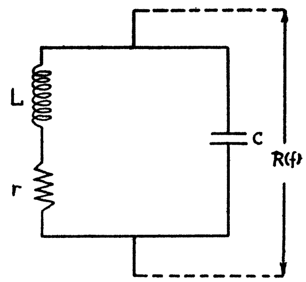

FIG. 1. Equivalent circuit of a crystal.

formula, however, is not valid except for frequencies sufficiently high to exclude the incidence of "flicker effect."[3,4] Indeed, below a few hundred cycles/sec., flicker effect becomes entirely dominant and the fluctuations may rise to the order of 10^2–10^3 times the theoretical value of "true" shot noise as predicted by the formula above. For this reason all modern fluctuations experiments, apart from those designed expressly to examine "flicker" effect, are carried out at frequencies of the order of 10^6 c/sec. or higher to exclude the relatively slow, gross variations characteristic of "flicker" phenomena.

Since the high resistance required must now be *dynamic* in character the question arises whether such a resistance can be provided and utilized. Fortunately, the high conductivity of pure metals near absolute zero favors the provision of a coil of low ohmic resistance and hence a tuned circuit of very high dynamic impedance. Indeed, one might visualize the use of a super-conductor for this purpose.

The next difficulty that arises is the effect of the dynamic input resistance of the valve amplifier itself resulting from reaction. This can be estimated from the expression for the input admittance to the valve

$$Y_i = i\omega\{C_{gc}+C_{ga}(1-m_a)\}, \tag{1}$$

where m_a is the complex amplification of the amplifier stage. It appears very unlikely that it would be possible to maintain the real part of Y_i (i.e., the dynamic input conductance) sufficiently small; (in this connection, however, a cathode follower might prove feasible, the input admittance being very low). In addition, it would of course be essential to ensure that the over-all ohmic leakage between grid and earth was also large compared with the dynamic resistance of the tuned circuit.

A rather more fundamental criticism must be directed against the second part of Lawson and Long's letter. They suggest that effectively increased Brownian voltage might be derived from the piezoelectric fluctuations of a quartz crystal arising primarily from the random energy of mechanical vibration. They make a detailed numerical estimate of the voltage generated, namely

$$\langle V^2\rangle_{\mathrm{Av}} = 1.4\times 10^{-12}\cdot T, \tag{2}$$

(reference 1, Eq. (5)) and suggest that the internal resistance of the quartz is only a few thousand ohms, so that an input resistance of 10^5 ohms is adequate.

It must be pointed out that for *any* system in complete thermal equilibrium, *irrespective of the particular electrical or electromechanical mechanisms involved or of the number of causes of the irregular movement*, the Brownian voltage generated between any two terminals is given by Nyquist's formula

$$\langle V^2\rangle_{\mathrm{Av}} = 4R(f)kT\Delta f, \tag{3}$$

(reference 1, Eq. (1)) where $R(f)$ is the resistance (at frequency f) measured between the two terminals concerned. The classical example of the galvanometer has been analyzed in detail by Ornstein,[5] and Professor M. H. L. Pryce kindly informs us that similar conclusions can be drawn for the more general case of a linear inter-connected system of n degrees of freedom.

It must, therefore, be possible to calculate Eq. (2) directly from the crystal impedance as viewed from its (electrical) terminals. It is evident, however, that this step cannot be reconciled with Eq. (3), (bearing in mind that the bandwidth ($\Delta f \doteq f/Q$) is only of the order of one cycle/sec.) unless the crystal impedance is in fact very high. The paradox is resolved by considering the equivalent electrical circuit of the crystal; to a sufficient approximation this may be represented as in Fig. 1.

It is true that r is only of the order of $10^3\Omega$, but at resonance $R(f)=Q^2\cdot r$ and Q is in the order of 10^3–10^4, so immediately showing that the relevant impedance is exceedingly high. Thus taking $R(f)\doteq 2.5\times 10^{10}\Omega$ (clearly a reasonable value on the quoted figures) and $\Delta f \sim 1$ c/sec., we immediately arrive at (our) Eq. (2) from the fundamental formula (3) without further ado.

It therefore appears that, contrary to Lawson and Long's conclusion, the electrical fluctuations of a crystal, despite its mechanical Brownian movement, do not differ fundamentally from those of any other electrical network and can offer no inherent superiority as a low temperature thermometer. The only possible advantage is of a practical nature, since the crystal may more readily provide a very high electrical dynamic resistance, in the use of which, however, all the difficulties mentioned above would still have to be overcome.

Similar conclusions must of course apply to any other physical system of coupled elements in thermal equilibrium.

[1] A. W. Lawson and E. A. Long, Phys. Rev. **70**, 220 (1946).
[2] D. O. North, *et al.* R. C. A. Rev. **4**, 471 (1940).
[3] J. B. Johnson, Phys. Rev. **26**, 71 (1925).
[4] W. Schottky, Phys. Rev. **28**, 74 (1926).
[5] L. S. Ornstein, *et al.* Proc. Roy. Soc. **A115**, 391 (1927).

Further Remarks on the Possible Use of Brownian Motion in Low Temperature Thermometry

A. W. LAWSON AND EARL A. LONG
Institute for the Study of Metals, University of Chicago, Chicago, Illinois
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IN a recent letter, Brown and MacDonald[1] have reviewed our discussion concerning the possibility of using Brownian motion for low temperature thermometry and have raised a number of objections concerning which we would like to make some additional remarks.

Brown and MacDonald point out that the estimates made by us of the lowest temperatures measurable with the thermal noise generated by a large resistance at the control grid of an electrometer tube neglect the flicker effect. For this reason, they believe that such a device would only be practical after a "drastic modification." The effect mentioned was omitted from our previous discussion because it appears likely that with an electrometer tube such as the Western Electric D-96475 operating at the necessarily low frequency of 5 cycles/second, the flicker noise can be substantially cancelled out by utilizing the screen grid, as in the familiar balanced circuit of Dubridge and Brown.[2]

The belief entertained by Brown and MacDonald that we have "apparently overlooked" a fundamental principle, namely the universal applicability of Nyquist's theorem, we can only attribute to the occurrence of an erroneous statement in our previous note, namely that an input resistance of 10^5 ohms is adequate for the successful operation of a piezoelectric thermometer. The statement in question should read that an input resistance of 10^{13} ohms should be adequate for such operation. Except for this error, we believe that there is no basis for drawing the conclusion of Brown and MacDonald that we claim any fundamental difference in the behavior of a piezoelectric thermometer from that of a resistance thermometer; the advantage is that of practicability. Although it is certainly more expeditious to make numerical estimates by applying Nyquist's theorem to experimentally determined dynamic resistances, it appeared worth while to derive our formula (4) expressing the noise voltage directly in terms of crystal constants, since this expression not only permits estimates of optimum crystal dimensions, but also reveals the inherent frequency limitation in such devices in a manner not immediately apparent from a consideration of Nyquist's formula itself.

The fact that the advantages of a piezoelectric thermometer are merely of a practical nature is not unimportant. Thus the piezoelectric thermometer avoids the following intrinsic difficulties associated with the first device described: (1) the long measurement time associated with low frequencies, owing to limitations imposed on band width by input capacities, (2) the necessity of minimizing microphonics, low frequency power supply variations, and flicker effects, and (3) the temperature-independence of the resistance, since the equivalent dynamic resistance of the piezoelectric crystal may be controlled by temperature-independent mechanical loading.

The principal disadvantage associated with a piezoelectric thermometer is the increased difficulty of obtaining a sufficiently high input resistance at the higher frequencies involved. However, recent developments in miniature tubes with small electron transit times (e.g., the RCA 6AK5) appear to offer some promise of meeting the required specifications.

In conclusion, it appears worth while to mention that Professor W. W. Hansen[3] has kindly pointed out to us that it may be advantageous to consider the use of a modification of the ingenious method recently described by Dicke[4] for the determination of temperature by the measurement of microwave radiation. A preliminary study by us of the design factors involved indicates that it should be possible to measure temperatures of the order of 0.1°K to 0.01°K using ordinary vacuum tubes with a noise generating resistance as low as 10^5 ohms.

1 James B. Brown and D. K. C. MacDonald, Phys. Rev. **70**, 976 (1946).
2 L. A. Dubridge and H. Brown, Rev. Sci. Inst. **4**, 532 (1933).
3 Private communication.
4 R. H. Dicke, Rev. Sci. Inst. **17**, 268 (1946).

The Angular Distribution of Gamma-Rays in Na^{24}, Co^{60}, Y^{88}*

WILFRED M. GOOD**
Massachusetts Institute of Technology, Cambridge, Massachusetts
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WHEN two gamma-rays are emitted successively in the process of nuclear de-excitation, a distribution in angle is expected for the second gamma-ray relative to the first.[1,2] Coincidence studies to observe this angular correlation have been largely unsuccessful in showing any distribution apart from isotropy.[3,4]

The radioactive nuclei Na^{24}, Co^{60}, Y^{88} are particularly suited for deciding whether or not the above effect can be observed for the following reasons: (i) each of the radioactive transformations is followed by two successive gamma-ray transitions; (ii) the gamma-ray transitions occur, respectively, in the nuclei Mg^{24}, Ni^{60}, Sr^{88}. These nuclei are of the even-even type, and hence possess in their ground state no angular momentum. Zero to zero transitions are forbidden for gamma-ray transitions, hence the intermediate state cannot have an angular momentum zero. But theory shows that in the cases of these nuclei, only an intermediate state with an angular momentum of zero can give rise to an isotropic distribution. It follows, therefore, that in the cases of these nuclei, a distribution apart from isotropy is known to exist. It can further be said that as far as theory goes, the minimum asymmetry to be expected from all permissible angular momentum and parity assignments is seven percent; (iii) the respective half-lives of 14.8 hr., 5 yr., and 105 days are sufficiently long so that complications do not arise due to short half-lives.

The expected distribution is a power series in $\cos^2\theta$. The instrument used in the investigation subtended a solid angle of 0.3 steradian at the source. Analysis showed that an observed distribution would differ, because of finite solid angles, from the real distribution by only a couple of percent. Because of the low coincidence rates, and because the distribution has its extremes at the angles 90° and 180°, counting was restricted to these angles. The results are tabulated in Table I.

TABLE I. Observations of γ-ray emission at 90° and 180°.

Nucleus	180°	90°	Difference	Net rate at 90°	Percent asymmetry
Y^{88}	1.12±0.02 c.p.m.	1.08±0.02 c.p.m.	0.04±0.03 c.p.m.	0.78 c.p.m.	5.1±3.9
Co^{60}	1.67±0.04	1.63±0.04	0.04±0.06	0.56	7.1±10.
Na^{24}	3671±60*	3613±60*	58±84*	1800*	3.2±4.7

* Total coincidence