

## Letters to the Editor

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### Neutron Diffraction and Nuclear Resonance Structure

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THE phenomena of neutron diffraction have been described and the present status of studies with monochromatic neutrons produced by this means have been given recently.<sup>1</sup> The first work in this laboratory in 1944 led to the characterization of the prominent resonance level in samarium. This level was found to obey the Breit-Wigner resonance formula within experimental error. Figure 1 shows the original data and the best fitting resonance curve. The deviation of some of the points at low energies are caused by higher order reflections.

Table I summarizes the characteristics of some of the resonances studied. The cross section  $\sigma_0$  given is the total cross section at resonance and includes contributions from resonance and potential scattering. Gamma is the half-width of the resonance at half-maximum. The integral under the resonance may be considered as an index of the importance of the resonance.

In the cases of indium and rhodium, the resolution of the instrument was not sufficient to evaluate the true constants of the resonance. The cross sections tabulated, therefore, represent lower limits, and the resonance widths represent upper limits.

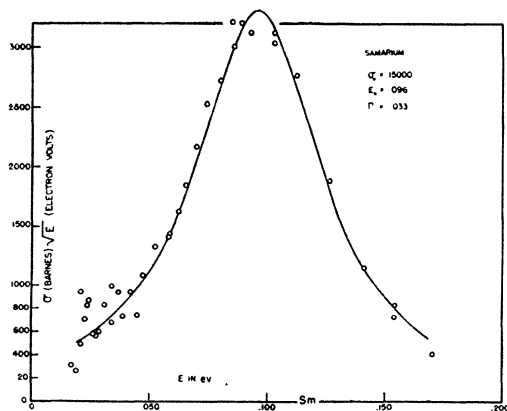


FIG. 1. Experimental results on the resonance scattering of neutrons on samarium. The solid line is the best fitting Breit-Wigner resonance curve.

TABLE I. Constants of neutron resonance levels.

Absorbing isotope	$E_0$	$\sigma_0 \times 10^4$	$\Gamma$	$\int \sigma E^{\frac{1}{2}} dE$
In <sup>115</sup>	1.39	>15,000	<0.15	8300
Rh <sup>103</sup>	1.30	>2,500	<0.13	1200
Sm <sup>149</sup>	.096	93,000	0.035	3200
Eu <sup>151</sup>	<.03	>3,000	?	?
Eu <sup>153</sup>	.54	20,000	0.075	3500
Gd <sup>157</sup>	.044	190,000	0.05	6300

The resonance in gadolinium and the low lying resonance in europium fall in a region where the second order of the Maxwellian continuum interferes with exact measurement. The resonance energies in these cases are approximations only.

The isotopic assignments in the cases of samarium and gadolinium are based upon the work of Lapp.<sup>2</sup> The assignments for europium are based upon the activations in the spectrometer of europium samples with monoenergetic neutrons of the exact resonance energies and subsequent identification of the induced radioactivity. Neutrons of 0.03 ev produced a 9-hr. period (Eu<sup>152</sup>,  $T_{\frac{1}{2}} = 9.4$  hr.) in an amount consistent with the measured flux and cross section. Neutrons of 0.54 ev produced a long-lived activity estimated from the flux and cross section to have a half-life of between 5 and 10 years (Eu<sup>154</sup>,  $T_{\frac{1}{2}} = 5-8$  yr.).

The resonance integral  $\int \sigma E^{\frac{1}{2}} dE$  appears to be a method of characterizing these neutron resonances in terms of relative importance. The range covered by these integrals is relatively large, so that the occurrence of three resonances (whose integrals differ only by a factor of two) in three isotopes differing by one proton and three neutrons is surprising. Further extension of this apparent series is prevented because of the instability of the nuclei involved.

<sup>1</sup> W. H. Zinn, Phys. Rev. **70**, 102A (1946); W. J. Sturm and S. Turkel, Phys. Rev. **70**, 103A (1946); E. Fermi and L. Marshall, Phys. Rev. **70**, 103A (1946); L. B. Borst, A. J. Ulrich, C. L. Osborne, and B. Hasbrouck, Phys. Rev. **70**, 108A (1946).

<sup>2</sup> R. E. Lapp, J. R. Van Horn, and A. J. Dempster, Phys. Rev. **70**, 104A (1946).

### The Liquid Drop Model for Nuclear Fission\*

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THE first extensive calculations on nuclear fission using the liquid drop model were made by Bohr and Wheeler.<sup>1</sup> The principal problem was to determine the potential energy surface, i.e., the potential energy as a function of the deformation parameters, and in particular the location of the saddle point giving the activation energy for fission. The deformation parameters were taken to be the coefficients  $a_n$  in the expansion of the radius vector in zonal harmonics (the surface of the drop is the locus  $r = R \sum a_n P_n(\cos \theta)$ ). Bohr and Wheeler calculated the surface and Coulomb energies of the deformed drop out to fourth-order terms in  $a_2$ , including the coupling terms with  $a_4$ . Even in the case of the uranium isotopes, however, large deformations are needed to reach the critical shape corresponding to the saddle point. The few terms considered

by Bohr and Wheeler are inadequate to treat such large deformations and while their activation energy curve is not justified by their calculations, it can be regarded as a reasonable interpolation between the computed results for small deformations and the experimental results for uranium. Further calculations were made by Present and Knipp<sup>2</sup> who extended the potential energy series to higher order terms in  $a_2$  and  $a_4$  and included the coupling terms between even and odd harmonics in order to see whether the observed asymmetry of fission could be explained in this way. The series was extended to terms as high as  $a_2^8$ ,  $a_2^4a_4$  and  $a_2^3a_4^2$  without yielding satisfactory convergence for the large values of  $a_2$  at which the saddle point of the energy surface is located in the case of uranium fission. The use of ellipsoidal coordinates was found to offer no appreciable advantage.

Since the values of  $a_2$  alone are large in the initial stages of fission (because of an accurate representation of the deforming drop by  $P_2(\cos \theta)$  alone), it was decided to carry out computations for fixed values of  $a_2$  expanding in terms of  $a_4$ . Separate calculations were made for  $a_2=0.3$  and 0.4 which are partly beyond the range of validity of the earlier power series calculation. The method broke down for larger values of  $a_2$  because of convergence difficulties with the Coulomb series for the coefficient of  $a_4^2$ . The results for the deformation  $a_2=0.4$  make possible a determination of the activation energy and the saddle point deformation parameters for a nucleus with  $x=0.80$  (where  $x$  represents half the ratio of the electrostatic energy of the undeformed drop to its surface energy). It will be recalled that if  $x \geq 1$  the drop is unstable against small second harmonic deformations and divides spontaneously. The new results which hold for  $x \geq 0.80$  can be extrapolated to the uranium isotopes ( $x \approx 0.75$ ) and indicate an activation energy of about 5 or 6 Mev in agreement with experiment. Coupling terms with the sixth harmonic were included in the calculation, and the effect of  $a_6$  was sufficiently small that  $a_8$  and higher even harmonics could be neglected.

The above calculations, with even harmonics only, correspond to a symmetric fission. The primary odd harmonics  $a_1$  and  $a_3$  were next introduced in all possible low coupling terms with  $a_2$  up to fifth order (e.g.,  $a_2^3a_1^2$ ,  $a_2^3a_1a_3$ ,  $a_2^3a_3^2$ ). The introduction of these terms raises the energy and their positive contribution increases with increasing  $a_2$ . The series ceases to be accurate at about  $a_2=0.3$  but the uncertainties do not become large enough to affect the sign of the energy until  $a_2 \geq 0.4$ . It is concluded that the critical shape of a nucleus with  $x \geq 0.80$  is symmetrical according to the liquid drop model. However, the possibility is not excluded that for some large deformation beyond the critical shape, the contribution of the odd harmonics may change sign and the fission path deviate from symmetry. Since the cases of uranium and plutonium fission lie so close to  $x=0.80$ , it is highly probable that the critical shape of these nuclei is symmetrical on the liquid drop model. This justifies the calculation of the preceding paragraph in which the activation energy is obtained from even harmonics alone.

An attempt has also been made to calculate the life-

time against spontaneous fission from the Gamow formula. The multiple integral over the coordinates of all nuclear particles is transformed into an integral with respect to the deformation parameters  $a_n$ . The Jacobian of this transformation is evaluated from the velocity potential for a streamline motion of the nuclear particles. The results are at variance with experiment in predicting too short a lifetime for  $U^{235}$ .

We hope to publish a fuller account of these calculations within the near future.

\* This work was begun in 1940 and completed in 1943. It was voluntarily withheld from publication until the end of the war.

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<sup>1</sup> N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).

<sup>2</sup> R. D. Present and J. K. Knipp, Phys. Rev. **57**, 751, 1188 (1940).

### Water Spectrum Near One-Centimeter Wave-Length

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BECKER and Autler<sup>1</sup> and Kyhl, Dicke, and Beringer<sup>2</sup> have measured the  $H_2O$  ( $5_{-1}-6_{-5}$ ) line at 1.35-cm wave-length for mixtures of water in air at atmospheric pressure. Under these conditions the line is many thousands of megacycles wide. Using a technique described earlier<sup>3</sup> of sweeping an oscillator in frequency across the line, this line has been detected and measured in pure water vapor at pressures near one-tenth mm Hg. It is a few megacycles wide at this pressure and its frequency may be measured with great accuracy.

Equal quantities of  $H_2O$  and  $D_2O$  were mixed in order to obtain a vapor with 50 percent HDO molecules, and to look for the several HDO lines predicted near 1-centimeter wave-length by Hainer, King, and Cross.<sup>4</sup> A search was made between 22,100 and 25,400 megacycles frequency. One line was found. Data on this and the  $H_2O$  line are given in Table I.

Becker and Autler did not measure the half-widths of the  $H_2O$  line in pure water vapor, but it may be deduced from their measurements in air at high and low water concentrations. Likewise the intensities quoted for them are not their actual measurements, but their intensities reduced to allow for the half-widths measured in pure vapor at low pressure. The half-widths and intensities given for HDO are for a mixture of 50 percent HDO, 25 percent  $D_2O$ , and 25 percent  $H_2O$ . Agreement between the data of Becker and Autler and measurements at low pressure ap-

TABLE I.

	Frequency (megacycles) Becker and Autler	Present measurement	Maximum intensity Becker and Autler	(nepers/cm) Present measurement
$H_2O$		22,237 $\pm$ 5		4 $\pm$ 2 $\times 10^{-6}$
HDO	22,320 $\pm$ 150	22,309 $\pm$ 5	5.7 $\times 10^{-6}$	15 $\pm$ 7 $\times 10^{-6}$
	Half-width at half- maximum (megacycles)	Pres- sure (mm Hg)	Half-width at half-maximum for 760 mm pressure Predicted from Becker and Autler	(megacycles) Predicted from present meas.
$H_2O$	1.45	0.103	11,600 $\pm$ 6000	10,700 $\pm$ 200
HDO	1.3	0.12		8,200 $\pm$ 1500