β-Ray Spectrum of K^{40} and Theory of β-Decay

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In principle, the highly forbidden β-ray transition $K^{40} \rightarrow Ca^{40} + \beta^-$, associated with a spin change of four units, should provide a sensitive test of the possible theories of β-decay. At first it was thought[1] that the known lifetime of K^{40} was best explained by the tensor or pseudo-vector interactions. It was argued that the other three interactions, in particular the vector interaction, led to *minimum* lifetimes comparable to the observed lifetime and could therefore be ruled out on the supposition that the nuclear matrix element is small compared to unity. Implicit in this argument, however, was an estimate of the magnitude of the Dirac α-operator.[2] Grueling[3] has shown that this estimate may be in error by a rather large factor and thereupon pointed out that the vector interaction could also explain the observed lifetime of K^{40}.

Since the lifetime calculations are inconclusive, it is of interest to consider the evidence on the energy spectrum. Recently, Dželepow, Kopjova, and Vorobjov[4] published a measurement of the β-ray spectrum of K^{40}. It seems worth while to compare the predictions of the theory with this admittedly crude measurement. The results are given in Fig. 1, taking $W_0 = 3.64$ (the total energy in units of mc^2) as the upper limit of the spectrum. Curves I–IV represent the ratios of D_i to D where D is the allowed energy distribution function and is plotted as Curve V. The circles represent the ratio of the experimental points to D. The D_i's are defined as follows (cf. Eq. (11) of Greuling's paper[3]):

$$C_{3T} = D_2|Q_4(\beta\sigma, r)|^2; \; C_{3A} = D_2|Q_4(\sigma, r)|^2$$

$$C_{4S} = D_1'|Q_4(\beta r, r)|^2; \; C_{4P} = D_1'|Q_4(\beta\gamma_5 r, r)|^2$$

$$C_{4V} = D_1''|Q_4(r, r)|^2 + D_2|Q_4(\alpha, r)|^2 + D_3 i[Q_4(\alpha, r)Q_4^*(r, r) - \text{c.c.}]$$

$$C_{4T} = D_1|Q_4(\beta\sigma\times r, r)|^2 + D_2|Q_4(\beta\alpha, r)|^2 - D_3[Q_4(\beta\sigma\times r, r)Q_4^*(\beta\alpha, r) + \text{c.c.}]$$

$$C_{4A} = D_4|Q_4(\sigma\times r, r)|^2$$

The quantities D_1' and D_1'' are not identical with D_1 but are so closely represented by D_1 over the entire range of W (within a few percent) that Curve I may be used. Curves I–IV are all fitted at $W = 2$, the maximum of D.

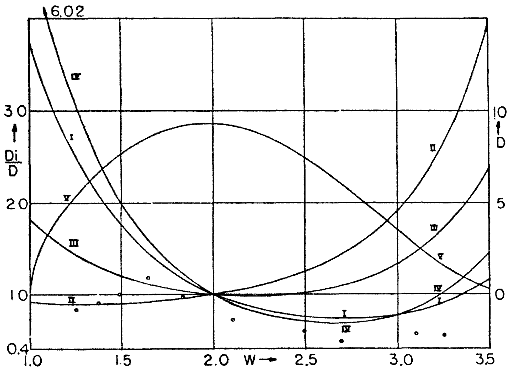

FIG. 1. Comparison of the predictions of theory and experiment for the β-ray spectrum of K^{40}.

The third forbidden transitions C_{3T} and C_{3A} require a parity change. Hence, if the parities of K^{40} and Ca^{40} were different, the energy dependence would be unique and given by Curve II. The disagreement is so striking in the high energy region—where the experimental results are supposedly most accurate[4]—that C_{3T} and C_{3A} may be excluded. If the parities of K^{40} and Ca^{40} are the same, the transition must be fourth forbidden. C_{4S}, C_{4P}, and C_{4A} lead to unique energy spectra—expressed by Curves I and IV—which deviate sharply from the experimental points in the low energy region. C_{4V} and C_{4T} are more flexible because one may choose the two matrix elements arbitrarily, but here too a satisfactory fit is impossible to achieve if one takes seriously the "peaked" character of the measured spectrum. Since the low energy experimental points are poor and the entire spectrum is preliminary, the question of which fourth forbidden transition is correct—if any—is still undecided. The most impressive feature of Fig. 1 is the sensitivity of the energy spectrum to the different interactions. A good measurement of the β-ray spectrum of K^{40}—using enriched K^{40}—would thus be extremely valuable. In particular, if it turned out that the clustering of the experimental points about the allowed spectrum is real (cf. Fig. 1), a satisfactory fit could only be obtained with C_{4V} or C_{4T}.

Figure 1 was prepared for the author by Messrs. B. Carlson and A. Wightman while all three were at the Research Laboratories of the General Electric Company last summer.

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[1] R. E. Marshak, Phys. Rev. **61**, 431 (1942).
[2] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).
[3] E. Grueling, Phys. Rev. **61**, 568 (1942).
[4] B. Dželepow, M. Kopjova, and E. Vorobjov, Phys. Rev. **69**, 538 (1946).

Possibility of "Conditional" Saturation in Nuclei

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Nuclear forces are commonly supposed to possess a saturation property in the sense that, within a given nucleus, one nucleon is presumed, under all circumstances, to interact effectively only with a limited number of other nucleons.[1] Actually the empirical uniformities in binding energy, spin, isotopic number, size, etc., do not require saturation in such an absolute sense, but only that this property holds for all nuclear states not too far different (in energy, etc.) from those heretofore found.

A physical analogy may be helpful in making the situation clear. Consider, for example, a microcrystal of solid deuterium consisting of 90 atoms. There is a theoretical possibility for this crystal to make a transition to the state of a single Th atom ($Z = 90$, $A = 180$) with release of nearly 10^9 ev of energy. The corresponding transition probability is, of course, immeasurably small, because the individual deuterons in the crystal are separated by distances large compared to the range of the nuclear forces and because an electronic transition of very high order is required. However, the essential point for our discussion is that this probability does not entirely vanish.

This example demonstrates the existence of a property which may be called "conditional" saturation. Thus, the system under consideration (the microcrystal) exists in a metastable state of extremely long life with binding energy and volume proportional to the number of a certain type of structural unit (the deuterium molecule) while another state of the same system is possible (the single heavy atom) with far greater binding energy, greatly reduced volume, and without the saturation property with respect to the previously defined unit. It is evident that such "conditional" saturation is not an uncommon occurrence in ordinary matter.

We now wish to suggest that perhaps a similar phenomenon may occur entirely on the nuclear level. Wigner and Eisenbud,[2] Bethe,[3] and particularly Volkoff[4] have discussed the possibility of the nuclear tensor force causing the failure of saturation and have pointed out some of the restrictions to be placed on the tensor force if this failure is not to occur. Let us here suppose, however, that the conditions required for the breakdown of "absolute" saturation in nuclei are actually present in nature, i.e., the ratio of the magnitudes of the tensor and central forces exceeds a certain critical value. Then the observed nuclear saturation (with its attendant properties of small and fairly uniform binding energy per particle, spherical shape, small spin, and isotopic number, etc.) is of the "conditional" type, and some nuclei (presumably the heavier ones) can exist in much lower energy states associated with a (average) total potential energy more or less proportional to the square of the mass number, a total binding energy comparable with the sum of the rest masses of the original nucleons, greatly reduced dimensions, shape highly distorted from the spherical, very large spin, and possibly very large isotopic number. If this supposition is correct, transitions may occur from the comparatively loosely bound "saturation" states to the much more tightly bound "collapsed" states with the emission of (a) γ-radiation or (b) one or more (but not too many) nucleons. The product "collapsed" nucleus will then undergo a certain number of $\beta^{\pm}$-transitions until it attains an equilibrium value of charge.[5] Because of the large spin difference between the original and product nuclei, the transition via (b) is perhaps more likely than (a), particularly in view of the strength of the nucleon-nucleon interaction, the order of magnitude equality of the nuclear force range and the nuclear dimensions, and the comparatively large statistical weight associated with the emission of several nucleons. Finally it should be mentioned that nucleons released in the transitions under consideration would have energies of the order 10^9–10^{10} ev and so could play a role as cosmic-ray primaries.

As a test of these ideas one could search for "collapsed" nuclei constituting extremely rare isotopes of known elements. Of course, the terrestrial concentrations of the collapsed nuclei might be much greater than the value expected from the above described transition probability and the age of the earth, since it is quite possible that some collapsed nuclei were formed at the time of formation of ordinary saturated nuclei. The extremely large binding energy of the collapsed nuclei would probably make it difficult to establish the exact number of neutrons in them; on the other hand, distinguishing features of these nuclei (in addition to their characteristic charge mass ratios) would be their completely non-integral mass values, and a tremendous hyperfine structure.

If the collapsed nuclei exist it is to be expected that their relative concentration is greatest in (terrestrially) rare elements since the final product of the above mentioned series of $\beta^{\pm}$-transitions is just as likely to have a value of Z associated with a rare as with a common element. The possibility of collapsed transuranic nuclei should also be considered.

Emphasis has here been placed on the possible role of the tensor force in breaking down "absolute" saturation, but other causes of "conditional" saturation can be found within the bounds of current field theories of nuclear forces; for example, in the existence of short range, attractive many body forces[6] which become fully effective only in collapsed nuclei.

[1] G. Breit and E. Wigner, Phys. Rev. **53**, 998 (1938).

[2] E. Wigner and L. Eisenbud, Phys. Rev. **56**, 214 (1939).

[3] H. A. Bethe, Phys. Rev. **57**, 260 (1940).

[4] G. M. Volkoff, Phys. Rev. **62**, 126, 134 (1942).

[5] The equilibrium value of charge appropriate to a collapsed nucleus possibility corresponds to a much greater neutron excess than is found in ordinary conditionally saturated nuclei, so that the $\beta^{\pm}$-transitions are largely β^{+}. This conclusion is a consequence of the fact that in collapsed nuclei an exchange type tensor force will have a potential energy which decreases (algebraically) more rapidly with increasing isotopic number than the kinetic energy increases; the Coulomb energy then favors a large neutron excess.

H. Primakoff and T. Holstein, Phys. Rev. **55**, 1218 (1939).

On the Dynamics of the RbI Crystal

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THE alkali halogenide ionic crystals can be dealt with by the statistical atomic model without the aid of empirical or semi-empirical parameters, especially in the case of ions containing many electrons. Jensen has made calculations on the whole alkali halogenide series by a combination of the Thomas-Fermi-Dirac and the Fermi-Amaldi models, called in the following the Jensen model.[1] Jensen took into account the van der Waals correction also, using the approximating formula $-1330\times10^{-60}/\delta^6$ erg of Mayer.[2] His results can be improved essentially by using the Gombás model, which takes into account the correlation effect.[3] Jensen's calculations must be extended at two points. First, we must introduce the electron densities of Gombás instead of those of Jensen, and secondly we must take into account the energy term resulting from the correlation. This last can be done by taking $\kappa_a'=1.13\kappa_a$ for Jensen's κ_a in the exchange energy term, since the correlation gives a term proportional to the exchange energy. I have performed the calculations for the face-centered lattice of the RbI crystal.