Eq. (53) assumes the form

$$\langle L_\alpha' L_\beta' \rangle \sim \operatorname{cosec}^2\theta \int\int_{x,y} y(x+y\cot\theta) \times \nabla^2\nabla^2 N(x,y,z)dxdy. \quad (61)$$

It can be shown that the expression which remains under the integral sign is changed but little if the circular sector is expanded into a semicircle. Thus, the expression (57) can be further simplified into

$$\langle L_\alpha' L_\beta' \rangle \sim \frac{1}{2\theta^2}\int\int_{x,y=-\infty}^{\infty} xy\nabla^2\nabla^2 N dxdy + \frac{1}{2\theta^3}\int\int_{x,y=-\infty}^{\infty} y^2\nabla^2\nabla^2 N dxdy. \quad (62)$$

Of these two terms, the second one is large compared with the first, so that the final expression is

$$\langle L_\alpha' L_\beta' \rangle \sim \frac{1}{2\theta^3}\int\int_{x,y=-\infty}^{\infty} y^2\nabla^2\nabla^2 N dxdy, \quad (63)$$

where y is the direction perpendicular to **R** in the plane of source and receivers.

To obtain the usual correlation coefficient, the expression (63) has to be divided through by (50). It is found that the correlation coefficient is independent of the distance between source and receivers and inversely proportional to the cube of the distance between the two receiving stations perpendicular to the line connecting the source with the location of the receivers, δ, as indicated in Fig. 2.

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A Calculation of the Binding Energies of H^3 and He^4 with a New Potential

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The binding energies of the nuclei H^3 and He^4 are calculated by the method of equivalent two-body, using the potential suggested by Wang. The range at which the potential between two nucleons is cut off is the same as that for the case of the deuteron, and the same range for the equivalent two-body is deduced accordingly. The binding energies thus calculated are 7.3 Mev and 15.1 Mev, respectively.

I. METHOD OF CALCULATION

THE nuclear potential proposed by Wang[1] was previously applied by the author to the calculation of the binding energy of the deuteron and of the neutron-proton scattering.[2] In the present work the binding energies of the nuclei H^3 and He^4 are computed by using the same potential in the method of the equivalent two-body.

Let $C\phi(r)$ be the potential between any two nucleons. If the Gaussian wave function $N\exp[-\frac{1}{2}\nu(r_{12}^2+r_{13}^2+r_{23}^2)]$ is used, the variation energy for H^3 is[3]

$$E(H^3) = \frac{9\nu\hbar^2}{2M} - 3C\frac{4}{\pi^{\frac{1}{2}}}\int_0^\infty \exp(-x^2) \times \phi[x/(3\nu/2)^{\frac{1}{2}}]x^2dx, \quad (1)$$

where

$$x = (3\nu/2)^{\frac{1}{2}}r_{12} \quad (\text{or } r_{13}, r_{23}),$$

and r_{12} is the distance between particles 1 and 2. For the nucleus He^4, taking $N\exp[-\frac{1}{2}\nu(r_{12}^2+r_{13}^2+r_{14}^2+r_{23}^2+r_{24}^2+r_{34}^2)]$ as the wave function, we have

$$E(He^4) = \frac{9\nu\hbar^2}{M} - 6C\frac{4}{\pi^{\frac{1}{2}}}\int_0^\infty \exp(-x^2) \times \phi[x/(2\nu)^{\frac{1}{2}}]x^2dx, \quad (2)$$

[1] K. C. Wang and H. L. Tsao, Phys. Rev. **66**, 155 (1944); Nature **155**, April 28 (1945).
[2] Mu-Hsien Wang, Phys. Rev. **66**, 103 (1944).
[3] William Rarita and R. D. Present, Phys. Rev. **51**, 788 (1937).

TABLE I

ϵ, erg	$\frac{5.00\times10^{-5}}{2\epsilon}-1$	coth $(370\epsilon^{\frac{1}{2}})$
10.8×10^{-6}	1.32	1.20⁻
11.5×10^{-6}	1.17	1.18
11.7×10^{-6}	1.14	1.17

TABLE II.

ϵ, erg	$\frac{5.00\times10^{-5}}{2\epsilon}-1$	coth $(388\epsilon^{\frac{1}{2}})$
11.5×10^{-6}	1.18	1.16
11.6×10^{-6}	1.16	1.15+
12.0×10^{-6}	1.08	1.15

where

$$x=(2\nu)^{\frac{1}{2}}r_{12} \quad (\text{or } r_{13}, r_{14}, \cdots).$$

Now, let $D\Phi(r)$ be the potential for the equivalent two-body problem. Using the wave function $N\exp[-\mu r^2/2]$, we get for the variation energy for the equivalent two-body:

$$E_{\text{eq}}=\frac{3\mu\hbar^2}{2M}-D\frac{4}{\pi^{\frac{1}{2}}}\int_0^\infty \exp(-x^2)\times\Phi(x/\mu^{\frac{1}{2}})x^2dx, \quad (3)$$

where

$$x=\mu^{\frac{1}{2}}r.$$

In order that $E(H^3)$, as given by (1), may equal E_{eq}, we should have

$$\mu=3\nu, \qquad D=3C, \qquad \Phi(r)=\phi(2^{\frac{1}{2}}r). \quad (4)$$

Similarly, to have $E(He^4)=E_{\text{eq}}$, we must set

$$3\mu/2=9\nu, \qquad D=6C, \qquad \Phi(r)=\phi(3^{\frac{1}{2}}r). \quad (5)$$

In our case, the potential is

$$C\phi(r)=V_0=\text{const.} \quad \text{for } r\leq a,$$

$$C\phi(r)=-Ae^{K/r} \quad \text{or} \quad -\frac{B}{r}e^{K/r} \quad \text{for } r>a,$$

where

$$r=r_{12} \quad (\text{or } r_{13}, r_{14}, \cdots),$$

and where a has the same meaning as in the deuteron; its value was found in reference 2. To determine the cut-off distance, r_0, to be used in the equivalent two-body method, we compare Eq. (3) in turn with Eqs. (1) and (2), and find that

$$r_0=a/2^{\frac{1}{2}} \text{ for } H^3, \quad (6)$$

and

$$r_0=a/3^{\frac{1}{2}} \text{ for } He^4. \quad (7)$$

If the zero cut-off method is employed, the potential for H^3 has therefore the form (9a, b) or the form (10a, b), and the potential for He^4 has the form (11a, b) or the form (12a, b).

The binding energy, ϵ, of H^3 and of He^4 is then the proper value of the wave equation

$$M\Delta\psi-\hbar^2(\epsilon+V)\psi=0,$$

where V is the potential. The condition for the smooth joint of the wave function at r_0 is[2]

$$|V(r_0)|/2\epsilon-1=\coth(\beta r_0), \quad (8)$$

where $\beta=(M\epsilon/\hbar^2)^{\frac{1}{2}}$.

II. THE BINDING ENERGY OF H^3

(a) For the potential

$$D\Phi(r)=0 \text{ for } r\leq r_0=a/2^{\frac{1}{2}}, \quad (9a)$$

and

$$D\Phi(r)=-3A\exp(K/2^{\frac{1}{2}}r) \text{ for } r>r_0, \quad (9b)$$

where[2] $A=4.78\times10^{-45}$, $K=3.84\times10^{-11}$ cm, $a=4.21\times10^{-13}$ cm, Eq. (8) becomes:

$$5.00\times10^{-5}/2\epsilon-1=\coth(370\epsilon^{\frac{1}{2}}).$$

The numerical results are given in Table I. The estimated binding energy of H^3 is therefore 11.5×10^{-6} erg, or 7.23 Mev.

(b) For the potential

$$D\Phi(r)=0 \quad \text{for } r\leq r_0=a/2^{\frac{1}{2}}, \quad (10a)$$

and

$$D\Phi(r)=-\frac{3B}{2^{\frac{1}{2}}r}\exp(K/2^{\frac{1}{2}}r) \quad \text{for } r>r_0, \quad (10b)$$

where[2] $a=4.42\times10^{-13}$ cm, $B=1.84\times10^{-55}$, we have

$$5.00\times10^{-5}/2\epsilon-1=\coth(388\epsilon^{\frac{1}{2}}).$$

The results are given in Table II. The estimated binding energy of H^3 is therefore 11.6×10^{-6} erg, or 7.30 Mev. These results are in good agreement with the experimental value 8.3 Mev.

III. THE BINDING ENERGY OF He^4

(a) For the potential

$$D\Phi(r)=0 \text{ for } r\leq r_0=a/3^{\frac{1}{2}}, \quad (11a)$$

TABLE III

ϵ, erg	$\frac{1.00\times10^{-4}}{2\epsilon}-1$	coth $(302\epsilon^{\frac{1}{2}})$
23.6×10^{-6}	1.12	1.12⁻
23.8×10^{-6}	1.11	1.11
24.0×10^{-6}	1.08	1.10

TABLE IV

ϵ, erg	$\frac{1.00\times10^{-4}}{2\epsilon}-1$	coth $(317\epsilon^{\frac{1}{2}})$
23.8×10^{-6}	1.11	1.10⁻
24.0×10^{-6}	1.08	1.09
24.2×10^{-6}	1.07	1.09⁻

and

$$D\Phi(r)=-6A\exp(K/3^{\frac{1}{2}}r) \text{ for } r>r_0, \quad (11b)$$

where the constants have the same values as in Section II (a), Eq. (8) turns out to be

$$1.00\times10^{-4}/2\epsilon-1=\coth(302\epsilon^{\frac{1}{2}}),$$

and yields Table III. According to this table, the binding energy of He^4 is 23.8×10^{-6} erg, or 15.0 Mev.

(b) For the potential

$$D\Phi(r)=0 \quad \text{for } r\leq r_0=a/3^{\frac{1}{2}}, \quad (12a)$$

and

$$D\Phi(r)=-\frac{6B}{3^{\frac{1}{2}}r}\exp(K/3^{\frac{1}{2}}r) \quad \text{for } r>r_0, \quad (12b)$$

where the constants have the same values as in Section II (b), Eq. (8) becomes

$$1.00\times10^{-4}/2\epsilon-1=\coth(317\epsilon^{\frac{1}{2}}),$$

and yields Table IV. According to this table, the binding energy of He^4 is 24.0×10^{-6} erg, or 15.1 Mev.

The values of the binding energy of He^4 computed above are about 45 percent less than the experimental value 27.8 Mev. The results for He^4 are therefore not as good as those for H^3. This may mean that the method of the equivalent two-body tends to be a poor approximation as the number of particles increases.

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Recurrence Phenomena in Small Cosmic-Ray Bursts*

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Chree's method of superposed epochs was employed in the statistical investigation of variations in the frequency of occurrence of small cosmic-ray bursts which produced 2.9 to 3.6 millions of pairs of ions in a shielded spherical ionization chamber of 13.3 liters effective volume containing air at 160 atmospheres. The data employed were obtained by Long and Whaley in the same investigation (during a little more than 18 months in 1938 and 1939) which supplied the data for the author's work on recurrences in variations of cosmic-ray intensity and their relation to geomagnetic and heliophysical activities. The analysis was carried out only for the range of day numbers from −45 to +45. Irregular secondary pulses were found both preceding and subsequent to both positive and negative primary pulses. Both subsequent and preceding difference curves and the combination difference curve displayed secondary pulses with peaks at about 27 days preceding and subsequent to the primary pulses. The secondary pulses amounted to about 3 or 4 percent of the average frequency of approximately 37 small bursts per day, and about 10 percent of the larger variations constituting the primary pulses.

THE data regarding bursts which are discussed in this paper were observed by Long[1] and Whaley[2,3] in the same investigation during eighteen months in 1938 and 1939, which yielded data used heretofore by the writer.[4,5]

* Presented at the meeting of the American Physical Society at St. Louis, November 30–December 1, 1945; Phys. Rev. 69, 46 (1946).

[1] V. A. Long, Ph.D. Thesis, University of Colorado, August 14, 1940.

[2] R. M. Whaley, M.A. Thesis, University of Colorado, June 3, 1940.

[3] V. A. Long and R. M. Whaley, Phys. Rev. 59, 470 (1941).

[4] J. W. Broxon, Phys. Rev. 59, 773 (1941).

[5] J. W. Broxon, Phys. Rev. 62, 508 (1942).