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Slow Neutron Resonances in Indium*

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The neutron spectrometer previously described has been modified and improved to permit the use of 12 detector timing channels. It has been used to determine the neutron cross section for absorption in indium as a function of neutron energy in the range extending from 0.017 to 50 ev. Five absorber thicknesses were used in the measurements. These were 0.0227, 0.246, 0.907, 2.754, and 4.05 g/cm². The strong resonance previously observed was found to be located at 1.44 ev and two weaker resonances were found at about 3.7 and 9.0 ev. The resonance at 1.44 ev was carefully studied. An analysis was made to take into consideration the effective resolution and to determine the parameters of the Breit-Wigner one-level formula which best describe the resonance. The constants which provided the best fit with the data were $\sigma_0 = 26,400 \times 10^{-24}$ cm² and $\Gamma = 0.085$ ev. The resolution did not permit a similar analysis for the levels at higher energy and it was only possible to place rough lower limits for the values of the cross section at resonance.

THE strong resonance for neutron absorption in indium at about 1.4 ev has been examined by many different workers. It was first studied by boron absorption and self-absorption methods, while more recently the slow neutron velocity spectrometer, based upon the time of flight method, has been used in examining this resonance.[1-3]

A beta-activity, having a 54-minute period, is induced by the absorption of neutrons in In^{115}. This strong activity has permitted indium to be used extensively as a standard monitor of neutron intensity. Consequently an accurate knowledge of the variation of the cross section as a function of neutron energy is of considerable importance. In addition it is desirable to know if the Breit-Wigner one-level formula can be successfully used to describe the variation of the cross section in the region of the resonance.

At the time this work was begun, spectrometer apparatus and techniques had been developed to the extent that it was feasible to make measurements on indium with a resolution much better than any previously used. A study was made of the variation of the cross section over a wide range of neutron energy. In the region of the important resonance, sufficient accuracy and resolution were attained to permit the desired comparison of experimental results with the theory.

APPARATUS

The time of flight method and the equipment which was used have been described earlier.[1,2] The equipment has been modified to permit detection in 12 time of flight intervals; except for this it is essentially the same.

The available frequencies of arc modulation were 2500, 1000, 500, 200, 100 c.p.s. The available "on times" of modulation for both arc and detector ranged from 5 to 500 μsec. The standard of frequency was provided by a 100-kc crystal oscillator which was known to be accurate to at least 0.1 percent.

The detection system consisted of an ionization chamber filled with BF_3 gas, and a pulse amplifier employing cathode degeneration to provide gain stability. The "rise time" of the amplifier was about 2 μsec.

The beam collimation and shielding for the detector are shown in Fig. 1a. The collimation in front of the ionization chamber was in the form of a cylindrical shield of B_4C of density 1.0 g/cm². A similar shield was placed behind the chamber while immediately surrounding the chamber was a B_4C shield of about 0.5 g/cm. Except for 17.3 cm at the front end, the collimator was surrounded by at least 14 cm of paraffin. The absorber was placed at the front end of the collimator.

* This work was performed late in 1942 and was presented as a Ph.D. thesis in February, 1943 at Cornell University. It was withheld from publication as classifiable material for the duration of the war.
[1] R. F. Bacher, C. P. Baker, and B. D. McDaniel, Phys. Rev. **69**, 443 (1946).
[2] C. P. Baker and R. F. Bacher, Phys. Rev. **59**, 332 (1941).
[3] W. W. Havens, Jr. and James Rainwater, Phys. Rev. **70**, 154 (1946).

For slowing down the neutrons, the Be target of the cyclotron was surrounded by a copper tank as illustrated in Fig. 1b. The tank was filled with water to its capacity of 11 liters. The face of the tank was in the direction of the detector. The arrangement of the equipment relative to the cyclotron room is shown in Fig. 1c.

PROCEDURE

In most respects the procedure for making the measurements of the transmission were similar to those used in experiments described in the references.[1,2]

The measurements which were made in the course of these experiments were taken with the detector at 2 meters and 3 meters from the source. In making the measurements on neutrons of energy 0.5 ev and greater, a thick layer of Cd was placed immediately in front of the detector to remove the slow, Cd absorbed neutrons from the beam to allow a higher repetition frequency to be used.

When measurements were made below 0.5 ev, they were taken with a source-detector distance of 2 meters. The source was then modified by adding 13.7 g/liter of borax ($Na_2B_4O_7 \cdot 10\,H_2O$) to the water in the tank. In addition, a 0.026 g/cm² B_4C filter was placed in the beam. In this manner the number of very low energy neutrons detected was reduced and the borax in the tank also reduced the mean life of thermal neutrons in the source. From previous measurements,[1] it was found that the mean life in the tank with this amount of boron, would be about 50 μsec. The resolution, as calculated from the assumed distribution of velocities,[4] was not greatly affected by the delayed emission from the source. Since the transmission of the absorber varied only slowly with the time of flight in this region, no correction was made for the effect of the mean life in the source on the resolution.

In performing the experiment it was necessary to consider the various sources of background and to make measurements for determining the magnitude of the corrections which should be applied. Two types of background exist. One of these is due to noise in the amplifier and pulses due to radioactive contamination. Since this background is proportional to the counting time, a correction is readily made. This correction, in individual channels, amounted in most cases to two percent or less of the incident beam intensity.

The other type of background is the fast neutron background which is proportional to the beam intensity and is due either to incomplete modulation of the cyclotron or to neutrons which have been scattered into the chamber without passing through the absorber, or else have passed through the absorber and have been delayed in arriving at the chamber and are recorded in the wrong time group.

This background was measured by interposing 2.0 g/cm² of B_4C and a 10-cm length of paraffin in the beam, with the boron closest to the detector, and observing the number of counts recorded in the timed channels as well as the total number of counts. From these measurements, the fraction of all counts which are due to this type of background may be computed. The background due to this cause was of the order of 3 percent.

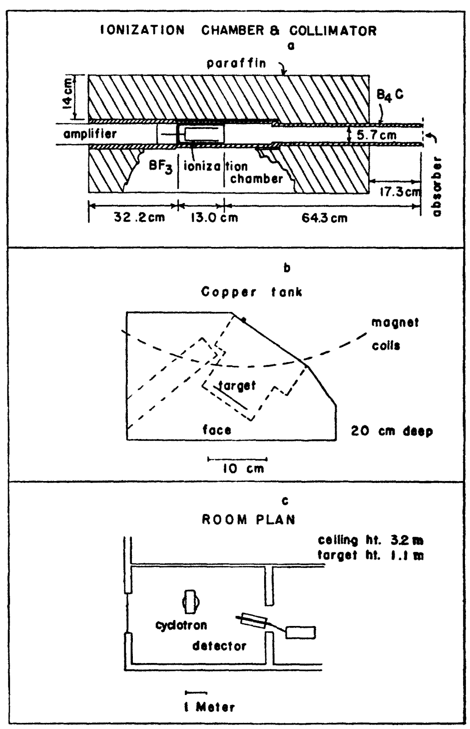

FIG. 1. Geometrical arrangements.

[4] C. P. Baker, B. D. McDaniel, and R. F. Bacher, "Velocity Distribution of Neutrons Emitted from a Slow Neutron Source," unpublished.

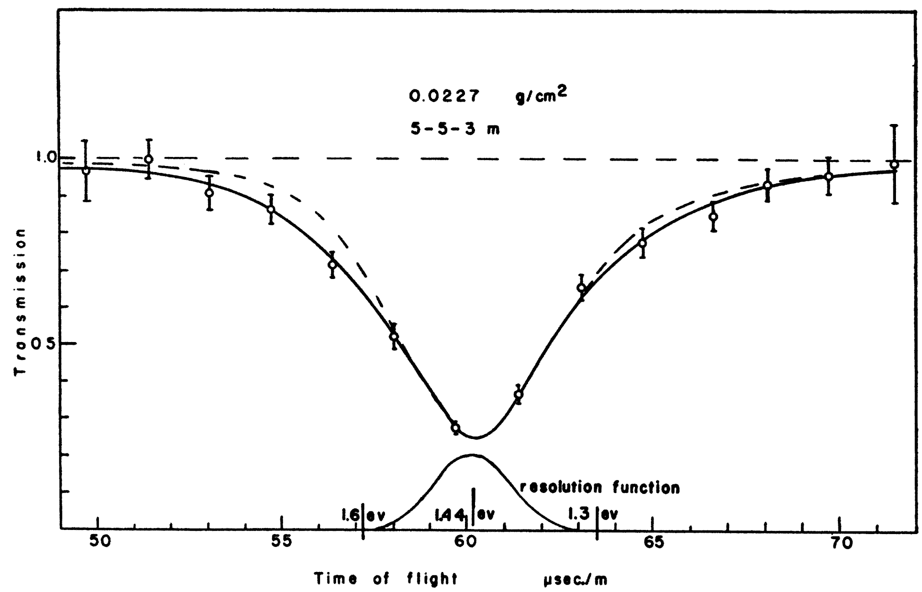

FIG. 2. Transmission of 0.0227 g/cm² of indium as a function of neutron time of flight.

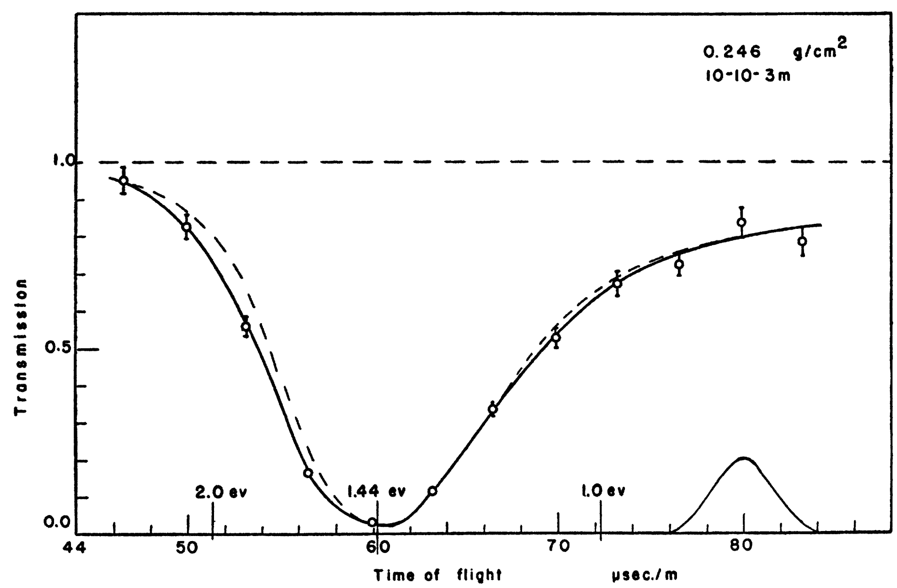

FIG. 3. Transmission of 0.246 g/cm² of indium as a function of neutron time of flight.

TABLE I. Experimental conditions.

Figure	On time μsec.	Distance meters	Rep. rate c.p.s.	Source	Filter g/cm²	Absorber thickness g/cm²	Time of flight μsec./m	Energy ev
2	5	3	2500	water only	0.45 Cd	0.0227	50–70	1.1 – 2.1
3	10	3	2500	water only	0.90 Cd	0.246	46–84	0.74 – 2.5
4	50	2	500	water and boron	0.026 B_4C	0.907	100–650	0.012– 0.52
4	20	2	500	water and boron	0.026 B_4C	0.907	50–175	0.17 – 2.1
5, 6	10	3	2500	water only	0.90 Cd	2.754	3.3–110	0.42 –480
5, 6	5	2	2500	water only	0.90 Cd	2.754	5–55	1.7 –210
7	10, 5	3	2500	water only	0.90 Cd	4.05	15–33	4.8 – 23

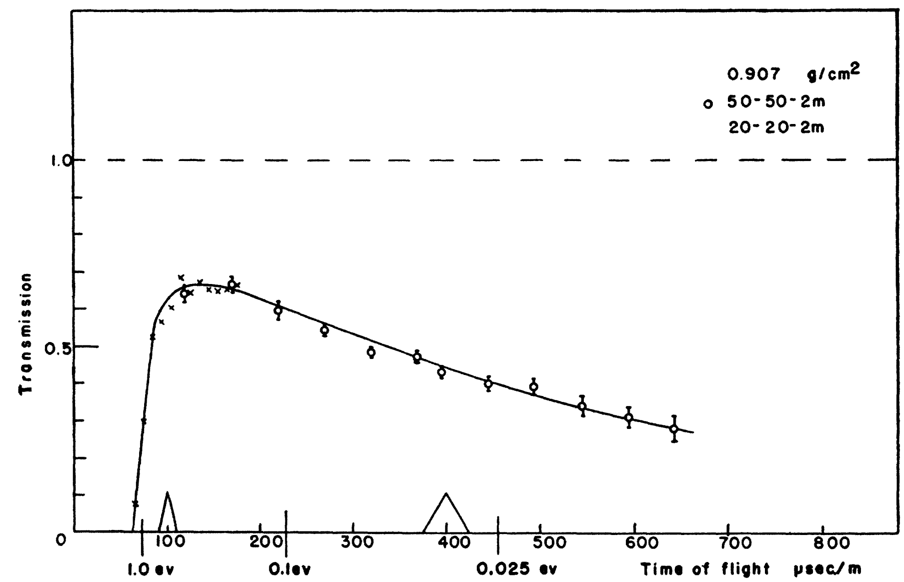

FIG. 4. Transmission of 0.907 g/cm² of indium as a function of neutron time of flight.

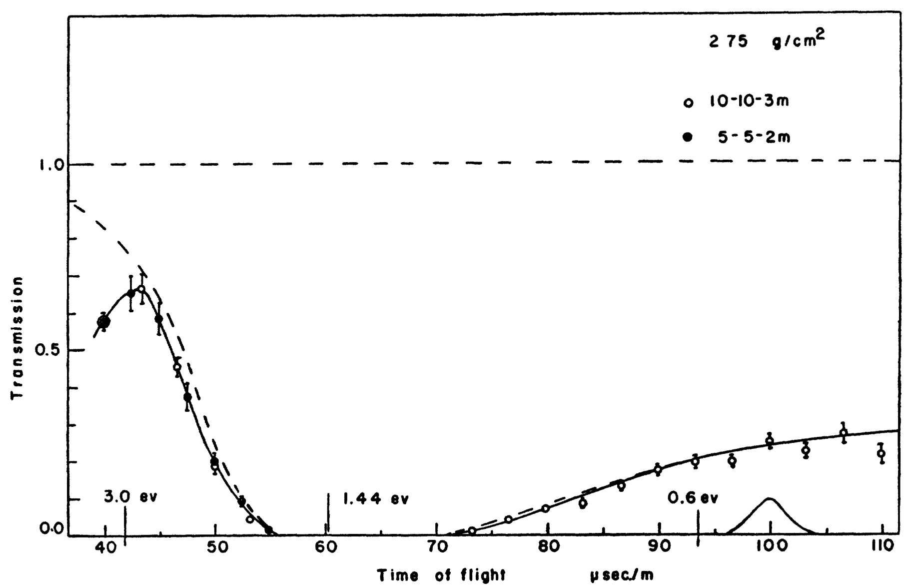

FIG. 5. Transmission of 2.75 g/cm² of indium as a function of neutron time of flight.

In the vicinity of the lowest and most important resonance, the background could also be measured by inserting an absorber of indium so thick that the transmission was known to be essentially zero over a long time of flight interval about the resonance. The number of counts recorded in the individual channels near exact resonance was found for a given number of total counts. Then by multiplying the ratio of these two numbers by the total transmission of the thick absorber, one finds the fraction of all neutrons in the incident beam which appear as background counts in the individual channels. The magnitude of the correction when measured in these two different ways agreed within the experimental error. However, it was felt that the latter method was best for determining the correction to be used in the vicinity of the resonance so that the value determined in this manner was used.

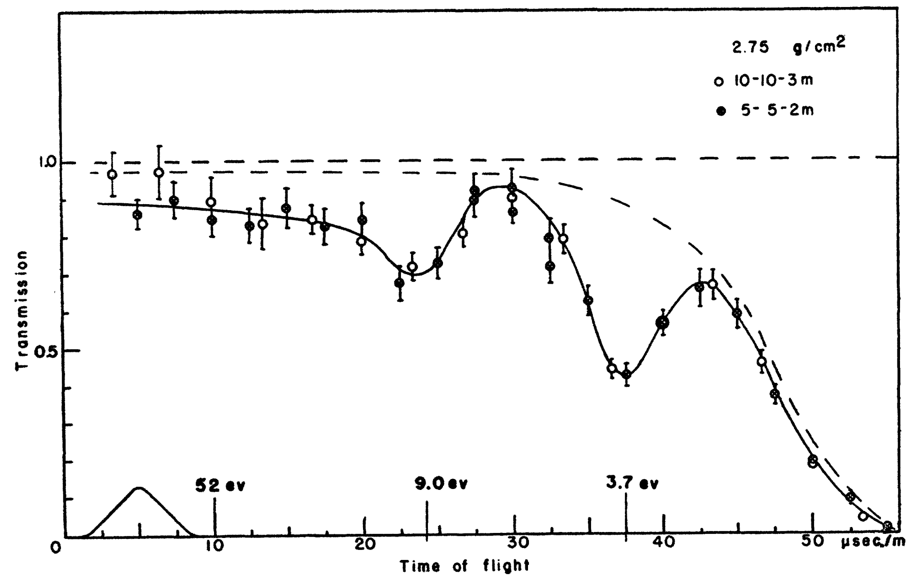

FIG. 6. Transmission of 2.75 g/cm² of indium as a function of neutron time of flight.

Several series of measurements of the time lag were made with different modulation schemes using a method similar to that described previously.[1] The values so determined were in good agreement with each other and gave an average of 10.5 μsec.

RESULTS

The transmissions of five indium absorbers were measured and the combined range of reliable measurements extended over a time of flight interval from 10 to 550 μsec./meter. This corresponds to an energy range from 0.017 to 50 ev. The main resonance was found at a time of flight of 60.2 μsec./meter (1.44 ev). A much weaker resonance was observed in the region of 37.5 μsec./meter (3.7 ev) and a still weaker one was detected at about 24 μsec./meter (9 ev).

The thicknesses of the absorbers were 0.0227, 0.246, 0.907, 2.754, and 4.05 g/cm². The uniformity of the various absorbers was measured. The thickest absorber was found to have a thickness constant to within one percent of the average thickness. The thinnest absorber, except for a few small pin holes in it, was uniform to within 10 percent of the average and calculations were made which indicated that for transmissions greater than 25 percent the error in transmission which would be made as a result would be less than one percent. The metal was analyzed and the analysis indicated that the total amount of impurity was less than 0.005 percent.

In Table I the experimental conditions are tabulated for the sets of measurements which were made. The results of these measurements are plotted in the Figs. 2, 3, 4, 5, and 6. The figure number corresponding to a given observation is given in the table. The results have been corrected for fast neutron background, noise, and chamber contamination. The solid lines have been drawn to fit the points. The statistical probable errors of the points are indicated by the lines extending above and below the points. The dotted curves, where they are shown are curves derived from theory and will be discussed later. The sequence of three numerals written above the graphs in the figures designate, respectively, the arc "on time," detector "on time" in microseconds, and distance of flight in meters The triangular shaped figures drawn along the horizontal axes are the resolution functions for the measurements given in the graphs. These functions are discussed in detail below.

In Figs. 2 and 3, the results taken with the two thinnest absorbers show the presence of the main resonance at 60.2 μsec./meter or 1.44 ev. Figure 4 shows the transmission of the 0.907 g/cm² sample in the energy region below the resonance extending through the thermal energy region.

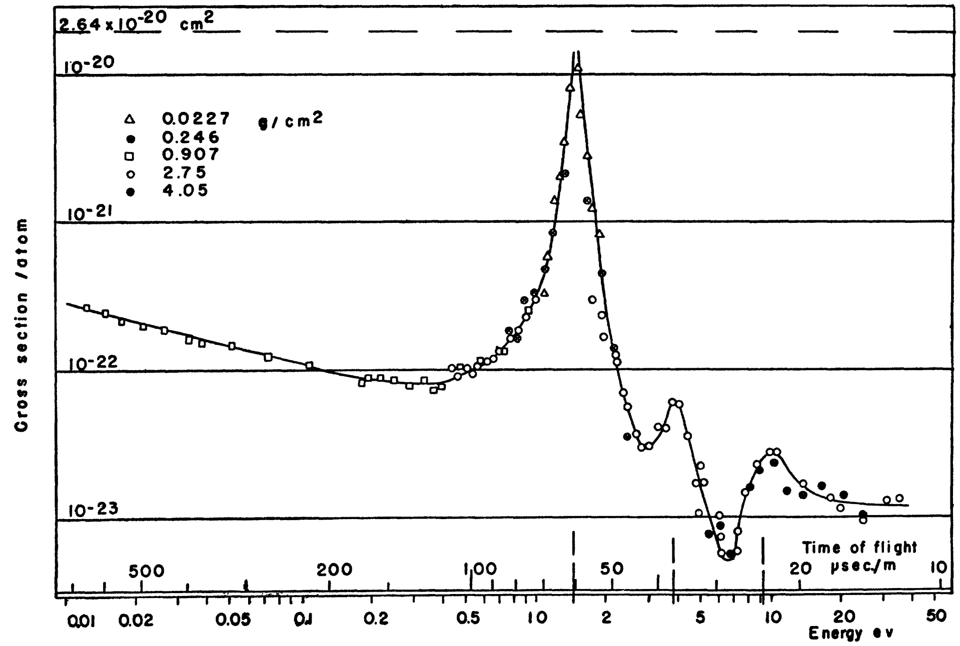

FIG. 7. Cross section of indium as a function of neutron energy. Logarithm of the cross section is plotted against the logarithm of the neutron energy. These data have not been corrected for the effect of finite resolution.

Figure 5 shows the results for the 2.75 g/cm² absorber in the vicinity of the main resonance while Fig. 6 is a plot presenting the results in the high energy region. In plotting the points for Fig. 5, the values for the corrected transmission in the region from 57 to 70 μsec./m time of flight were omitted since the points scattered on both sides of the axis because of statistical errors, and little significance could be placed upon these values.

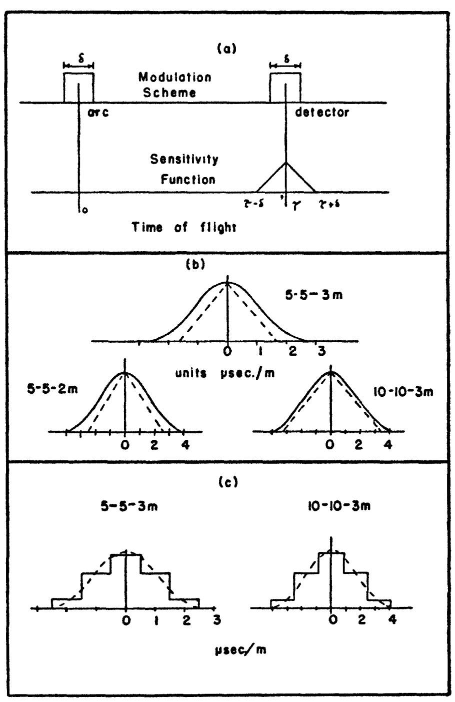

FIG. 8. Resolution functions: (a) ideal resolution, (b) experimental resolution, (c) step function approximation to the experimental resolution.

In Fig. 6, it is observed that in addition to the strong resonance at 1.44 ev there are two regions of strong absorption. These are at about 3.7 and 9 ev. While these resonances are much weaker and more poorly resolved, they are quite definite. Since there are only two known isotopes of indium, In^{113} and In^{115}, and since there are a total of three resonances, it is clear that at least two of these resonances occur in the same isotope. Even though the abundance of isotopes is about 21 to 1 for In^{115} and In^{113}, respectively, a definite isotopic assignment of the higher resonance is difficult. Since the lowest resonance is so much stronger than the others, and since thermal neutron bombardment strongly excites the 54 minute beta-period, it is reasonable to conclude that this resonance occurs in In^{115}.

To verify further the presence of the weak resonance near 9 ev, additional observations were made of the transmission of the 4.05 g/cm² absorber. The range of this set of measurements only extended from 15 to 33 μsec./m time of flight so that the results were not plotted separately, but instead, the cross section values were calculated from the observed transmissions and plotted in Fig. 7 along with the cross sections calculated from each of the transmission measurements for the five absorbers. The horizontal axis is a logarithmic energy scale and the vertical axis a logarithmic cross-section scale. In addition, certain times of flight are marked along the horizontal axis to permit comparison.

In plotting this figure, no attempt was made to correct the cross sections for the effect of the finite resolving power of the apparatus. However, three of the points were omitted from the graph because the corrections which should have been made for resolution were obviously large, and as a result, the uncorrected values fell far off the curve.

DISCUSSION

It is first desired to obtain some idea of the probable effect which the resolution produced on the measurements. A rather complete discussion of the ideal case of equal "on time" for the arc and detector is given by Baker and Bacher.[2] For a given setting, the function which measures the sensitivity of one of the channels to neutrons of various times of flight is given in Fig. 8a. (τ) is the average time separation between the "on time" of the arc and the "on time" of the detector channel. (δ) is the length of the "on times." The sensitivity is zero for any neutron having a time of flight less than ($\tau-\delta$) or greater than ($\tau+\delta$). At these two times the sensitivity begins to rise linearly to a maximum at (τ).

However, several factors enter to distort this sensitivity function even at energies well above thermal energies. One is a straggling in the production of the slow neutrons. This straggling may arise in either the acceleration of the ions in the cyclotron or in the slowing down of the fast neutrons that are produced at the target. Probably neither of these effects is sufficiently large to be considered very seriously. The straggling in the acceleration of the ions in the cyclotron is less than a microsecond as was indicated by earlier measurements,[1] which were made with a fast neutron detector. In these earlier measurements the sensitive time of the detector was moved progressively over the "on time" of the arc. The shape of the resultant curve indicated no large straggling in the acceleration. As far as straggling is concerned in the slowing down of neutrons to the energy of 1 ev, it is not significant because the mean slowing down time is probably of the order of 1.5 μsec.

Another cause for distortion of the sensitivity function is improper settings of the timing equipment. Small errors in setting up the modulation system or drifts of the timing during observations would cause a distortion of the average sensitivity function. It is thought that the uncertainty in the modulation times from this source was limited to 0.5 μsec.

Probably the largest factor in distorting the ideal sensitivity function is the time required for the neutron to traverse the length of the ionization chamber. A neutron having an energy of 1.44 ev, the energy of the main indium resonance, has a time of flight through the effective length (11.0 cm) of the chamber of about 6.6 μsec. Since the neutron may produce a disintegration anywhere over the length of the chamber, this introduces an uncertainty in the distance traveled, or an equivalent uncertainty in the time of flight to associate with a given neutron. Although a similar uncertainty exists in determining the origin of the neutrons at the source, since the mean free path in the source is fairly short (1.0 cm) this effect is not important. In Fig. 8b is plotted the sensitivity function for 5 μsec. "on times" at both 2 and 3 meters as well as one for 10 μsec. at 3 meters. The functions have been corrected for the effect produced by the depth of the chamber and depth of the source. It is noted that the effective width of the sensitivity function is considerably increased for the case of 5 μsec. "on time" and is still significant for the 10 μsec. "on time." It is also seen that the function for 5 μsec. "on time" at 2 meters corresponds very closely to that of 10 μsec. "on time" at 3 meters, so that in treating the data taken with the 2.75 g/cm^2 absorber, the resolution was considered effectively the same for both sets of data.

As was pointed out by Baker and Bacher,[2] it is very difficult to proceed from the observed transmission curves to calculate the transmission curves which would be expected if the resolution were infinitely sharp. Such a calculation is made especially difficult because of the statistical uncertainties associated with the observations. Because of this, it seemed more reasonable to determine from the experimental results the most probable values of the parameters of the Breit-Wigner one-level formula for neutron absorption when it is applied to the main resonance.

The Breit-Wigner one-level formula[5] is given by

$$\sigma(E)=\sigma_0\left(\frac{E_R}{E}\right)^{\frac{1}{2}}\frac{\Gamma^2}{\Gamma^2+4(E-E_R)^2},$$

$\sigma(E)$ = cross section for neutron of energy E,
σ_0 = cross section at exact resonance,
E_R = resonance nergy,
E = energy of incident neutron,
Γ = total width.

Because of the Doppler effect, the effective cross section will be given by the following expression

$$\sigma(E)=\frac{\sigma_0}{\pi^{\frac{1}{2}}\Delta}\int_0^\infty\left(\frac{E_R}{E'}\right)^{\frac{1}{2}}\frac{\exp\left[-(E-E'/\Delta)^2\right]}{1+(4/\Gamma^2)(E'-E_R)^2}dE',$$

Δ = Doppler width $=2(E_RE_{th}m/M)^{\frac{1}{2}}$,
m = neutron mass,
M = mass of target atom,
E_{th} = thermal energy.

The effective thermal energy was assumed to be $1.2\,kT$ rather than kT, to take into account the lattice binding. This is based on the calculations of Lamb.[6] With this value the Doppler width was found to be 0.039 ev.

The determination of the most probable value for the Breit-Wigner parameters is made in the following manner. The time of flight of neutrons at exact resonance was determined from the transmission curve for the 0.0227 g/cm^2 absorber. Reasonable estimates were made of the value for the constants σ_0 and Γ. By use of these values for the constants, and by making the correction for the Doppler effect, the corresponding effective cross section was calculated as a function of time of flight. From these, the transmissions were calculated for absorbers of the same thickness as those used in the experiments. It was next desired to see what transmissions one would expect to observe if absorbers having these calculated transmissions were to be measured by the apparatus. The resolution was assumed to be the same as that which was used in making the experimental determinations and step function approximations to the sensitivity curves were used to calculate the expected observed transmissions. The step functions are shown in Fig. 8c. The expected observed transmissions, hereafter described as "derived" transmission curves, were then compared with the experimental data. Successive choices of σ_0 and Γ were made to determine the set which yielded the best fit of the derived curves to the experimental curves. In determining the quality of the fit two regions were weighted heavily. One of these was at the maximum cross section and the other at the minimum in the cross section which occurs between exact resonance and zero energy (near 135 μsec./meter time of flight). The value of the scattering cross section was assumed to be 2×10^{-24} cm^2/atom, and a correction was made.

The values which seemed to produce the best fit are given below. The probable errors listed were estimated and include consideration of statistical errors, lack of resolution, and other factors which are discussed below.

Resonance energy	1.44 ± 0.03 ev
Maximum cross section at resonance	$(26{,}400\pm6500)\times10^{-24}$ cm^2/atom
Total width	0.085 ± 0.01 ev.

Because of the Doppler effect, the maximum cross section given by the Breit-Wigner formula is effectively lowered. For the parameters chosen, the effective maximum cross section was 32 percent less than the cross section uncorrected for the Doppler effect.

The derived curves may be directly compared to the experimentally determined curves. The comparison is made in the Figs. 2, 3, 5, and 6 which show the observed transmissions. The dotted lines are the derived curves. In Fig. 4 the curve which would be drawn to fit the points differed so slightly from the derived curve that only one curve was drawn and it was the derived curve.

[5] H. A. Bethe, Rev. Mod. Phys. 9, 116 (1937), Eq. (428).
[6] W. E. Lamb, Phys. Rev. 55, 190 (1939).

In general, the agreement between the experimental results and those predicted by theory is quite good. This is particularly true if one considers the great range in cross section and energy over which the measurements were made.

In Fig. 2, for the 0.0227 g/cm^2 absorber, it is seen that the experimental curve fits the derived curve moderately well except on the high energy side of the resonance. Though the probable errors are relatively large compared with the difference between the two curves it seems that there may be a real difference between these observed values and the expected values The difference between the observed curve and the derived curve on the low energy side is not considered significant with the probable errors as shown

To show the sensitivity of the shape of the transmission curve of this absorber to the value of the constants that were chosen, two other derived curves were calculated. One assumed a $\Gamma=0.094$, the other $\Gamma=0.075$ ev. In the region of greatest difference, the difference between the two curves was found to be about twice the probable error associated with the experimental points in the same region.

Both in Figs. 3 and 5, corresponding to 0.246 and 2.75 g/cm^2 absorbers, the observed curves and the derived curves fit well on the low energy side, but are in slight disagreement on the high energy side.

The agreement between the experimentally determined points and the derived curve for the 0.907 g/cm^2 absorber (Fig. 4) is excellent. This is to be expected for two reasons. One reason is that except at the very high energy, the transmission does not vary rapidly so that the effect of the resolution is not very pronounced. The other reason is that the point of maximum transmission was taken as one of the points at which the Breit-Wigner formula was adjusted to the experimental data. The points at still lower energy would be expected to agree well with the derived curve because the shape of the derived curve is not sensitive to the choice of the constants in this region, where the cross section dependence is proportional to the time of flight. The value of the total cross section at 0.025 ev is found to be 191×10^{-24} cm^2/atom.

The differences between the experimental curves and the derived curves in the region just above the resonance may indicate that the one-level formula does not accurately describe the variation of the cross section. Because of the presence of higher levels, it would not be surprising if this were the case. However, it cannot be stated with any certainty that this is the explanation.

It is to be noted that a shift in the time axis of less than one microsecond per meter (3 microseconds at 3 meters) would greatly improve the agreement on the high energy side of the resonance, but this would be at the expense of the agreement on the low energy side. It does not seem that this could serve as a satisfactory explanation for the disagreement.

Because the resolution used was more likely to be poorer rather than better than the assumed resolution, it seems that if there is any large error in the values of Γ and σ_0, it is most probable that they should be smaller and larger, respectively.

The results of the self-absorption measurements of Hornbostel, Goldsmith, and Manley[7] give for the values of the constants:

$$E_R=1.0 \text{ ev},$$
$$\sigma_0=23{,}000\times10^{-24} \text{ cm}^2/\text{atom},$$
$$\Gamma=0.07 \text{ ev}.$$

In this early measurement the value of 1.0 ev for the resonance energy was determined by the boron method assuming a value of 0.025 ev for the average energy of neutrons emitted from a slow neutron paraffin source. The average energy from such a source is now known to be considerably higher.[1] Since their result for the values for σ_0 and Γ are dependent on the assumed resonance energy, correction of the calculation by using the correct resonance energy might give better agreement on the width and maximum cross section The value of $E_R=1.44$ ev reported by Havens and Rainwater[3] is in agreement with that found here.

Relatively little may be learned from the measurements of the two weaker resonances because of insufficient information. The existence of both resonances seems to be established without doubt. It is not surprising that these reso-

[7] J. Hornbostel, H. H. Goldsmith, and J. H. Manley, Phys. Rev. 58, 18 (1940).

nances were not observed earlier, since comparable resolution has not previously been used. Even the resolution used here is not sufficient to make it possible to determine the strengths of the resonances. However, minimum values may be ascribed to the cross section at resonance by taking into account the resolution. These values are about 220×10^{-24} and 67×10^{-24} cm^2/atom for the 3.7 and 9 ev levels, respectively.

ACKNOWLEDGMENT

The author wishes to express his gratitude to Professor R. F. Bacher for his encouragement and guidance of the work, to Dr. C. P. Baker for much valuable assistance, and to Professor H. A. Bethe for his many helpful suggestions.

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The Density Spectrum of the Extensive Cosmic-Ray Showers of the Air

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The paper describes measurements performed at 2200 meters above sea level for the purpose of studying the density of particles in the extensive air showers. From the results obtained, one finds that the dependence of the number of showers on the density can be expressed over a wide range by an exponential low.

1

THE so-called "extensive air showers" are produced by cosmic-ray particles which arrive at the top of the atmosphere with very high energies (10^{13}–10^{16} ev) and give rise to a large number of electrons and photons through a process of multiplication extending to the bottom of the atmosphere. The large number of particles observed indicates a large subdivision of the primary energy, while the length of the path between two successive elementary processes produces a considerable spread of the shower particles.

The problem of the spread of air shower has been fully investigated by Auger and co-workers with experiments carried out at different heights above sea level. These experiments led to the establishment of the well-known "decoherence curve" which relates the number of showers with their extension.[1]

The same experimenters,[2] as well as others, have also investigated the dependence of the number of air showers on their density, i.e., on the number of particles per m^2. The results, however, are uncertain and contradictory. Therefore new experiments were undertaken in order to contribute to the solution of this problem.

2

The experimental determination of the particle density in an air shower detected with a set of G-M counters is based upon a classical probability formula which was first applied to this specific problem by Auger, and was later used by other authors. This formula reduces the computation of the density to the determination of the relative number of coincidences, C_n and C_{n-1}, recorded in equal times with n and $n-1$ counter, respectively. The counters are arranged in a horizontal plane, over an area small compared with the spread of the shower.

Let $p(S, \Delta)$ be the probability that a counter of area S is *not* struck by a shower of average density Δ. If the area is increased by dS, this probability becomes

$$p(S+dS, \Delta)=p(S, \Delta)(1-\Delta\cdot dS),$$

[1] P. Auger, R. Maze, P. Ehrenfest, and A. Freon, J. de phys. et rad. **10**, 39 (1939).
[2] H. Geiger, Abhandl. Preuss. Akad. Wiss. **10** (1941). J. Daudin, University of Paris, Thesis, 1942. J. Clay, Physica **9**, 897 (1942).