

### Binding Energy of Polyelectrons

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THE question as to the existence of groups of electrons and positrons having temporary stability has recently been raised by J. A. Wheeler,<sup>1</sup> who shows that clusters of the form ++ and ++- are stable with respect to decay into their elementary constituents. With regard to more complex structures no definite judgment could be reached.

Wheeler found, as a first approximation, the upper limit of -9.93 ev for the ground state energy of the hypothetical "electromeson" or "polyelectron" composed of two electrons and two positrons. A binding energy of 9.93 ev is, however, about 3.6 ev below the energy  $me^4/2\hbar^2$  necessary to prevent disintegration into two systems each of which contains one electron and one positron.

The variational function in Wheeler's calculation is, aside from a difference in notation and in the values of the parameters, the ground state function employed by H. Margenau and W. A. Tyrrell in their variational calculations of the binding energy of the alpha-particle,<sup>2</sup> in which orthogonal Hermite functions were used to approximate the state of the particle.

The symmetry considerations in the alpha-particle problem may be transferred unaltered to the case of the four-component polyelectron in question.

If we choose the notation of Margenau and Tyrrell the Hamiltonian of the polyelectron may be written

$$H = -\frac{\hbar^2}{2m}(2\nabla_1^2 + \nabla_2^2 + \nabla_3^2) + 2^{-1}e^2 \left( \frac{1}{\rho_2} + \frac{1}{\rho_3} - \sum \frac{2}{|\boldsymbol{\rho}_1 \pm \boldsymbol{\rho}_2 \pm \boldsymbol{\rho}_3|} \right).$$

The summation extends over four different terms indicated by the double signs. In this form the Hamiltonian does not include the center of mass term. The relative coordinates are the scalar components along the Cartesian axes of the vectors

$$\boldsymbol{\rho}_1 = (\frac{1}{2})^{\frac{1}{2}}(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_a - \mathbf{r}_b), \quad \boldsymbol{\rho}_2 = (\frac{1}{2})^{\frac{1}{2}}(\mathbf{r}_1 - \mathbf{r}_2), \quad \boldsymbol{\rho}_3 = (\frac{1}{2})^{\frac{1}{2}}(\mathbf{r}_a - \mathbf{r}_b).$$

The ground state harmonic oscillator function which yields the energy value presented by Wheeler then takes the form

$$\psi_0 = \text{const.} \exp -\frac{1}{2}[p\rho_1^2 + q(\rho_2^2 + \rho_3^2)], \quad p = 1.134q.$$

It follows from the form of the Hamiltonian above that functions of odd degree of excitation do not combine with the ground-state function. Nor do the two doubly excited functions combine when  $p$  and  $q$  are given the values which minimize  $H_{00}$ .

The total second-order perturbation energy resulting from the eight quadruply excited functions comes out to be roughly 0.9 ev of which 0.65 ev originates from the third and fourth functions in the enumeration of reference 2.

The three first sextuply excited functions add altogether about 0.1 ev.

While there is no clear indication of convergence at this stage, the small magnitude of the contributions from these

orthogonal functions suggests with reasonable assurance that this structure is not stable against disintegration into two bielectrons.

This conclusion is perhaps somewhat strengthened by the following consideration. If the binding energy of the bielectron (+-) is computed variationally with the use of the analog of our  $\psi_0$  there results a binding energy of 5.75 ev, i.e., 85 percent of the true value of 6.77 ev. The calculations on light nuclei show that  $\psi_0$  represents a better approximation to the lowest state of the four-particle system than to that of the two-particle system. If, then, we assume the value of 9.93 ev to represent 85 percent of the true energy of the cluster, we arrive at a limit of 11.7 ev, which is still 1.8 ev below stability.

For these reasons the preceding perturbation calculations were not extended by more refined variational methods.

It might seem that while Hermite functions are adapted to the alpha-particle problem with its short range forces, the Coulomb potentials in polyelectrons require more hydrogenlike functions. To investigate this point, we choose a new set of coordinates, replacing  $\boldsymbol{\rho}_1$ ,  $\boldsymbol{\rho}_2$  and  $\boldsymbol{\rho}_3$  by

$$\mathbf{s} = (\frac{1}{2})^{\frac{1}{2}}(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_a - \mathbf{r}_b), \quad \mathbf{t} = (\frac{1}{2})^{\frac{1}{2}}(\mathbf{r}_1 - \mathbf{r}_2 - \mathbf{r}_a + \mathbf{r}_b) \\ \text{and} \quad \mathbf{u} = (\frac{1}{2})^{\frac{1}{2}}(\mathbf{r}_1 - \mathbf{r}_2 + \mathbf{r}_a - \mathbf{r}_b).$$

The symmetry in identical particles leads to functions symmetric and of even degree in the  $t$  and  $u$  coordinates. When we take this fact into account we may write the proper Hamiltonian for the calculation of matrix elements in the form

$$H = -\frac{\hbar^2}{m}[\Delta_s^2 + 2\Delta_t^2] + 2^{\frac{1}{2}}e^2 \left[ \frac{1}{|\mathbf{u} - \mathbf{t}|} - \frac{2}{|\mathbf{t} - \mathbf{s}|} \right].$$

Let us consider functions of the kind

$$\psi_i = \text{const.} (s \cdot t \cdot u)^i \exp -\frac{1}{2}[\alpha s + \beta(t + u)],$$

and take for  $\alpha$  and  $\beta$  the values that minimize the respective  $H_{ii}$ .  $i=0$  yields in this way a binding energy of 9.5 ev, that is, less than the ground-state oscillator function.  $i=\frac{1}{2}$  however, yields 10.0 ev for  $\alpha=1.5505\beta$ ,  $\beta=0.81888 \cdot me^2/\hbar^2$ . This energy value is 73.8 percent of the minimum binding energy required for stability. With five functions in linear combination,  $i=1/2, 3/2, \dots, 9/2$ , and the same  $\alpha$  and  $\beta$  which minimize  $H_{11}$ , this percentage is increased to about 75 percent.

This is in fact below the value of 10.3 ev or 76 percent which we may get variationally by combining the ground-state harmonic oscillator function with the rather effective fourth quadruply excited function. Changes in the values of the parameters have been found to have very little effect on this result for the binding energy.

Although the evidence here presented against the stability of the polyelectron composed of two electrons and two positrons is not conclusive in a strict mathematical sense, it counsels against the assumption that clusters of this (or even of higher) complexity can be formed.

<sup>1</sup>J. A. Wheeler, Ann. New York Acad. Sci. to be published. The bielectron has also been discussed by A. E. Ruark, cf. Phys. Rev. **68**, 278 (1945).

<sup>2</sup>H. Margenau and W. A. Tyrrell, Jr., Phys. Rev. **54**, 422 (1938).