the pieces of badly cracked solid, and that uncracked pieces are super-conducting.

The potassium methylamine system has been reported[3] to separate into two liquid phases at concentrations higher than $0.01N$. The writer has observed this effect with methylamine to which several percent of ammonia has been added but not with pure methylamine. Rapid freezing of these solutions has yielded changes in resistance similar to those occurring in the sodium-ammonia system. When a $0.05N$ solution of potassium in ten to one methylamine-ammonia was rapidly chilled from 260°K to 90°K and then "healed" at 170°K (ten degrees below the melting point of the mixture), the resistance dropped from a value of 700 ohms at the highest temperature to a steady value of 0.3 ohm in the solid state. The solid possessed a lustrous, metallic, blue-gray appearance. When this solid was melted, two liquid phases, one blue and one bronze, resulted; the resistance of this mixture at a temperature just above the melting point was 2600 ohms.

Ogg has suggested an interpretation of the great decrease in resistance in the sodium-ammonia system in terms of a Bose-Einstein condensation of electron pairs. While the decrease for the above solutions occurs at a temperature well below the B-E condensation temperature for an ideal gas of density corresponding to the concentration of the solution, the discontinuity in resistance of the potassium methylamine-ammonia system occurs at a somewhat higher temperature than the corresponding B-E condensation temperature. For a solution $0.05N$ in potassium ($0.025N$ in electron pairs) the corresponding condensation temperature would be about 90°K, while the decrease in resistance occurs at about 180°K.

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On the Level Scheme of Mg^{24} and the Mass of Na^{24}

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SIEGBAHN has recently reported[1] the γ-ray spectrum associated with the disintegration of Na^{24} to consist of two lines, one at 1.38 Mev and the other at 2.76 Mev. In determining the corresponding level scheme of Mg^{24}, he assumes that these γ-transitions are in cascade. It has already been pointed out[2] that this assumption leads to a mass for Na^{24} which is undesirably high compared to the masses of similar nuclei in the same part of the periodic table. In particular, the mass of 23.99893 given by Siegbahn is about 1.5 Mev greater than that calculated by Barkas[3] for Na^{24}.

Although this is by no means certain evidence that the level assignment is wrong, it should be kept in mind that it is possible to construct a level scheme consistent with the data and giving a mass of Na^{24} which is in close agreement with the Barkas value. In this scheme,[2] it is assumed that the two levels of Mg^{24} are at 1.38 Mev and 2.76 Mev. Then the 1.38 γ-ray would actually consist of two cascading γ-rays of very nearly equal energy and the 2.76 radiation would be produced by a direct transition from the higher level to the ground state. This assumption is in good agreement with the results of Wilkins[4] and of Dicke and Marshall[5] on the inelastic scattering of protons by Mg. The level at 1.30 Mev obtained by Little, Long, and Mandeville[6] from the inelastic scattering of neutrons on Mg also agrees reasonably well with this scheme. The corresponding mass of Na^{24} would be 23.99745.

In this alternate scheme, the intensities of the radiations of different energies would not be expected to be equal so the application to the determination of the efficiency curve of G-M counters proposed by Siegbahn[1] would not be possible.

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Expanding Universe and the Origin of Elements

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IT is generally agreed at present that the relative abundances of various chemical elements were determined by physical conditions existing in the universe during the early stages of its expansion, when the temperature and density were sufficiently high to secure appreciable reaction-rates for the light as well as for the heavy nuclei.

In all the so-far published attempts in this direction the observed abundance-curve is supposed to represent some equilibrium state determined by nuclear binding energies at some very high temperature and density.[1-3] This point of view encounters, however, serious difficulties in the comparison with empirical facts. Indeed, since binding energy is, in a first approximation, a linear function of atomic weight, any such equilibrium theory would necessarily lead to a rapid exponential decrease of abundance through the entire natural sequence of elements. It is known, however, that whereas such a rapid decrease actually takes place for the first half of chemical elements, the abundance of heavier nuclei remains nearly constant.[4] Attempts have been made[2] to explain this discrepancy by the assumption that heavy elements were formed at higher temperatures, and that their abundances were already "frozen" when the adjustment of lighter elements was taking place. Such an explanation, however, can be easily ruled out if one remembers that at the temperatures and densities in question (about 10^{10}°K, and 10^6 g/cm^3) nuclear transformations are mostly caused by the processes of absorption and re-evaporation of free neutrons so that their rates are essentially the same for the light and for the heavy elements. Thus it appears that the only way of explaining the observed abundance-curve lies in the assumption of some kind of unequilibrium process taking place during a limited interval of time.

The above conclusion finds a strong support in the study of the expansion process itself. According to the general theory of expanding universe,[5] the time dependence of any linear dimension l in it is given by the formula

$$\frac{dl}{dt}=\left(\frac{8\pi G}{3}\rho l^2-\frac{C^2}{R^2}\right)^{\frac{1}{2}}, \tag{1}$$

where G is the Newton constant, ρ the mean density, and R (real or imaginary) a constant describing the curvature of space. It may be noticed that the above expression represents a relativistic analog of the familiar classical formula

$$v=\left(2\cdot\frac{4\pi l^3}{3}\rho\cdot\frac{G}{l}-2E\right)^{\frac{1}{2}} \tag{2}$$

for the inertial expansion-velocity of a gravitating dust sphere with the total energy E per unit mass. The imaginary and real values of R correspond to an unlimited expansion (in case of superescape velocity), and to the expansion which will be ultimately turned into a contraction by the forces of gravity (subescape velocity). To use some definite numbers, let us consider in the present state of the universe (considered as quite uniform) a cube containing, say, 1 g of matter. Since the present mean density of the universe is $\rho_{\text{present}}\cong 10^{-30}$ g/cm^3, the side of our cube will be: $l_{\text{present}}\cong 10^{10}$ cm. According to Hubble,[6] the present expansion-rate of the universe is 1.8×10^{-17} cm/sec. per cm, so that $(dl/dt)_{\text{present}}\cong 1.8\times 10^{-7}$ cm/sec. Substituting the numerical values in (1) we obtain

$$1.8\times 10^{-7}=(5.7\times 10^{-17}-C^2/R^2)^{\frac{1}{2}}, \tag{3}$$

showing that at the present stage of expansion the first term under the radical (corresponding to the potential energy of gravity) is negligibly small as compared with the second one. For the numerical value of the (constant) radius of curvature we get from (3): $R=1.7\times 10^{17}\sqrt{-1}$ cm or about 0.2 imaginary light year.

In the past history of the universe, when l was considerably smaller, and ρ correspondingly larger, the first term in (1) was playing an important role corresponding physically to the slowing-down effect of gravity on the original expansion. The transition from the slowed down to the free expansion took place at the epoch when the two terms were comparable, i.e., when l was about one thousandth of its present value. At this epoch the gravitational clustering of matter into stars, stellar clusters, and galaxies, probably must have taken place.[7]

Applying our formula (2) with $C^2/R^2=-3.3\times 10^{-14}$ to the earlier epoch when the average density of masses in the universe was of the order of 10^6 g/cm^3 (as required by the conditions for the formation of elements), we find that at that time $l\cong 10^{-2}$ cm, and $dl/dt\cong 0.01$ cm/sec. This means that *at the epoch when the mean density of the universe was of the order of 10^6 g/cm^3, the expansion must have been proceeding at such a high rate, that this high density was reduced by an order of magnitude in only about one second.* It goes without saying that one must be very careful in extrapolating the expansion formula to such an early epoch, but, on the other hand, this formula represents nothing more than the statement of the law of conservation of energy in the inertial expansion against the forces of gravity.

Returning to our problem of the formation of elements, we see that *the conditions necessary for rapid nuclear reactions were existing only for a very short time*, so that it may be quite dangerous to speak about an equilibrium-state which must have been established during this period. It is also interesting to notice that the calculated time-period during which rapid nuclear transformations could have taken place is considerably shorter than the β-decay period of free neutrons which is presumably of the order of magnitude of one hour. Thus if free neutrons were present in large quantities in the beginning of the expansion, the mean density and temperature of expanding matter must have dropped to comparatively low values *before* these neutrons had time to turn into protons. We can anticipate that neutrons forming this comparatively cold cloud were gradually coagulating into larger and larger neutral complexes which later turned into various atomic species by subsequent processes of β-emission. From this point of view the decrease of relative abundance along the natural sequence of elements must be understood as being caused by the longer time which was required for the formation of heavy neutronic complexes by the successive processes of radiative capture. The present high abundance of hydrogen must have resulted from the competition between the β-decay of original neutrons which was turning them into inactive protons, and the coagulation-process through which these neutrons were being incorporated into heavier nuclear units.

It is hoped that the further more detailed development of the ideas presented above will permit us to understand the observed abundance-curve of chemical elements giving at the same time valuable information concerning the early stages of the expanding universe.

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The Problem of Quantization of Higher Order Equations

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SUPPOSE there is given a function L depending on a field function $\psi(x)$ and its first and second derivatives:

$$L(\psi, \psi x_\mu, \psi x_\mu x_\nu). \tag{1}$$

This may be regarded as the generalized Lagrangian function. We may require, as usual

$$\delta\int L dx=0. \tag{2}$$

By introducing variations which vanish together with their derivatives on the surface S of the integration volume