$\alpha=5/3$ in the combination of the Weber and Riemann formulas in the hands of Tisserand[2] and Lévy,[3] $W=\alpha W_R+(1-\alpha)W_W$. The Ritz[4] theory and O'Rahilly's[5] preference $\lambda=3$ $(=4A-1$, corresponding to $A=1$ in the reciprocal force) failed to account for Mercury's advance because of the lack of certain acceleration terms. The reciprocal energy formula,[1] however, with the preferred value $A=1$ (and hence with $B=-\frac{1}{2}$), namely

$$W=(ee'/r)(1+u^2/c^2-(\mathbf{u}\cdot\mathbf{r})^2/2c^2r^2+\cdots), \qquad (1)$$

when applied to gravitation, predicts closely the advance of the perihelion of mercury. Using the value 14.4″ per century given by Tisserand for the Weber formula[2] and twice that for the Riemann energy,[3] and setting $\alpha=2$, which reduces Lévy's expression to Eq. (1), one finds for the advance of the perihelion of Mercury the value 43.2″ per century. The observational advance[6] is listed as 43.5″ per century, while the value predicted by the partially relativistic theory of Einstein is given as 42.9″ per century. A recheck using more recent values of the measured quantities gives the advance of the perihelion of Mercury as 43.0″ per century, both on the standard relativity basis and on the basis of the reciprocal force formula. The observed advance of the perihelion of the planets thus does not distinguish between Einstein relativity and an electrical theory conforming the Newtonian relativity.

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Energy-Angle Distribution of Betatron Target Radiation†

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An approximate expression for the energy-angle distribution of the bremsstrahlung produced by fast electrons in a thin target has been given by Sommerfeld.[1] This is obtained by integration of the Bethe-Heitler formula over the angular coordinates of the outgoing electron, and is valid with neglect of screening when the energy of the incident electron is large in comparison with its rest energy. It is not applicable when the target is of the thickness used in betatrons, since the electron beam is spread out by multiple scattering in the target.

According to Williams[2] the normalized distribution in angle θ per unit solid angle of electrons of energy E after penetrating a thickness t of target containing N atoms of nuclear charge Ze per unit volume is:

$$(1/2\pi\theta_0^2)\exp(-\theta^2/2\theta_0^2), \quad \theta_0=(9.2Ze^2/E)(Nt)^{\frac{1}{2}}\equiv(\beta t)^{\frac{1}{2}};$$

the numerical coefficient 9.2 is nearly constant for heavy metal targets such as tungsten having thicknesses of the order of a tenth millimeter. The angular spread of the x-rays due to just the radiation process is of order mc^2/E. For tungsten, θ_0 is large compared to mc^2/E if $t\gg10^{-4}$ cm; this is usually the case.

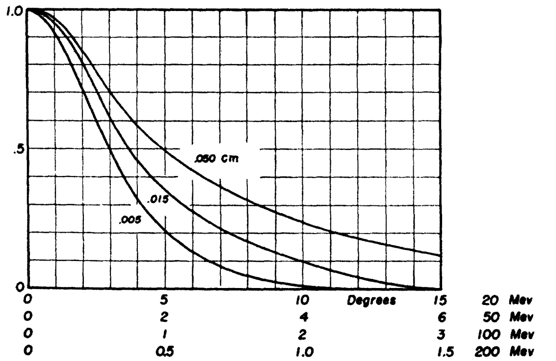

FIG. 1. Ratio of radiation intensity at angle θ to the intensity at $\theta=0$ for three thicknesses of tungsten target.

Since electrons are radiating at all values of t from zero to the total target thickness x, Williams' formula must be integrated over t to give the effective electron angular distribution per unit solid angle:

$$(1/2\pi\beta)[-Ei(-\theta^2/2\beta x)].$$

This assumes single traversal of the target, and is valid so long as the target is thin enough so that there is not excessive straggling of the electrons; for tungsten this corresponds to $x\gtrsim0.05$ cm. The energy-angle distribution of the x-rays is now obtained by combining this electron distribution with Sommerfeld's formula. For angles somewhat larger than mc^2/E this means simply that the angular distribution of the x-rays is the same as that of the electrons, and the energy spectrum is that obtained by integrating the Bethe-Heitler formula over the directions of both the outgoing electron and the quantum.[3] For small angles, however, the divergence in the electron distribution makes it necessary to carry through the combination in detail. This is readily done for $\theta=0$; in the absence of screening, the energy distribution is still the integrated spectrum, and this is a good approximation when screening is included.

The result is that to good approximation the energy distribution at all angles is that usually associated with the total radiation. The ratio of intensity at an angle θ somewhat larger than mc^2/E to the intensity at $\theta=0$ is per unit solid angle:

$$[-Ei(-\theta^2/2\beta x)]/[\ln(2\beta xE^2/m^2c^4)-0.5772].$$

Since β is proportional to $1/E^2$, the denominator is independent of E and curves for different energies differ only by a scale factor that is inversely proportional to E. Curves for three thicknesses of tungsten target are shown in Fig. 1; they are in good agreement with measurements reported by D. W. Kerst (private communication). Thanks are due T/5 C. B. Gass for help with the numerical computations.

* Now returned to the University of Pennsylvania from leave of absence.
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