nances were not observed earlier, since comparable resolution has not previously been used. Even the resolution used here is not sufficient to make it possible to determine the strengths of the resonances. However, minimum values may be ascribed to the cross section at resonance by taking into account the resolution. These values are about $220\times10^{-24}$ and $67\times10^{-24}$ cm$^2$/atom for the 3.7 and 9 ev levels, respectively.

**ACKNOWLEDGMENT**

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# The Density Spectrum of the Extensive Cosmic-Ray Showers of the Air

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The paper describes measurements performed at 2200 meters above sea level for the purpose of studying the density of particles in the extensive air showers. From the results obtained, one finds that the dependence of the number of showers on the density can be expressed over a wide range by an exponential low.

## 1

The so-called "extensive air showers" are produced by cosmic-ray particles which arrive at the top of the atmosphere with very high energies ($10^{13}$–$10^{16}$ ev) and give rise to a large number of electrons and photons through a process of multiplication extending to the bottom of the atmosphere. The large number of particles observed indicates a large subdivision of the primary energy, while the length of the path between two successive elementary processes produces a considerable spread of the shower particles.

The problem of the spread of air shower has been fully investigated by Auger and co-workers with experiments carried out at different heights above sea level. These experiments led to the establishment of the well-known "decoherence curve" which relates the number of showers with their extension.[1]

The same experimenters,[2] as well as others, have also investigated the dependence of the number of air showers on their density, i.e., on the number of particles per m$^2$. The results, however, are uncertain and contradictory. Therefore new experiments were undertaken in order to contribute to the solution of this problem.

## 2

The experimental determination of the particle density in an air shower detected with a set of G-M counters is based upon a classical probability formula which was first applied to this specific problem by Auger, and was later used by other authors. This formula reduces the computation of the density to the determination of the relative number of coincidences, $C_n$ and $C_{n-1}$, recorded in equal times with $n$ and $n-1$ counter, respectively. The counters are arranged in a horizontal plane, over an area small compared with the spread of the shower.

Let $p(S, \Delta)$ be the probability that a counter of area $S$ is *not* struck by a shower of average density $\Delta$. If the area is increased by $dS$, this probability becomes

$$p(S+dS, \Delta)=p(S, \Delta)(1-\Delta\cdot dS),$$

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[1] P. Auger, R. Maze, P. Ehrenfest, and A. Freon, J. de phys. et rad. **10**, 39 (1939).
[2] H. Geiger, Abhandl. Preuss. Akad. Wiss. **10** (1941). J. Daudin, University of Paris, Thesis, 1942. J. Clay, Physica **9**, 897 (1942).

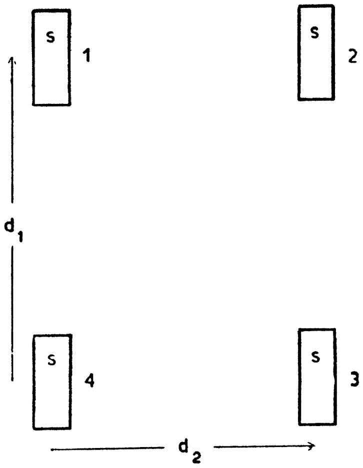


FIG. 1. Counter arrangement in the first series of measurements.

from which it follows:

$$p=e^{-S\Delta}.$$

Therefore, the probability for a counter to be struck is

$$P=1-p=1-e^{S\Delta},$$

and the probability per $n$ counter, all with the same area $S$, to be struck is

$$P^n=(1-e^{-S\Delta})^n.$$

The ratio $C_n/C_{n-1}$ of $n$-fold to $(n-1)$-fold coincidences is given by

$$C_n/C_{n-1}=(1-e^{-S\Delta})$$

from which it follows

$$\Delta=\frac{1}{S}\log\frac{1}{1-C_n/C_{n-1}}. \qquad (1)$$

3

All the experiments described in the present paper were performed in the periods July–October, 1942 and July–October, 1943 at Passo Sella (Bolzano), in a wide porch under a roof having a thickness 4 g/cm². The altitude was 2200 m above sea level and the average pressure 7.95 m $H_2O$.

The experiments consisted in recording threefold and fourfold coincidences between counters (or groups of counters in parallel) having different areas.

The first series of experiments (1942) was carried out with counters arranged as in Fig. 1; fourfold (1+2+3+4) and threefold (1+2+3) coincidences were recorded separately. Measurements were taken with counting units formed by one, two, or four individual counters, each of $30\times 4.3$ cm² $=0.0129$ m² sensitive area, so that the area of the unit was 0.0129, 0.0258, 0.0516 m², respectively. Different distances $d_1$ and $d_2$ between the counting units were used. Also a set of measurements was taken with $d_1=d_2=2$ m, using four counters of $10\times 2.4$ cm² $=0.0024$ m². The results are shown in columns 3 and 4 of Table I.

The second series of experiments (1943) was carried out with the counter arrangement shown in Fig. 2, by counting fourfold (1+2+3+4) and threefold (1+2+3) coincidences *simultaneously*, so as to minimize the statistical errors. The surface of each counting unit was varied between 0.0011 and 0.1032 m² by variously combining counters of three different sizes ($4.5\times 2.4$ cm², $11\times 2.4$ cm², $30\times 4.3$ cm²). The results are shown in Table II.

All the counters used had brass walls from 0.5 to 1 mm thick and were filled with a mixture of argon and alcohol. The threshold was approximately 1300 volts. During operation they were kept at a temperature of about 15°C by thermostatic control. The high voltage (1450–1500 volts) was supplied by an Evans model stabilizer. The recording was done with a conventional Rossi's coincidence circuit, having a resolving time $\tau=4\div 5\times 10^{-7}$ minutes. Chance coinci-

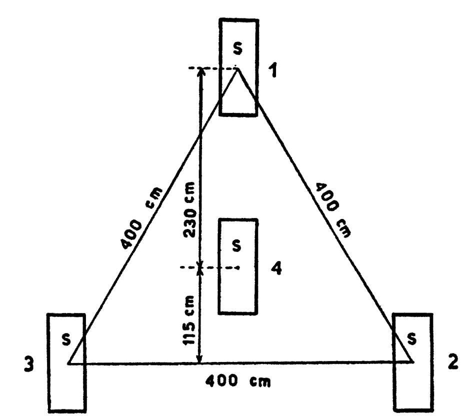


FIG. 2. Counter arrangement in the second series of measurements.

TABLE I. Distribution of shower density with counters arranged as in Fig. 1.

| 1<br>$d_1 \times d_2$ | 2<br>Area of the counter $S$ (m²) | 3<br>Fourfold coincidence (min.⁻¹) | 4<br>Threefold coincidence (min.⁻¹) | 5<br>Density $\Delta$ (m⁻²) |
|---|---|---|---|---|
| | 0.0024 | 204/34,565 =0.0059±0.0004 | 178/16,200 =0.0110±0.0008 | 325±85 |
| | 0.0129 | 386/4,799 =0.080±0.004 | 177/1,569 =0.113±0.008 | 96±36 |
| 2×2 m² | 0.0258 | 1,370/7,206 =0.190±0.005 | 590/1,887 =0.313±0.013 | 37± 8 |
| | 0.0516 | 1,074/2,065 =0.52±0.016 | 714/993 =0.726±0.027 | 24± 5 |
| | 0.0129 | 492/6,937 =0.071±0.003 | 294/2,560 =0.114±0.007 | 75±25 |
| 4×4 m² | 0.0258 | 462/2,875 =0.160±0.008 | 317/1,158 =0.270±0.016 | 35±10 |
| | 0.0516 | 1,005/2,132 =0.470±0.015 | 432/593 =0.730±0.035 | 20± 5 |
| | 0.0129 | 1,080/16,106 =0.067±0.002 | 729/6,919 =0.105±0.004 | 79±15 |
| 4×8 m² | 0.0258 | 1,243/6,647 =0.187±0.005 | 929/3,412 =0.273±0.009 | 45± 8 |
| | 0.0516 | 1,072/2,673 =0.402±0.013 | 1,239/1,743 =0.710±0.020 | 16± 3 |

dences were negligible in all measurements with fourfold coincidences and in most of the measurements with threefold coincidences. In the most unfavorable conditions (threefold coincidences with 8 counters in each unit, having a background of 3000÷3500 counts per minute) the number of chance coincidences was less than 2 percent of the number of true coincidences. The numbers given in the three last rows, column 4, of Table II have been corrected for chance coincidences.

The measurements were carried out uninterruptedly day and night. The operation of the counters and of the circuits was checked daily with a cathode-ray oscilloscope. Absence of electric pick-up was checked by verifying that no fourfold coincidences were recorded when the counter voltage was kept 50 to 100 volts below threshold. The errors due to the roof of the porch as well as the determination of the sensitive areas of the counters will be discussed below.

The first series of experiments was performed by A. Loverdo and V. Tongiorgi; the second by A. Loverdo.

## 4

By applying formula (1) to the results obtained, we have calculated the average densities of shower particles listed in Tables I and II. Examination of Table I shows that $\Delta$ varies considerably when the counting area $S$ changes, while it is approximately independent of $d_1$ and $d_2$. The latter result indicates that shower particles are uniformly distributed over all areas investigated. Therefore in the second series of experiments the distances between the four groups of counters were kept unchanged. Moreover, the more symmetrical "triangular" arrangement was used.

The variation of $\Delta$ with $S$ shown by Table I is confirmed beyond any doubt by the results contained in Table II, where $\Delta$ is seen to vary by a factor of about 100 when $S$ is changed from 0.0011 to 0.1032 m².

The dependence of $\Delta$ on $S$ can be explained in the following way. The showers striking the counters have different densities. This is so (a) because the number of particles in a shower depends on the energy of the primary particle by which the shower is produced and (b) because our counter array, whose dimensions are small compared with the dimension of the shower, may find itself either near the center of the

TABLE II. Distribution of shower density with counters arranged as in Fig. 2.

| 1<br>Area of counters $S$ (m²) | 2<br>Minutes | 3<br>Threefold coincidence (total) | 4<br>Threefold coincidence (min.⁻¹) | 5<br>Fourfold coincidence (total) | 6<br>Fourfold coincidence (min.⁻¹) | 7<br>Density $\Delta$ (m⁻²) |
|---|---|---|---|---|---|---|
| 0.0011 | 77,070 | 179 | 0.0023±0.0002 | 112 | 0.00145±0.00014 | 890±145 |
| 0.0027 | 69,153 | 818 | 0.0118±0.0004 | 511 | 0.0074±0.0004 | 370±28 |
| 0.0054 | 37,006 | 1,167 | 0.0316±0.0010 | 732 | 0.0198±0.0008 | 183±12 |
| 0.0108 | 17,905 | 1,474 | 0.0824±0.0022 | 956 | 0.0534±0.0017 | 97±5.5 |
| 0.0129 | 18,825 | 2,380 | 0.126±0.003 | 1,515 | 0.0806±0.0021 | 79.2±3.5 |
| 0.0258 | 13,030 | 3,885 | 0.299±0.005 | 2,517 | 0.193±0.004 | 40.4±1.4 |
| 0.0516 | 10,063 | 7,671 | 0.76±0.01 | 5,087 | 0.507±0.007 | 21.7±0.6 |
| 0.0774 | 6,688 | 7,910 | 1.165±0.013 | 5,330 | 0.796±0.011 | 14.9±0.4 |
| 0.1032 | 3,545 | 6,466 | 1.79±0.023 | 4,529 | 1.28±0.02 | 12.2±0.4 |

shower, where the density is larger or near the edge, where the density is smaller.

With $S$ constant, the probability of 3 (or 4) counters being struck by a shower varies considerably with $\Delta$. In a rough approximation, we may assume that all showers with $\Delta$ larger than a certain value $\Delta_1$ are recorded, while none of the showers with $\Delta<\Delta_1$ are recorded. Now $\Delta_1$ changes with $S$ and more precisely increases when $S$ decreases. Hence, the average density of the showers recorded must vary when the counter area $S$ is changed.

## 5

We shall now make our considerations more quantitative.

As explained in Section 1, the probability of a coincidence between $n$ counters each of area $S$ when a shower of density $\Delta$ falls upon them is given by

$$P^n=(1-e^{-S\Delta})^n=\varphi(S\Delta). \tag{2}$$

Let us now assume an exponential distribution for the density $\Delta$; i.e., let us assume that the probability of a shower of density $(\Delta, d\Delta)$ striking the counters is given by:

$$\nu(\Delta)d\Delta=K\Delta^{-\gamma}d\Delta. \tag{3}$$

Then the probability for an $n$-fold coincidence is

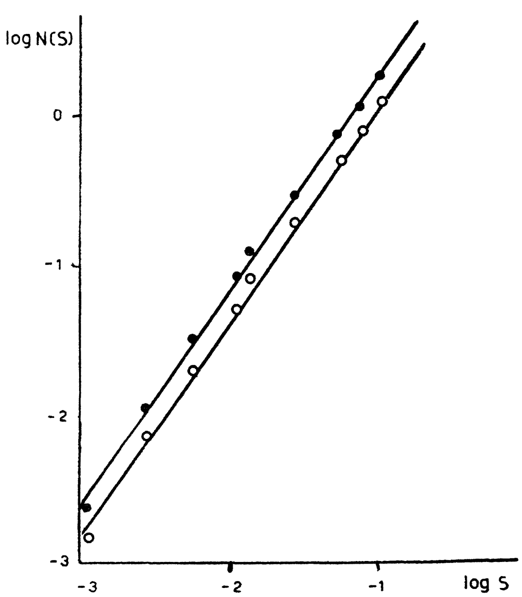


FIG. 3. Threefold and fourfold coincidences as a function of the counter area.

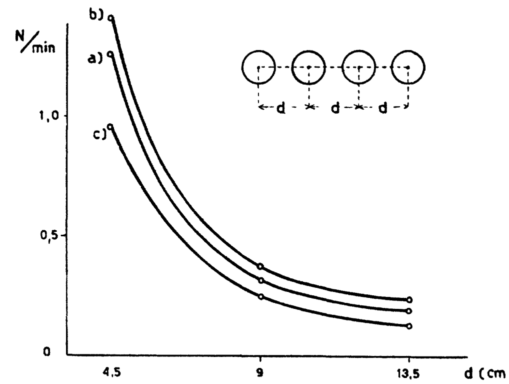


FIG. 4. Fourfold coincidences between counters: (a) immediately under the roof; (b) 2 meters from the roof; (c) in the open air.

represented by

$$N(S)=K\int_0^\infty \Delta^{-\gamma}\varphi(S\Delta)d\Delta \tag{4}$$

or, if we put $S\cdot\Delta=x$

$$N(S)=KS^{\gamma-1}\int_0^\infty x^{-\gamma}\varphi(x)dx. \tag{5}$$

Considering now two different values of $S$, we have:

$$N(S_1)/N(S_2)=(S_1/S_2)^{\gamma-1}. \tag{6}$$

All the quantities on the right side of Eq. (6) can be measured experimentally; hence $\gamma$ can be determined.

In Fig. 3 the numbers $N$ of threefold and fourfold coincidences listed in Table II are represented, on a double logarithmic plot, as a function of the corresponding area $S$. Within the statistical errors the experimental points lie on two straight lines with the same slope, thus confirming the dependence of $N$ on $S$ expressed by Eq. (6). From the value of this slope one obtains

$$\gamma=2.45.$$

The value of $K$ can also be determined from the experimental results, using Eqs. (2) and (5). One obtains

$$K\approx 80.$$

Hence Eq. (3) can now be written as follows:

$$\nu(\Delta)=80\Delta^{-2.45}, \tag{3'}$$

where $\Delta$ is measured in $m^{-2}$.

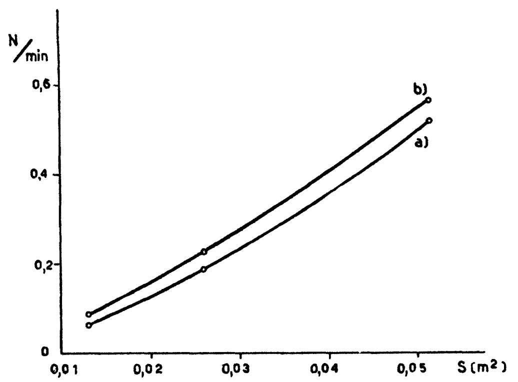


FIG. 5. Fourfold coincidences as a function of counter area. (a) Immediately under the roof; (b) 2 meters from roof.

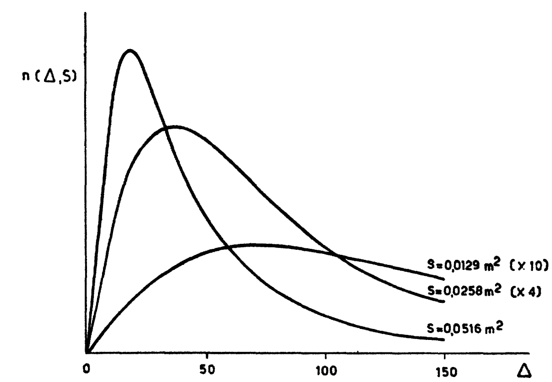


FIG. 6. Number of showers recorded with various fourfold coincidence arrangements as a function of shower density.

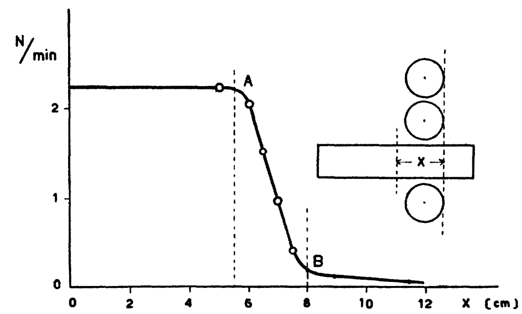


FIG. 7. Fourfold coincidence arrangement for the measurement of the sentitive area of a $2.4\times 11$ cm² counter.

### 6

As already pointed out, our measurements were carried out under a light roof. It had been shown by Cosyns[3] that the effect of a roof on the multiplication process cannot be neglected, and experiments were performed to check this result. Coincidences were recorded between four counters in a horizontal plane placed (a) immediately under the roof, (b) 2 meters below the roof, and (c) in the open air. Measurements were taken with 4.3, 9, 13.5 cm between the axes of the counters, and the results are shown in Fig. 4. The three curves are similar, but definitely distinguishable from one another.

In order to study the effect of the roof with a counter arrangement similar to that used in the actual experiments, coincidences were recorded between four counting units placed at the four corners of a square of 2-m sides, first with the counters immediately under the roof, second with the counters 2 meters below the roof. For each position, one, two, and four counters, respectively, were used in each counting unit. The results are shown in Fig. 5.

In our experiments on the air showers,[4] the counters were placed immediately under the roof, in order to minimize the coherence effect produced by the roof. Just the same, as shown by Fig. 4, the effect of the roof is not negligible. This effect may be described as producing an increase of the effective area of the counters, increase which is relatively more pronounced for the smaller counters. As a consequence, the showers of greater density appear to be more abundant when the measurements are carried out under the roof than when they are carried out in the open air. Therefore, the value of $\gamma$ as determined above is somewhat too large; we believe that its most likely value is:

$$\gamma = 2.6.$$

### 7

It is interesting to investigate the behavior of the function

$$n(S, \Delta)d\Delta = \Delta^{-2.6}(1-e^{-S\Delta})d\Delta,$$

which represents the probability of recording a shower with density between $\Delta$ and $\Delta+d\Delta$ with a given experimental arrangement. The curves in Fig. 6 give $n$ as a function of $\Delta$ for $n=4$ and three different values for $S$, namely, 0.0129, 0.0258, 0.0516 m². Each of these curves exhibits a pronounced maximum for a value of the density

[3] M. Cosyns, Nature **145**, 668 (1940).

[4] G. Cocconi and V. Tongiorgi, Ricerca Scient. **10**, 566 (1939).

which depends on the value of $S$. This means that any given experimental arrangement favors the recording of showers with density in the neighborhood of a certain value. This is so because showers with low density are numerous but have a very small probability of being recorded, while showers with high density have a large probability of being recorded, but are very rare.

In other words, every experimental arrangement operates like a monochromator, selecting a given band of the density spectrum of air showers. Thus our results are explained and the disagreement between the conclusions of different authors becomes clearly understandable.*

8

In the evaluation of our experiments it is important to know the sensitive area of the counters exactly. As effective width, we have used the diameter of the brass tube. However, we have not deemed safe to take the effective length as equal to that of the wire because of possible end effects due to modifications of the electric field near the supports of the wire.

Therefore, the effective length was determined experimentally by placing the counter in question between three other counters of $2.4\times 11$ cm$^2$, forming a vertical counter telescope (see Fig. 7). The axis of the counter to be examined was perpendicular to the axes of the other counters, and the number of fourfold coincidences was measured for different positions of the counter investigated. The results obtained with a counter of $2.4\times 11$ cm$^2$ are represented in Fig. 7. Similar curves were obtained with the other types of counters used. The effective lengths of the counters were taken as equal to twice the value of $x$ corresponding to the point $A$ in the diagram.

It may be noted that in all our experiments the distance $AB$ over which the counting rate drops from its full value to practically zero is nearly equal to the diameter (2.4 cm) of the counters forming the vertical telescope. This means that the efficiency of the counter drops very abruptly from one to zero as one moves along the axis. No region of appreciable width seems to exist where the efficiency has an intermediate value.

* By the same token, the results obtained by Auger and collaborators [P. Auger, Robley, and Culvinage, Comptes rendus **209**, 536 (1939)] comparing the average density of air showers at two different altitudes are valueless because the measurements were performed with counters of different areas. The differences found are probably due to the different nature of the experimental arrangement used at the two elevations.

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# Experimental and Theoretical Evaluation of the Density Spectrum of Extensive Cosmic-Ray Showers

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Measurements are made of the density spectrum of extensive cosmic-ray showers at an altitude of 120 m above sea level. The results together with the measurements previously made at 2200 m confirm the electronic nature of the primary radiation which produces the extensive showers. It is estimated that the denser showers are generated by electrons of $10^{15}$ to $10^{16}$ ev, the less dense showers by electrons of energies $10^{12}$ ev.

In a previous note[1] we gave the results on the density spectrum of extensive cosmic-ray showers at 2200 m above sea level (Passo Sella, atmospheric pressure 760 g/cm$^2$). We present here first the results of similar measurements carried out by two of us (A.L. and V.T.) at 120 m above sea level (Milan, 1000 g/cm$^2$). We then discuss the results obtained at the two stations in the light of the theory of cascade multiplication (G.C.).

[1] G. Cocconi. A. Loverdo, and V. Tongiorgi, Nuovo Cimento **2**, 14 (1944).