β-Ray Spectrum of K^{40} and Theory of β-Decay

R. E. Marshak
University of Rochester, Rochester, New York
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In principle, the highly forbidden β-ray transition $K^{40} \rightarrow Ca^{40} + \beta^-$, associated with a spin change of four units, should provide a sensitive test of the possible theories of β-decay. At first it was thought[1] that the known lifetime of K^{40} was best explained by the tensor or pseudo-vector interactions. It was argued that the other three interactions, in particular the vector interaction, led to *minimum* lifetimes comparable to the observed lifetime and could therefore be ruled out on the supposition that the nuclear matrix element is small compared to unity. Implicit in this argument, however, was an estimate of the magnitude of the Dirac α-operator.[2] Grueling[3] has shown that this estimate may be in error by a rather large factor and thereupon pointed out that the vector interaction could also explain the observed lifetime of K^{40}.

Since the lifetime calculations are inconclusive, it is of interest to consider the evidence on the energy spectrum. Recently, Dželepow, Kopjova, and Vorobjov[4] published a measurement of the β-ray spectrum of K^{40}. It seems worth while to compare the predictions of the theory with this admittedly crude measurement. The results are given in Fig. 1, taking $W_0 = 3.64$ (the total energy in units of mc^2) as the upper limit of the spectrum. Curves I–IV represent the ratios of D_i to D where D is the allowed energy distribution function and is plotted as Curve V. The circles represent the ratio of the experimental points to D. The D_i's are defined as follows (cf. Eq. (11) of Greuling's paper[3]):

$$C_{3T} = D_2 |Q_4(\beta\sigma, r)|^2; \; C_{3A} = D_2 |Q_4(\sigma, r)|^2$$

$$C_{4S} = D_1' |Q_4(\beta r, r)|^2; \; C_{4P} = D_1' |Q_4(\beta\gamma_5 r, r)|^2$$

$$C_{4V} = D_1'' |Q_4(r, r)|^2 + D_2 |Q_4(\alpha, r)|^2 + D_3 i[Q_4(\alpha, r) Q_4^*(r, r) - \text{c.c.}]$$

$$C_{4T} = D_1 |Q_4(\beta\sigma \times r, r)|^2 + D_2 |Q_4(\beta\alpha, r)|^2 - D_3 [Q_4(\beta\sigma \times r, r) Q_4^*(\beta\alpha, r) + \text{c.c.}]$$

$$C_{4A} = D_4 |Q_4(\sigma \times r, r)|^2$$

The quantities D_1' and D_1'' are not identical with D_1 but are so closely represented by D_1 over the entire range of W (within a few percent) that Curve I may be used. Curves I–IV are all fitted at $W = 2$, the maximum of D.

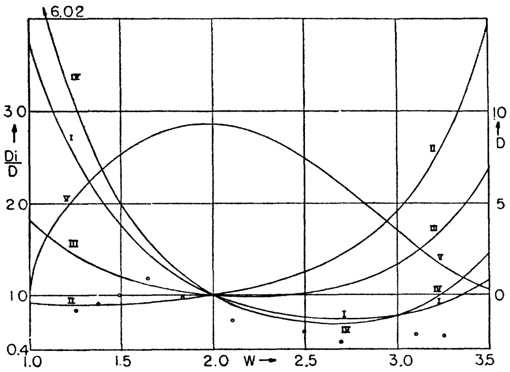

FIG. 1. Comparison of the predictions of theory and experiment for the β-ray spectrum of K^{40}.

The third forbidden transitions C_{3T} and C_{3A} require a parity change. Hence, if the parities of K^{40} and Ca^{40} were different, the energy dependence would be unique and given by Curve II. The disagreement is so striking in the high energy region—where the experimental results are supposedly most accurate[4]—that C_{3T} and C_{3A} may be excluded. If the parities of K^{40} and Ca^{40} are the same, the transition must be fourth forbidden. C_{4S}, C_{4P}, and C_{4A} lead to unique energy spectra—expressed by Curves I and IV—which deviate sharply from the experimental points in the low energy region. C_{4V} and C_{4T} are more flexible because one may choose the two matrix elements arbitrarily, but here too a satisfactory fit is impossible to achieve if one takes seriously the "peaked" character of the measured spectrum. Since the low energy experimental points are poor and the entire spectrum is preliminary, the question of which fourth forbidden transition is correct—if any—is still undecided. The most impressive feature of Fig. 1 is the sensitivity of the energy spectrum to the different interactions. A good measurement of the β-ray spectrum of K^{40}—using enriched K^{40}—would thus be extremely valuable. In particular, if it turned out that the clustering of the experimental points about the allowed spectrum is real (cf. Fig. 1), a satisfactory fit could only be obtained with C_{4V} or C_{4T}.

Figure 1 was prepared for the author by Messrs. B. Carlson and A. Wightman while all three were at the Research Laboratories of the General Electric Company last summer.

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[1] R. E. Marshak, Phys. Rev. **61**, 431 (1942).
[2] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).
[3] E. Grueling, Phys. Rev. **61**, 568 (1942).
[4] B. Dželepow, M. Kopjova, and E. Vorobjov, Phys. Rev. **69**, 538 (1946).

Possibility of "Conditional" Saturation in Nuclei

E. Feenberg and H. Primakoff
Washington University, St. Louis, Missouri
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NUCLEAR forces are commonly supposed to possess a saturation property in the sense that, within a given nucleus, one nucleon is presumed, under all circumstances, to interact effectively only with a limited number of other nucleons.[1] Actually the empirical uniformities in binding energy, spin, isotopic number, size, etc., do not require saturation in such an absolute sense, but only that this property holds for all nuclear states not too far different (in energy, etc.) from those heretofore found.

A physical analogy may be helpful in making the situation clear. Consider, for example, a microcrystal of solid deuterium consisting of 90 atoms. There is a theoretical possibility for this crystal to make a transition to the state of a single Th atom ($Z = 90$, $A = 180$) with release of nearly 10^9 ev of energy. The corresponding transition probability is, of course, immeasurably small, because the individual deuterons in the crystal are separated by distances large