THE
PHYSICAL REVIEW
A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 71, NO. 1 JANUARY 1, 1947

A Method for Measuring Half-Lives

A. C. GRAVES* AND R. L. WALKER**
University of California, Los Alamos Scientific Laboratory, Santa Fe, New Mexico
(Received August 29, 1946)

A method for measuring half-lives with Geiger counters which is independent of counting losses is described. Foils which have been irradiated for various times are counted after a lapse of time adjusted to make the counting rates of the foils the same. Saturated activities of the foils will be different if an incorrect value of the decay constant has been assumed. Equations are given for obtaining a more correct value of the decay constant. The method is illustrated by data on the 54-minute period of indium.

INTRODUCTION

MEASUREMENT of the half-life of an artificial activity with a counter has usually required a correction for counting losses.[1] This correction is time-consuming and is not easy to make with precision.[2] In the following a method of making such measurements which does not require this correction is reported.

THEORY

If the total number of counts N obtained by counting the emission of a sample for t_2 minutes is measured, the saturated activity A_s (the initial counting rate of a sample irradiated for an infinite time) is given by the formula,

$$A_s=\lambda N \exp(\lambda t_3)/(1-\exp(-\lambda t_1))\times(1-\exp(-\lambda t_2)), \quad (1)$$

where t_1 is the time the sample was irradiated, t_3 is the time between the irradiation and the start of the counting period, $\lambda=\ln 2/T$, and T is the half-life in minutes. If the derivative of A_s with respect to λ is calculated and counting times selected to make this derivative zero, the value of A_s will not depend strongly on a small error in λ. Then if other counting schedules are selected with which the derivative is not zero, the calculated value of A_s will differ from the above value if an incorrect value of λ has been assumed. Differentiation of Eq. (1) yields the formula,

$$dA_s/d\lambda=KA_s, \quad (2)$$

where

$$K=1/\lambda+t_3-\frac{t_1\exp(-\lambda t_1)}{1-\exp(-\lambda t_1)}-\frac{t_2\exp(-\lambda t_2)}{1-\exp(-\lambda t_2)}. \quad (3)$$

If one assumes that A_s is approximately a linear function of λ one can write

$$\lambda=\lambda'+(A_s-A_s')/K'A_s', \quad (4)$$

* Now on leave from University of Texas.
** Now at Cornell University.
[1] See for example: H. Volz, Zeits. f. Physik **93**, 539 (1935); L. I. Schiff, Phys. Rev. **50**, 88 (1936); W. B. Lewis, Proc. Camb. Phil. Soc. **33**, 549 (1937); A. E. Ruark and F. E. Brammer, Phys. Rev. **52**, 322 (1937); L. Alaoglu and N. Smith, Jr., Phys. Rev. **53**, 832 (1938); J. Kurbatov and H. B. Mann, Phys. Rev. **68**, 40 (1945).
[2] H. Lifschutz and O. Duffendack, Phys. Rev. **54**, 714 (1938); J. Cork and J. Lawson, Phys. Rev. **56**, 291 (1939); D. E. Hull and H. Seelig, Phys. Rev. **60**, 553 (1941).

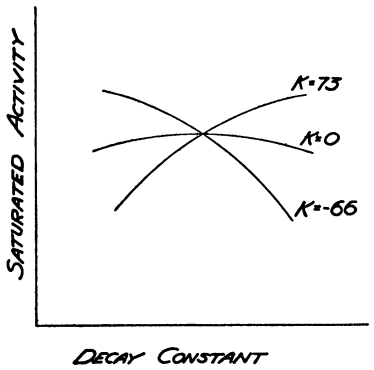

FIG. 1. Schematic representation of calculated saturated activity *vs.* decay constant.

where λ' is the assumed decay constant, A_s is the correct saturated activity, A_s' is the saturated activity calculated from Eq. (1) with the assumed value of decay constant, and λ is a more nearly correct value of the decay constant.

In order for this method to be useful enough counts must be recorded to make statistical errors small, K must have predetermined values, and counting rates with the various schedules must be nearly the same. Since there are three conditions to be fulfilled and three parameters, t_1 and t_2 and t_3 to adjust, appropriate schedules can always be obtained.

Since all foils have the same activity when counted, the counting loss correction will be the same for all foils and will not affect the value of λ as given by Eq. (4). From Eq. (2) it can be seen that the sensitivity of the method (the percent difference in A_s divided by the percent error in λ) is equal to $K\lambda$. Hence, for example, for indium where $\lambda=0.0128$ a value of about 80 for K is required to give a sensitivity of one. If one is attempting to detect an error in λ of one percent with a counting schedule adjusted to make $K=80$, one must measure activities to one percent. Figure 1 shows a schematic representation of A_s as a function of λ for three different counting schedules. Since A_s cannot vary with the counting schedule if the correct value of λ is used in calculating saturated activities, all such curves must intersect at one point as shown. In practice, because of experimental errors, the three curves will not be concurrent but will give three points of intersection and three values of λ. If Eq. (4) is applied to any two such curves and A_s eliminated from the resulting equations one obtains,

$$\lambda=\lambda'+(A_s''-A_s')/(K'A_s'-K''A_s''). \quad (5)$$

A_s'' and A_s' are saturated activities calculated with the assumed decay constant λ' for counting schedules corresponding to K'' and K'.

HALF-LIFE OF INDIUM

A test of this method was made with indium.[3] Data were taken on two Geiger counters with three different counting schedules. The three different schedules and corresponding values of K are given in Table I. K is calculated from Eq. (3) with $\lambda=0.012836$. The times t_1, t_2, and t_3 have the same significance as in Eq. (1). Table II gives the calculated activities corresponding to the three values of K. N_1 and A_{s1} are the number of foils measured with counter 1 and the calculated activities. N_2 and A_{s2} are the corresponding quantities measured on counter 2. A_s gives the average activity. A_{s1} and A_{s2} have been multiplied by appropriate factors to make the sensitivities of the two counters the same. Using the

TABLE I. Calculated values of K for three counting schedules.

K min.	t_1 min.	t_2 min.	t_3 min.
−66	9	10	3
0	20	10	63
+73	52	10	123

TABLE II. Activities calculated for three values of K.

K min.	N_1	A_{s1} counts/min.	N_2	A_{s2} counts/min.	A_s counts/min.
−66	13	34882±31	13	34867± 97	34875±50
0	9	35002±75	7	34973±136	34988±77
73	6	34904±90	7	34948±136	34926±90

TABLE III. Half-life of indium.

K'; K'' min.	min.	Half-life T min.
−66; 0	0.012787±0.000040	54.21±0.17
0; 73	0.012860±0.000047	53.90±0.20
−66; 73	0.012825±0.000021	54.05±0.09
	mean	54.05±0.16

[3] See also; E. Amaldi, O. d'Agostino, and E. Segrè, Ricerca Scient. **5**, 2 (1934); E. Amaldi, O. d'Agostino, E. Fermi, B. Pontecorvo, F. Rasetti, and E. Segrè, Proc. Roy. Soc. **149**, 522 (1935).

three saturated activities in pairs one obtains from Eq. (5) values for the half-life of indium shown in Table III. In this table errors given are probable errors calculated from the root mean square deviations of the data.

CONCLUSION

These measurements were undertaken with the primary purpose of determining whether or not the half-life of indium was sufficiently different from 54 minutes to produce a significant systematic error in experiments being performed with indium as a neutron detector. An error of a percent or more in T was considered sufficient to affect these experiments. Since the authors were not looking for errors of the order of a few tenths of a percent the measurement suffers from two uncertainties. First, while the indium used was known to be quite pure, no check of the purity was made. Hence, it is possible that an impurity with a large cross section may have been present and affected the result. Second, no attempt was made to time operations with the precision necessary to obtain an accuracy of a few tenths of a percent. It is believed that the result should be correct to one percent or better and that the half-life of indium is 54.05 ± 0.5. However, it is felt that the method is useful and it is hoped the measurements reported give an indication of the accuracy which may be obtained by this method of determining the half-life of an artificially induced activity.

This work was carried out under contract between the University of California and the Manhattan District, Corps of Engineers, War Department.

PHYSICAL REVIEW VOLUME 71, NUMBER 1 JANUARY 1, 1947

Photo-Fission in Heavy Elements*

G. C. BALDWIN AND G. S. KLAIBER
Research Laboratory, General Electric Company, Schenectady, New York

(Received October 4, 1946)

Measurements have been made of the yields of photo-fission in uranium and thorium together with a search for photo-fission in other heavy elements, using continuous x-rays from a 100-Mev betatron. Fission was detected in the presence of an intense background of x-rays by a differential ionization chamber and linear amplifier, the substance investigated being coated on an electrode of one chamber. A Victoreen r-thimble, surrounded by $\frac{1}{8}$-inch lead walls, was used to monitor the radiation. Curves were obtained of the number of fissions per roentgen unit for uranium and thorium. These are of similar shape, the uranium curve showing a rapid rise with increasing x-ray energy up to 18 Mev, followed by a gradual decrease as the maximum energy of the x-rays is further increased; the yield of fissions per roentgen at 100 Mev is about half that at 18 Mev. The ratio of uranium and thorium yields is very nearly two at all x-ray energies. No fissions were observed in intense 100-Mev irradiations of Bi, Pb, Tl Au, W, and Sm. Determination of cross sections from the yield curves is complicated by the continuous spectrum of the x-rays which has not been measured experimentally. A rough analysis of the data has been made in which a spectrum is assumed for which the intensity is constant in each unit energy interval and the r-meter efficiency calculated roughly from a simplified picture of the generation of secondaries in the lead walls. The resulting analysis of the yield curves shows that the cross section for photo-fission as a function of quantum energy passes through a maximum and then decreases and is extremely small above 30 Mev. The maximum cross section is of the order of 5×10^{-26} cm^2 for uranium and half that for thorium. In the other elements studied, the cross section must be below 10^{-29} cm^2.

INTRODUCTION

ACCORDING to Bohr and Wheeler's theory of the fission process,[1] fission should be possible for all heavy nuclei which lie well beyond the minimum of the packing fraction curve, provided sufficient excitation is provided to produce the necessary deformation of nuclear fluid which precedes division of the nucleus. Such excitation can be provided by particle

* A preliminary report of this work was presented in an invited paper at the January, 1946 meeting of the American Physical Society.

[1] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).