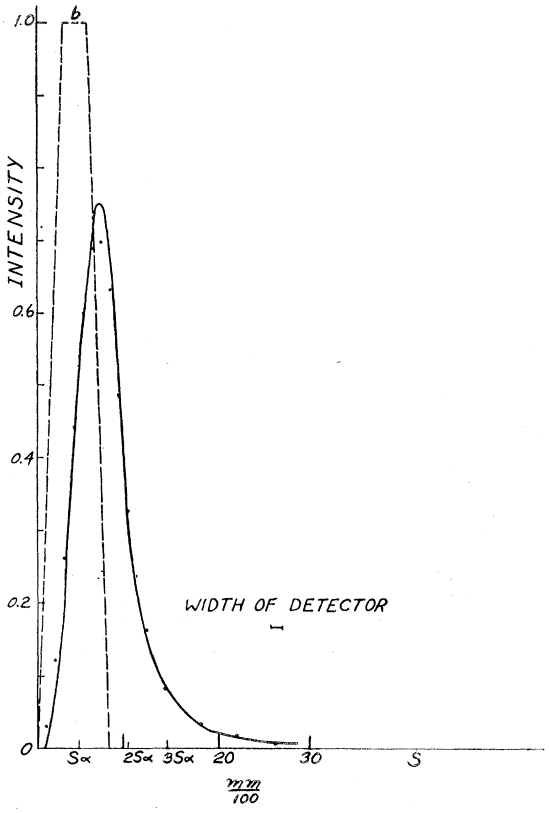

FIG. 11. Gravity deflection of a potassium beam.

(g) Ovens

The ovens were made of monel metal as shown in Fig. 9. The cesium was contained in sealed-off glass capsules holding about ½ g. They were broken inside the ovens in a nitrogen atmosphere, and the plugs were closed immediately thereafter. The ovens were mounted in a cylindrical copper tube heated by a Nichrome coil. Another small heater unit was screwed on to the front of the oven near the slit. A thermocouple was attached to the back of each oven. In a separate test, a second thermocouple was attached to the front of the oven near the slit and the temperature difference between the two thermocouples (about 5°C) was recorded for different temperatures. The ovens were mounted in a cradle which could be moved inside the vacuum in a vertical direction by means of a micrometer screw and a sylphon bellows. The cradle was also equipped with a level in order to make the oven slits horizontal. The size of the slits was 3 mm $\times$0.06 mm. Each oven rested on 3 pins which insured the reproducibility of its position.

(h) Assembly

After the alignment was completed, the horizontal glass plate carrying the copper tube was inserted in the vacuum apparatus. Then, the brass plate separating the beam room from the oven room was attached to the front end of this glass plate. The brass plate was equipped with a sylphon bellows and a bushing for the copper tube and with two channels for the beams. (See Fig. 5.) The cradles for the ovens were also attached to it. Next, the vertical glass plate carrying slits and detectors was slid into the vacuum envelope and moved into its correct position where it was held by two springs. Then the electrical connections between the detectors and the amplifier tube, etc. and the vacuum and pressure connections between the Bourdon tubes and the outside were made. A shutter operated

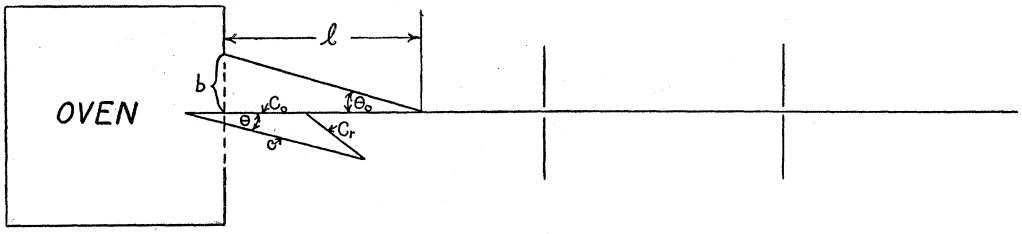

FIG. 12.

by pressure changes in a sylphon bellows was inserted through one of the side arms of the large glass tube near the collimating slit. Then the glass end plates provided with bellows for the copper tube were attached and tightened with Apiezon. Finally, the ovens were charged with Cs, inserted into the cradle through a side hole in the oven room and after the heater and thermocouple connections were made and the hole closed with a glass plate, the apparatus was evacuated as fast as possible.

4. PROCEDURE

After the necessary vacuum of 10^{-7} mm Hg or better was obtained, the oven was heated slowly to about 450°K and allowed to stabilize at a certain temperature in that region. The detector was moved to the position of maximum intensity and the oven was shifted up and down until the oven slit covered the collimating slit completely. The beam intensity was measured during this operation and the resulting "oven displacement curve" served as a check for the slit alignment. With the oven slit in the correct position, the detector was moved in the vertical direction through the beam in steps of one or several hundreds of a millimeter and the gravity deflection curve was obtained. For deflections of more than 0.1 mm from the maximum, it was necessary to move the oven slit slightly downwards with respect to the foreslit in order to keep all the slits, as well as the detector, on the parabolic trajectory of the atoms with the velocity corresponding to the detector position. The intensity of the beam was checked frequently by returning the detector to the maximum position, and a possible shifting of the position of the beam was controlled by checking a few points on the steep, upper side of the gravity deflection curve. The position of the "undeflected" beam was calculated from the measured points on the upper side of the gravity deflection curve, essentially from the position of the point at which the intensity is one-half of the maximum intensity. This distance $S_{\frac{1}{2}}$ from the upper edge of the undeflected beam (Fig. 2) can be calculated for given values of b and S_α. Another method for the determination of the position of the undeflected beam, which was also used occasionally, is to send a compensating current I_0 through the copper conductor tube. The magnetic field produced by this current will bring $\frac{1}{16}$ of the atoms into the position of the undeflected beam, which can thus be measured out directly.

5. RESULTS

Experiments were carried out with potassium and cesium at different oven temperatures. The gravity displacement curves obtained agreed in general with the calculated curves. Figure 10 shows the results for a cesium beam, Fig. 11 for a potassium beam. The curves are calculated, the dots show the measured points. For larger deflections, *viz.*, $S\alpha$, $2S\alpha$, and $3S\alpha$, the measured intensity values were definitely lower than the calculated values. This deficiency increased with increasing deflection (slower atoms) and also with increasing oven temperature (pressure).

TABLE II. Values of $F(\gamma)$.

$\gamma=c/c_0$	$\gamma'=c_0/c$	$F(\gamma)$	$F(\gamma)^*$	$\frac{F(\gamma)-F(\gamma)^*}{F(\gamma)}$
0	∞	0.5000	0.5000	0
0.250	4	0.4080	0.4045	0.87%
0.333	3	0.3804	0.3754	1.31%
0.500	2	0.3322	0.3248	2.24%
0.667	1.5	0.2952	0.2883	2.34%
0.833	1.2	0.2737	0.2703	1.24%
0.909	1.1	0.2711	0.2697	0.51%
1	1	0.2761	0.2761	0
1.1	0.909	0.2982	0.2967	0.51%
1.2	0.833	0.3285	0.3244	1.24%
1.5	0.667	0.4428	0.4324	2.34%
2.0	0.500	0.6645	0.6495	2.24%
3.0	0.333	1.1412	1.1261	1.31%
4.0	0.250	1.6320	1.6179	0.87%

Table 1 shows the deficiency; expressed as the ratio of the measured and calculated intensities I_{meas}/I_{calc} for the positions $S\alpha$, $2S\alpha$, and $3S\alpha$ for different experiments with cesium. $S\alpha$, $2S\alpha$ and $3S\alpha$ are measured from the center of the undeflected beam (not from the upper edge as in Fig. 2).

6. DISCUSSION

The experiments serve as a demonstration that individual atoms follow the laws of free fall in the same way as other pieces of matter. Moreover, they permit a more accurate determination of the velocity distribution in molecular rays than those carried out earlier. The knowledge of this distribution is of great importance for many molecular beam experiments. It has usually been assumed that the Maxwellian distribution law is valid as long as the mean free path of the molecules in the oven is several times as large as the width of the oven slit. These experiments show, however, that there is a considerable deficiency of slow molecule seven at much lower pressures. This deficiency is probably caused by collisions in the immediate vicinity of the oven slit (see appendix).

7. APPENDIX

(a) Calculation of the Distortion of the Velocity Distribution

To estimate the order of magnitude of the distortion in the velocity distribution in the beam through collisions in the neighborhood of the oven slit ("cloud"), we calculate this effect under the following simplifying assumptions: The number of collisions is calculated for rigid spheres on the basis of the classical kinetic theory. The cross section of the spheres is taken from the experimental determination of the mean free path. It is assumed that every collision throws the beam molecule out of the beam. This assumption is certainly justified in our case where a deflection as small as 10^{-5} radian (1/100 mm to 1 m) removes the molecule from the beam. Furthermore, we assume that the density of the molecules inside the oven is uniform and that it and the velocity distribution of the molecules inside the oven are undisturbed up to the oven slit. We also neglect the number of molecules thrown into the beam by collisions outside the oven.* Finally, we consider the case of a circular oven opening and regard the beam, which is defined by the foreslit and the collimating slit, as emerging from the center of the oven opening perpendicular to it and with a cross section which is small compared to the area of the oven opening.

TABLE III. Values of $\bar{F}$ as a function of c_0/α.

$x_0=c_0/\alpha$	$S/S\alpha=x_0^{-2}$	$\bar{F}$
3	1/9	0.3664
2	¼	0.3261
$\sqrt{3}$	⅓	0.3172
$\sqrt{2.5}$	1/2.5	0.3148
$\sqrt{2}$	½	0.3167
1.3	1/1.69	0.3225
1.2	1/1.44	0.3320
1	1	0.3707
$1/\sqrt{2}$	2	0.5232
$1/\sqrt{3}$	3	0.6690
½	4	0.8018
1/2.5	6¼	1.0624
⅓	9	1.3319
¼	16	1.8827

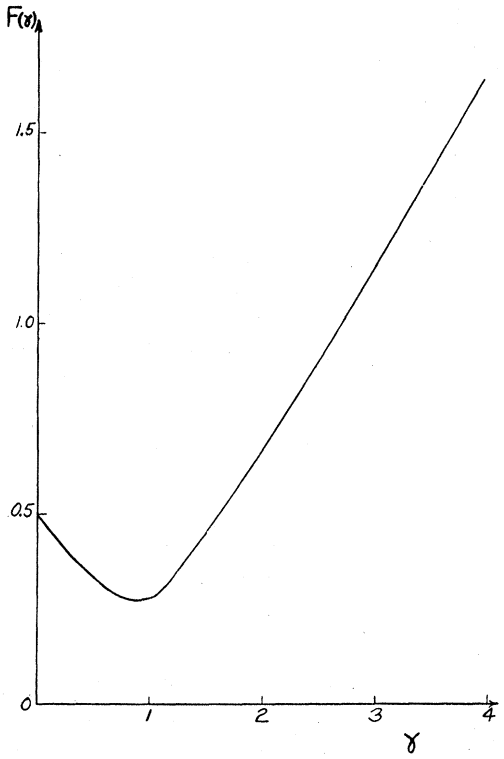

FIG. 13.

Let l be the distance from the oven hole (see Fig. 12), b the radius of the oven hole, n the number of molecules with the velocity c_0 passing the cross section of the beam at l per second, and N the number of molecules per cm³ in the oven with the velocity c. We consider first the case $c>c_0$. The angle between c and c_0 may be θ. Therefore the relative velocity is $c_r=(c^2+c_0^2-2cc_0\cos\theta)^{\frac{1}{2}}$. The number of collisions at l during the time dt is:

$$dn=ndt\cdot\pi\sigma^2N\int_0^{\theta_0}c_r\tfrac{1}{2}\sin\theta d\theta \qquad (4)$$

where $\pi\sigma^2$ is the collision cross section and $\tan\theta_0=b/l$. The decrease of n in traversing the distance $dl=c_0dt$ is therefore

* On the basis of some rough numerical estimates we believe that this assumption should not change the order of magnitude of our results.

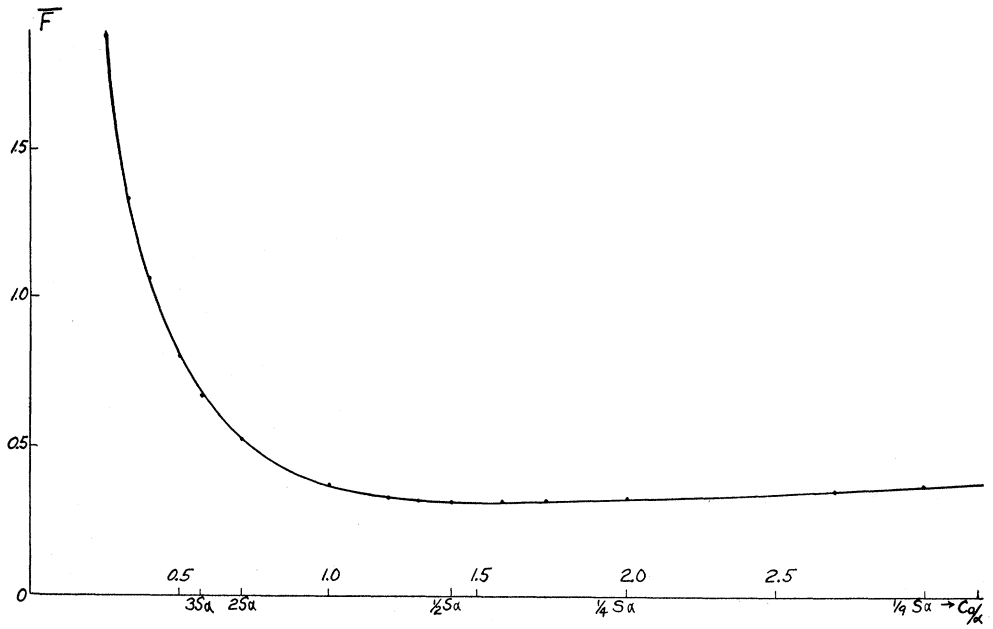

FIG. 14.

given by:

$$-d\ln n/dt=\pi\sigma^2(N/c_0)\int_0^{\theta_0}c_r\tfrac{1}{2}\sin\theta d\theta$$

$$=\pi\sigma^2(N/c_0)\int_{c_{r(0)}}^{c_{r(\theta_0)}}(c_r^2/2cc_0)dc_r$$

$$=[\pi\sigma^2N/(6cc_0)][c_{r(\theta_0)}{}^3-c_{r(0)}{}^3] \quad (5)$$

since $2c_rdc_r=2cc_0d\cos\theta$. Considering θ_0 as function of l ($\tan\theta_0=b/l$), and setting $(1/6cc_0)[c_{r(\theta_0)}{}^3-c_{r(0)}{}^3]=f(l)$ we get

$$n/n_0=\exp[-\pi\sigma^2N\int_0^{\infty}f(l)dl] \quad (6)$$

where n_0 is the number of beam molecules with the velocity c_0 leaving the oven slit per second. We are allowed to integrate to $l=\infty$ because at larger distances ($l\gg b$) the number of collisions is negligible. Substituting again θ_0 for l, setting $dl=(-b/\sin^2\theta_0)d\theta_0$, we obtain finally:

$$n/n_0=\exp[-\pi\sigma^2Nb\int_0^{\pi/2}f(\theta_0)d\theta_0] \quad (6a)$$

with

$$f(\theta_0)=[(c^2+c_0^2-2cc_0\cos\theta_0)^{\frac{3}{2}}-(c-c_0)^3]/6cc_0\sin^2\theta_0.$$

For the case $c_0>c$ we have to replace $(c-c_0)$ by (c_0-c).

The numerical value of the integral

$$\int_0^{\pi/2}f(\theta_0)d\theta_0=F(\gamma)$$

depends only on $\gamma=c/c_0$. For $\gamma>1$, we have:

$$f(\theta_0)=[(\gamma^2+1-2\gamma\cos\theta_0)^{\frac{3}{2}}-(\gamma-1)^3]/6\gamma\sin^2\theta_0 \quad (7)$$

for $\gamma<1$ with $\gamma'=1/\gamma=c_0/c>1$;

$$f(\theta_0)=[(\gamma'^2+1-2\gamma'\cos\theta_0)^{\frac{3}{2}}-(\gamma'-1)^3]/6\gamma'^2\sin^2\theta_0 \quad (7a)$$
$$F(\gamma)=F(\gamma')/\gamma'$$

and for $\gamma=1$,

$$f(\theta_0)=(2-2\cos\theta_0)^{\frac{3}{2}}/6\sin^3\theta_0. \quad (7b)$$

Limiting expressions are:

$\gamma\gg1$ (slow beam molecules) $F(\gamma)=\frac{1}{2}\gamma-\pi/8=\frac{1}{2}\gamma-0.3927$

$\gamma\ll1$ (fast beam molecules) $F(\gamma)=\frac{1}{2}-\pi\gamma/8=\frac{1}{2}-0.3927\gamma$.

Table II and Fig. 13 give some values of $F(\gamma)$.* The weakening of the beam molecules with the velocity c_0 by collisions with oven molecules of the velocity c is therefore given by:

$$n/n_0=\exp(-\pi\sigma^2bNF);\quad F=F(\gamma)=F(c/c_0), \quad (8)$$

where N is the number of oven molecules per cm³ with the velocity c. If we take c as the average velocity and N as the total number N_0 of molecules per cm³ we get already a fair approximation, especially for the case of slow beam molecules. It is, however, not difficult to take into account the velocity distribution of the oven molecules. We simply replace N according to Maxwell by

$$dN=N_0\frac{4}{\sqrt{\pi}}\exp[-c^2/\alpha^2](c^2/\alpha^2)d(c/\alpha).$$

---

* $F(\gamma)$ was calculated by numerical integration and by the formula

$$F(\gamma)=\tfrac{1}{3}(\gamma^2+1)^{\frac{1}{2}}+[(\gamma+1)/6\gamma](\gamma-1)^2\delta\mathbf{F}$$
$$-[(\gamma+1)/6\gamma](\gamma^2+1)\delta\mathbf{E},$$

where

$$\delta\mathbf{F}=\mathbf{F}(\kappa,\pi/2)-\mathbf{F}(\kappa,\pi/4)$$
$$\delta\mathbf{E}=\mathbf{E}(\kappa,\pi/2)-\mathbf{E}(\kappa,\pi/4);$$

and

$$\kappa^2=4\gamma/(\gamma+1)^2.$$

$\mathbf{F}$ and $\mathbf{E}$ are the elliptic integrals of the first and the second kind.

Since the e-factors of the molecules with different velocities multiply, we get:

$$n/n_0=\prod_{c=0}^{c=\infty}\exp[-\pi\sigma^2 bFdN]=\exp\left[-\pi\sigma^2 b\sum_{c=0}^{\infty}FdN\right]$$
$$=\exp[-\pi\sigma^2 bN_0\bar{F}] \quad (9)$$

where

$$\bar{F}=\int_0^\infty F\frac{4}{\sqrt{\pi}}\exp[-c^2/\alpha^2](c^2/\alpha^2)d(c/\alpha).$$

To evaluate $\bar{F}$, the following approximation formula F^* was used for F:

$$\text{For } \gamma>1;\quad F^*=\frac{1}{2}\gamma-\frac{\pi}{8}+\frac{0.16884}{\gamma^2};$$

$$\text{For } \gamma<1;\quad F^*=\frac{1}{2}-\frac{\pi}{8}\gamma+0.16884\gamma^3;$$

$$\gamma=c/c_0.$$

This formula gives the right limiting expressions for small and large values of γ. The factor 0.16884 of the third term is so chosen that the value of F for $\gamma=1$ is correct. In the intervals $0<\gamma<1$ and $1<\gamma<\infty$, the largest deviation from the correct value is less than $2\frac{1}{2}\%$ (compare Table II).

The resulting values of $\bar{F}$ as function of c_0/α are given in Table III and Fig. 14.

(b) Comparison with the Experiments

Our theoretical formula gives the "weakening factor" Φ

$$\Phi=n/n_0\exp[-\pi\sigma^2 N b\bar{F}].$$

The cross section for collisions of Cs atoms with Cs atoms was measured by Foner:*

$$\pi\sigma^2=2.35\times10^{-13}\text{ cm}^2.$$

The number N of Cs atoms per cm³ in the oven is:

$$N=p/kT=2.1\times10^{14}\times p',$$

where p' is the pressure of the Cs vapor in the oven in units of 10^{-2} mm Hg. For the temperature we took an average value $T=460°$K.

The width of the oven slit in our experiments was 5×10^{-3} cm. In our theoretical equation, b is the radius of a circular oven opening. If we set $b=5\times10^{-3}$ cm we should get the right order of magnitude.

With these numerical values we have:

$$\Phi=(n/n_0)\exp[-0.25p'\bar{F}].$$

To compare our experimental results with this formula we have to consider that our I_{meas}/I_{calc} (see Table I) refers to an I_{calc} which sets arbitrarily $\Phi=1$ in the neighborhood of the maximum of the deflection curve. In reality we know only that Φ is approximately constant in this region. This follows directly from our experimental result that in the neighborhood of the maximum the shape of the measured intensity curve agrees with the shape calculated from the modified Maxwellian distribution. It also agrees with our theoretical result for $\bar{F}$ (see Fig. 14). We assume therefore that our I_{calc} corresponds to the minimum value of $\bar{F}=0.32$. If we call the weakening factor in this region Φ_0, we have:

$$\Phi_0=\exp[-0.25p'\times0.32]=\exp[-0.080p'].$$

For the deflection $S\alpha$, the velocity of the beam molecules is α, and from our curve for $\bar{F}$ we take $\bar{F}=0.37$, and finally obtain

$$\Phi_{S\alpha}=\exp[-0.25\times p'\times0.37]=\exp[-0.0925p'].$$

Hence, for the deflection $S\alpha$, our I_{meas}/I_{calc} is

$$I_{meas}/I_{calc}=\Phi_{S\alpha}/\Phi_0=\exp[-0.0125p'].$$

Correspondingly we have for the deflections $2S\alpha$ and $3S\alpha$:

$2S\alpha$: $c_0=\alpha/\sqrt{2}$; $\bar{F}(1/\sqrt{2})=0.52$;
$I_{meas}/I_{calc}=\Phi_{2S\alpha}/\Phi_0=\exp[-0.050p']$

$3S\alpha$: $c_0=\alpha/\sqrt{3}$; $F(1/\sqrt{3})=0.67$;
$I_{meas}/I_{calc}=\Phi_{3S\alpha}/\Phi_0=\exp[-0.0875p']$.

The following Table IV gives some calculated values of I_{meas}/I_{calc}:

TABLE IV. Calculated values of I_{meas}/I_{calc}.

p'	$S\alpha$	$2S\alpha$	$3S\alpha$
2	0.98	0.90	0.84
4	0.95	0.82	0.70
8	0.90	0.67	0.50

A comparison with the measured values in Table I shows agreement in the dependence on pressure and velocity as well as in the order of magnitude of the numerical values. Generally, the measured deviations are somewhat larger than the calculated ones. That may be due to additional collisions, either with foreign gas molecules in the oven room, or directly at the oven slit with foreign gas molecules originating in the oven. The fluctuations in the measurements corroborate this explanation. The discrepancy could also be due, at least in part, to the choice of b. A slightly larger value of b ($7\frac{1}{2}\times10^{-3}$) would give agreement within the limits of error of the measurements. Considering the number of serious simplifications made in the calculations we cannot expect a better agreement than in the order of magnitude.

* S. N. Foner, Thesis, Carnegie Institute of Technology, Pittsburgh, 1945 (see also the following paper).