The only other well-established case of inverse predissociation is the observation by Stenvinkel[11] of the emission of the predissociated lines of AlH under conditions that strongly favor formation of AlH molecules from Al and H atoms in two-body collisions.

Independent of the specific mechanism of the recombination process it follows that the dissociation energy of the C_2 molecule is less than the excitation energy of the $v'=6$ level and larger than that of the $v'=5$ level that is between the limits 3.4 and 3.6 volts—probably closer to the upper limit. This value depends of course on the correctness of the assumption of a recombination process for the explanation of the selective emission of the $v'=6$ progression of the Swan bands (i.e., the high pressure carbon bands).

[11] G. Stenvinkel, Zeits. f. Physik **114**, 602 (1939).

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Racetrack Stability*

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In a recent Letter to the Editor, Serber[1] has pointed out that the existence of the straight, field-free sections of the racetrack synchrotron introduces a condition for the stability of the orbits which depends upon (a) the ratio of the length of a straight section to the radius of curvature of the circular portions of the orbit and (b) upon the frequencies of oscillation in the circular portions. These latter, the frequencies of the Z and of the radial motion, are proportional to $(n)^{\frac{1}{2}}$ and to $(1-n)^{\frac{1}{2}}$, respectively, where n is the coefficient determining the fall-off law for the magnetic field.

The calculation made by Serber was for a racetrack with two equal straight sections, and it was found that the proposed dimensions of the Michigan racetrack, while leading to stable orbits, were uncomfortably close to a region of instability.

We have reexamined the problem and have extended the formulas to include the case of N equal straight sections, each of length L, connected by N circular arcs of length $2\pi r/N$. The method of solution was as follows. Consider the mth passage of the electron through a circular portion of the orbit. Let the coordinate describing the particle motion within the tube be x and let the frequency of oscillation be ω. (x may be identified with either the Z or the radial displacement from equilibrium.)

$$x = A_m \sin\omega t + B_m \cos\omega t.$$

At the end of the mth circular portion of the orbit the electron enters the mth straight section. Throughout this section $\dot{x}$ remains constant but the particle has received a displacement of amount $L\dot{x}/v$ where v is its forward velocity. The particle now enters the $(m+1)$th circular portion and will again be described by

$$x = A_{m+1} \sin\omega t + B_{m+1} \cos\omega t.$$

Recursion formulas connecting A_{m+1} and B_{m+1} with A_m and B_m may be easily found and solved.

$$A_m = A \sin m\pi\mu + B \cos m\pi\mu,$$
$$B_m = C \sin m\pi\mu + D \cos m\pi\mu,$$

where A, B, C, and D are constants which are related through recursion formulas.

It is thus seen that the amplitudes vary sinosoidally and that the motion of the x coordinate approximates to that of a modulated wave. This statement would be rigorously correct if m increased uniformly with the time. Actually m is an integer and therefore discontinuous. However the orbit, which is in reality composed of short sinusoidal arcs connected by straight lines, will closely resemble a modulated sine

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[1] R. Serber, Phys. Rev. **70**, 434 (1946).

curve. The frequency of modulation may be determined through the quantity $\pi\mu$ and the equation which fixes its value also constitutes the stability condition, since $\cos\pi\mu$ must lie between ± 1. In agreement with Serber,[1] we find,

$$\cos\pi\mu = \cos\pi\nu - p\sin\pi\nu.$$

In this expression $\nu = 2n^{\frac{1}{2}}/N$ and $p = n^{\frac{1}{2}}L/d$ for the Z motion where d is twice the radius of curvature of the circular arcs. For the radial motion $\nu = 2(1-n)^{\frac{1}{2}}/N$ and $p = (1-n)^{\frac{1}{2}}L/d$. From our earlier study[2] it appears that the magnetic fall-off parameter n should lie in the range from 0.56 to 0.75. In the following calculations a value of $n = \frac{2}{3}$ has been used. For the number of straight sections, $N = 2$, 3, 4, 6, and 8, the ratio L/d must be less than 0.363, 1.06, 1.64, 2.69, and 3.69, respectively, if the resulting orbits are to be stable. Originally the design of the Michigan synchrotron called for a ratio $L/d = 0.30$ which is therefore just inside the range of stability for the case of $N = 2$. When the number of gaps is increased to 3 or more, however, it is clear that $L/d = 0.30$ is far from the instability region and indeed may be somewhat increased with safety.

A second way of viewing the phenomenon is to calculate the degree of modulation. This quantity, R, may be defined as the ratio of the greatest amplitude of oscillation which the particle achieves to its least amplitude. As L/d is decreased, R will increase and become infinite at the point of instability. A calculation yields the formula,

$$R^2 = (\sin\pi\nu + p\cos\pi\nu + p)/(\sin\pi\nu + p\cos\pi\nu - p).$$

In the following table R is listed for various values of N and for both the radial and Z motions. It has been assumed that $L/D = 0.30$ and $n = \frac{2}{3}$.

N	R (Z motion)	R (radial motion)
2	2.45	1.21
3	1.30	1.19
4	1.27	1.22
6	1.32	1.28
8	1.38	1.35

Since the final yield of high energy electrons will probably be greatest when R is the least, the logical choice for the number of gaps is 4, and the Michigan synchrotron will be so constructed.

A study has been made to show how the stability is affected by an inequality in the lengths of the straight sections. It was assumed that the racetrack contains two opposite sections of lengths L_1 and two sections of lengths L_2, each section being connected by a quadrant of circular arc. The stability condition becomes,

$$|2(\cos\pi\nu - p_1\sin\pi\nu)(\cos\pi\nu - p_2\sin\pi\nu) - 1| < 1,$$

where for the Z motion $\nu = \frac{1}{2}n^{\frac{1}{2}}$, $p_1 = L_1 n^{\frac{1}{2}}/d$ and $p_2 = L_2 n^{\frac{1}{2}}/d$. A new region of instability is thus revealed, namely when the values of p_1 and p_2 bracket $\cot\pi\nu$. It would appear easy to use dimensions for the 4-gap track so as to avoid this region. (Incidentally the value $p = \cot\pi\nu$ is just that point where instability sets in for the 2-gap racetrack.)

[2] D. M. Dennison and T. Berlin, Phys. Rev. 69, 542 (1946).