TABLE III

ϵ, erg	$\frac{1.00\times10^{-4}}{2\epsilon}-1$	coth $(302\epsilon^{\frac{1}{2}})$
23.6×10^{-6}	1.12	1.12^{-}
23.8×10^{-6}	1.11	1.11
24.0×10^{-6}	1.08	1.10

TABLE IV

ϵ, erg	$\frac{1.00\times10^{-4}}{2\epsilon}-1$	coth $(317\epsilon^{\frac{1}{2}})$
23.8×10^{-6}	1.11	1.10^{-}
24.0×10^{-6}	1.08	1.09
24.2×10^{-6}	1.07	1.09^{-}

and

$$D\Phi(r)=-6A\exp(K/3^{\frac{1}{2}}r)\ \text{for}\ r>r_0, \quad (11b)$$

where the constants have the same values as in Section II (a), Eq. (8) turns out to be

$$1.00\times10^{-4}/2\epsilon-1=\coth(302\epsilon^{\frac{1}{2}}),$$

and yields Table III. According to this table, the binding energy of He^4 is 23.8×10^{-6} erg, or 15.0 Mev.

(b) For the potential

$$D\Phi(r)=0 \quad \text{for}\ r\leq r_0=a/3^{\frac{1}{2}}, \quad (12a)$$

and

$$D\Phi(r)=-\frac{6B}{3^{\frac{1}{2}}r}\exp(K/3^{\frac{1}{2}}r) \quad \text{for}\ r>r_0, \quad (12b)$$

where the constants have the same values as in Section II (b), Eq. (8) becomes

$$1.00\times10^{-4}/2\epsilon-1=\coth(317\epsilon^{\frac{1}{2}}),$$

and yields Table IV. According to this table, the binding energy of He^4 is 24.0×10^{-6} erg, or 15.1 Mev.

The values of the binding energy of He^4 computed above are about 45 percent less than the experimental value 27.8 Mev. The results for He^4 are therefore not as good as those for H^3. This may mean that the method of the equivalent two-body tends to be a poor approximation as the number of particles increases.

In conclusion, the author wishes to express his thanks to Dr. K. C. Wang for helpful discussions.

PHYSICAL REVIEW VOLUME 70, NUMBERS 7 AND 8 OCTOBER 1 AND 15, 1946

Recurrence Phenomena in Small Cosmic-Ray Bursts*

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(Received July 11, 1946)

Chree's method of superposed epochs was employed in the statistical investigation of variations in the frequency of occurrence of small cosmic-ray bursts which produced 2.9 to 3.6 millions of pairs of ions in a shielded spherical ionization chamber of 13.3 liters effective volume containing air at 160 atmospheres. The data employed were obtained by Long and Whaley in the same investigation (during a little more than 18 months in 1938 and 1939) which supplied the data for the author's work on recurrences in variations of cosmic-ray intensity and their relation to geomagnetic and heliophysical activities. The analysis was carried out only for the range of day numbers from −45 to +45. Irregular secondary pulses were found both preceding and subsequent to both positive and negative primary pulses. Both subsequent and preceding difference curves and the combination difference curve displayed secondary pulses with peaks at about 27 days preceding and subsequent to the primary pulses. The secondary pulses amounted to about 3 or 4 percent of the average frequency of approximately 37 small bursts per day, and about 10 percent of the larger variations constituting the primary pulses.

THE data regarding bursts which are discussed in this paper were observed by Long[1] and Whaley[2,3] in the same investigation during eighteen months in 1938 and 1939, which yielded data used heretofore by the writer.[4,5]

* Presented at the meeting of the American Physical Society at St. Louis, November 30–December 1, 1945; Phys. Rev. 69, 46 (1946).

[1] V. A. Long, Ph.D. Thesis, University of Colorado, August 14, 1940.

[2] R. M. Whaley, M.A. Thesis, University of Colorado, June 3, 1940.

[3] V. A. Long and R. M. Whaley, Phys. Rev. 59, 470 (1941).

[4] J. W. Broxon, Phys. Rev. 59, 773 (1941).

[5] J. W. Broxon, Phys. Rev. 62, 508 (1942).

These data were obtained at an altitude of 5400 ft., latitude 40° N, longitude 105° 15′ W. After correction for bursts and barometric variations, the ionization currents produced by the cosmic radiation displayed an annual variation of about 1.3 percent amplitude with maximum in late January or February, almost diametrically out of phase with the annual variation of local outdoor temperature. Also, a preliminary analysis of one month's data by Long[1] indicated a diurnal variation of about 0.2 percent amplitude. The author's[4] analysis by Chree's method of superposed epochs showed recurrences in fluctuations of the (corrected) cosmic-ray intensity at intervals of about 28 days, with secondary pulses deviating some 0.2 percent from the mean. Further analysis[5] showed that the primary pulses in the intensity of the cosmic radiation were in general phase opposition to corresponding pulses in the terrestrial magnetic character and in sunspot areas. The peaks of the opposed pulses in magnetic character occurred one day prior to those of the primary pulses of the cosmic radiation, while the corresponding lead in the case of the pulses in sunspot area was three or four days.

Because of the sensitivity of the apparatus, Long and Whaley were able to distinguish cosmic-ray bursts which were appreciably smaller than those commonly observed with recording apparatus. After careful consideration of criteria for distinguishing between bursts and statistical fluctuations, they concluded that bursts corresponding to a deflection of 0.5 mm on the photographic record could be distinguished quite reliably. They concluded that there was only about 0.1 percent probability that a 1-mm burst (the smallest they read and considered in their study of magnitude-frequency relations) as selected by them might actually be a statistical fluctuation. A 1-mm burst corresponds to the formation of 2.9×10^6 pairs of ions in the chamber. The 13.8-liter chamber containing air at 160 atmospheres was spherical, with a concentric spherical collecting electrode (with 620 volts collecting P.D., compensated) of 9.5 cm diameter, leaving an effective volume of about 13.3 liters. This was shielded by 5 inches of lead and the stone building in which it was housed. Considering the dimensions of the chamber, and arbitrarily assuming a mean linear ion density of 100 pairs per cm of path of a cosmic-ray particle in air at N.T.P., Long[1] concluded that a 1-mm burst might correspond to as few as 65 cosmic-ray particles passing through the chamber.

For their investigation of magnitude-frequency relations, Long and Whaley arranged the bursts in groups or classes corresponding to 0.5 mm differences in the deflections produced on the photographic record, the first class comprising a narrower range than the others. Thus all bursts from 1.0 to 1.2 mm, inclusive, were classified as 1-mm bursts. The next group, corresponding to deflections from 1.3 to 1.7 mm, inclusive, were classified as 1.5-mm bursts, and so on for larger bursts. While the frequency of occurrence of large bursts (which extended to 56 mm) appeared to be entirely random in time, there were indications of both diurnal and annual variation in the frequency of the 1-mm bursts. Long[1] found the frequency of this class to be some 25 or 30 percent greater in the neighborhood of (local) noon than around midnight, on the average, for the first 12 months. Also, their average frequency during February and March was about 2.5 times as great as it was during July, August, and September of either year. In this connection, it may be mentioned that the monthly average frequency of occurrence of the 3-mm bursts (which were less frequent than the 1-mm bursts in the ratio of 1 to 58; the difference would be greater if the 3-mm class were restricted to the narrower range) varied somewhat more irregularly with the season, and was greatest in September of 1938 and August of 1939.

In view of the fact that the smallest bursts did not appear to occur entirely at random, but did show indications of diurnal and annual variations corresponding somewhat to those of the cosmic-ray intensity as indicated by the ionization current, it was decided to investigate the possibility that the method of Chree might indicate recurrences in the frequency of these small bursts. The high frequency of occurrence of the 1-mm bursts was also an item considered by the writer in deciding to apply the method to data extending over only 18 months. Burst data were used for precisely the same (Greenwich) days as in the investigations[4, 5] of the pulses in cosmic-ray intensity, with the exception of the last day, December 1, 1939. The frequencies designated represent the number of bursts per 24-hour day, corrections being applied for the days when data were not available for all 24 hours. When such correction was necessary, the nearest integral number was employed; no fractions were retained in designating the number of 1-mm bursts per 24-hour day. Also no correction factor was applied on account of the fact that the central system was grounded during two minutes of each hour. The extreme frequencies of occurrence of the 1-mm bursts were 5 and 115 per day. Both of these represented remarkably large departures from the mean, particularly the latter. The grand average of the corrected values for all days was 36.15 per 24-hour day for the 1-mm bursts. If further corrected for the two-minute grounding period during each hour, this would represent a mean frequency per day of 37.4 bursts in the range representing the formation of about 2.9 to 3.6 millions of pairs of ions in the chamber.

In classifying bursts and restricting attention to those within specified limits it is important that these limits be definite and constant. Changes of sensitivity of the apparatus could affect the boundaries between classes if corrections were not applied. Although the temperature of the apparatus varied only about 1°C during the year and a half of observation, some changes of sensitivity as great as 5 percent did occur. In classifying the bursts, corrections for variations of sensitivity were not made. In the case of the 1-mm bursts, it is estimated that observational errors of this order of magnitude were quite likely to occur. This is apparent if one considers the difficulty of distinguishing accurately among bursts at the lower limit of this narrow range, the finite width and distinctness of the photographic record as well as the time of collection of the ions being involved.

Although considerations of accuracy of observation made the undertaking far from promising in that random variations might be expected to outweigh possible regularities in the variation of frequency, it was decided to proceed with an investigation over a limited range of day numbers by Chree's method of superposed epochs. Accordingly, the five days with the greatest

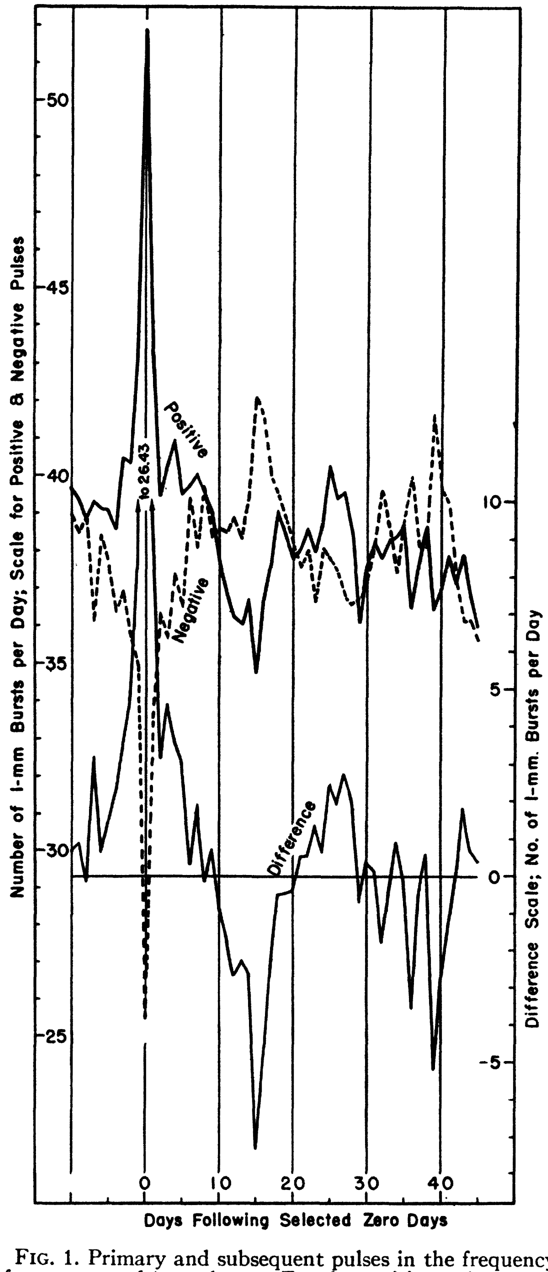

FIG. 1. Primary and subsequent pulses in the frequency of occurrence of 1-mm bursts. For the positive-pulse curve, the zero days are the five of greatest frequency of 1-mm bursts in each of the first fifteen months, June, 1938, through August, 1939. For the negative-pulse curve, the zero days are the five of least frequency of 1-mm bursts in each of the same months. The difference curve was obtained by subtracting ordinates of the negative-pulse curve from those of the positive-pulse curve for the same day numbers.

number of 1-mm bursts in each of the 18 complete months of the investigation were selected as the zero days for the positive pulses, while the

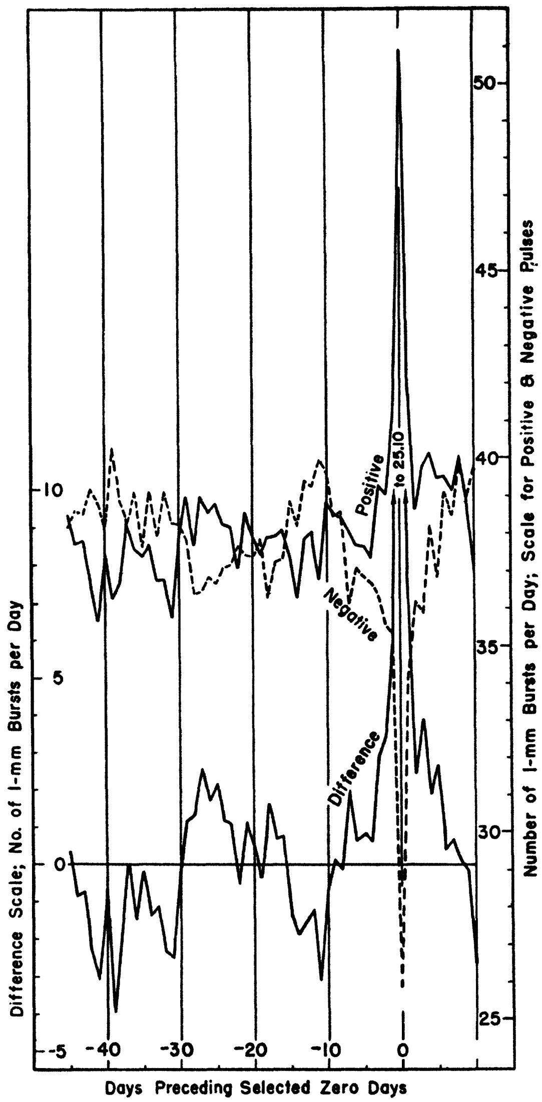

FIG. 2. Primary and previous pulses in the frequency of occurrence of 1-mm bursts. For these curves, zero days were selected from the last fifteen months, September, 1938, through November, 1939.

five with the smallest number of 1-mm bursts in each month were selected as the zero days for the negative pulses. When dealing with the large numbers of very small bursts, it might be supposed that, if they did not occur at random, then the frequency of occurrence would be greatest when the cosmic-ray intensity (as measured by the average ionization current) was greatest, and least when the intensity was least. If so, then one might suppose that upon investigating the four groups of 90 zero days each, there would be found a considerable degree of coincidence among the two groups selected for the positive pulses, and likewise among those selected for the negative pulses. On the other hand, one might suppose that a converse relation could exist, for the ionization currents had been corrected for bursts as well as for variations of barometric pressure, and the number of ions formed by thirty-seven 1-mm bursts exceeds 0.2 percent of the number of ions formed in the chamber in one day as represented by the average ionization current of 38.2 ions per cc per sec.; the secondary pulses in the ionization current as yielded by the Chree method amounted to about 0.2 percent of the average. Actually, 17 zero days selected for their large number of 1-mm bursts were identical with days selected on the basis of large average cosmic-ray ionization current and 22 zero days selected for their small number of 1-mm bursts were identical with those selected for their small average current. On the other hand, 9 zero days selected for their large number of 1-mm bursts were identical with those selected on the basis of small average current, and 13 zero days selected for their small number of 1-mm bursts were identical with those selected for their large average current. While the evidence appears slightly to favor the first of the two suppositions suggested above, there appears to be no considerable correlation between zero days selected upon the basis of large or small average currents, and those selected upon the basis of large or small number of 1-mm bursts.

Proceeding with the analysis by Chree's method as in the investigation of average ionization currents,[4] primary and subsequent pulse curves were constructed for the limited range of day numbers, $n=-10$ to $n=45$, for the first 15 months. These are shown in Fig. 1. For the solid line marked "positive," the zero days were the five with the greatest number of 1-mm bursts in each month from June, 1938, through September, 1939. For the broken line marked "negative," the zero days were the five with the least number of 1-mm bursts in each of the same months. As usual, the "difference" curve was obtained by subtracting ordinates of the "negative" curve from the corresponding ordinates of the "positive" curve.

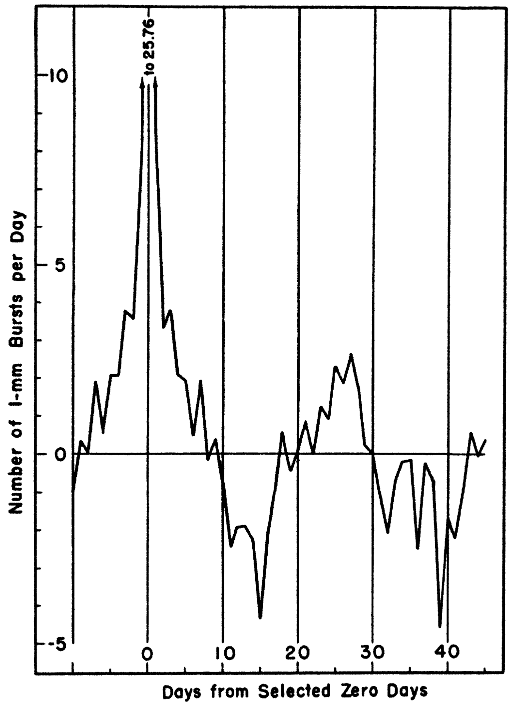

FIG. 3. Combination of difference pulses of Fig. 1 and Fig. 2. The ordinate for any day n is the average of the "difference" value for day number n in Fig. 1 and that for day number $-n$ in Fig. 2.

Though the curves are quite irregular, the "positive" and "negative" curves of Fig. 1 do display rather definite loops, indicating subsequent secondary pulses in both the positive and negative curves. The highest peak in the subsequent positive pulse is at day number $n=25$, though the shape of the pulse makes it somewhat difficult to decide where it is centered. The greatest dip in the subsequent negative pulse is at $n=28$. The subsequent difference pulse has its highest tip at $n=27$, though its shape might be taken as indicating that it is centered at a slightly smaller day number.

Figure 2 shows the corresponding curves for day numbers $n=-45$ to $n=10$ with zero days selected from the last 15 months, September, 1938, through November, 1939. These curves are also quite irregular, but again there are loops indicating preceding secondary pulses. The preceding positive pulse has a peak at $n=-27$, with another of the same height at $n=-29$. The shape of the pulse appears to lend weight to the former value. The preceding negative pulse has its greatest dip at $n=-28$, though the shape of the pulse indicates that it is centered at a smaller negative day number. The preceding difference pulse shows a rather definite peak at $n=-27$.

Figure 3 shows the combination difference curve formed from the difference curves of Figs. 1 and 2 in the usual manner. This also displays a secondary pulse of about the same magnitude and shape as the subsequent and preceding secondary pulses in the individual difference curves, with its highest peak at $n=27$. The magnitude indicates preceding and subsequent secondary positive and negative pulses amounting to some 3 to 4 percent of the average frequency for all days, and about 10 percent of the primary pulses.

While irregularities are perhaps somewhat greater than in the case of daily average ionization currents produced by the cosmic radiation, this investigation appears to the author to provide definite indication of recurrences of variations in the frequency of these small cosmic-ray bursts at intervals in the neighborhood of 27 days preceding and subsequent to the larger variations which determine the primary pulses. This is perhaps somewhat surprising in view of the experimental difficulties associated with the precise measurement of burst magnitudes, as indicated above. Perhaps observational errors are fairly well compensated in this statistical treatment of such a large number of bursts.