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Initiation of Discharge in Arcs of the Thyratron Type

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When a grid overvoltage is applied to a thyratron, the resulting electron current produces positive ions which move back toward the cathode of the tube. Some of these ions are "trapped" in the potential well formed near the cathode by the electron space charge. The "trapped" positive ions diminish the depth of the potential well and thus permit a larger current to be emitted from the cathode. When the number of positive ions "trapped" in the potential well becomes sufficient to decrease the depth of the well to zero, the saturation value of the cathode current is emitted, and the initiation of the discharge is completed. From this model of the initiation of the discharge, an equation for the anode current as a function of time during the initiation of the discharge has been derived. This equation yields current *versus* time curves which are in good agreement with experiment. From this equation, estimates of the ionization time can be made. The calculations give ionization times of the correct order of magnitude. The calculated ionization times show the experimentally observed dependence upon the grid overvoltage, gas pressure, and anode voltage.

I. INTRODUCTION

ALTHOUGH a rather detailed study has been made of the processes involved in the deionization of thyratrons, relatively little work yielding information about the physical phenomena involved in the initiation of the discharge in thyratrons has been done. The ionizing time is of the order of microseconds; hence, until recently it was usually safe to assume that the thyratron discharge initiated instantaneously after the application of the triggering voltage to the grid. However, with the advent of "microsecond techniques," circuits containing thyratrons are called upon to generate or analyze pulses whose durations are comparable to the ionizing time of the thyratrons. Thus in many applications the ionization time places a greater limitation on the use of the thyratron than does the "dead" (deionization) time. For this reason a more detailed study of the mechanism involved in the initiation of the thyratron discharge has been made in this paper.

II. SUMMARY OF EXPERIMENTAL DATA ON IONIZATION TIME

Experimental studies of the initiation of the thyratron discharge have been made by several workers. Wheatcroft and co-workers[1] have studied the "pre-striking" characteristics; Harrison[2] and Snoddy[3] have used high speed oscilloscopes to observe the current build-up. Very recently Germeshausen[4] has made a study of the initiation of discharge in hydrogen-filled thyratrons. The results obtained by these experimenters may be summarized as follows:

1. The current (or voltage) *versus* time curves for the initiation of the discharge are of the type shown in Fig. 1.[5] After application of the grid overvoltage (at $t=0$) there is little increase in current for a time t_1, which may be called the

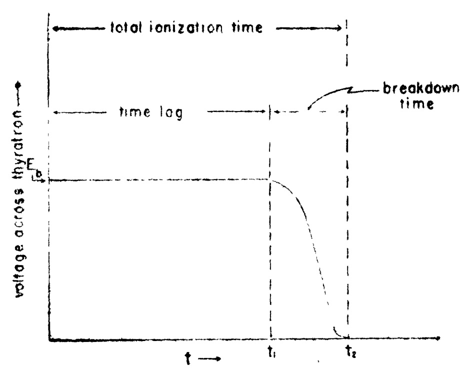

FIG. 1. Typical curve for voltage drop across thyratron as a function of time after application of grid overvoltage (at $t=0$). E_b is the power supply voltage.

[1] E. T. Wheatcroft, R. B. Smith, and J. Metcalfe, Phil. Mag. 25, 649 (1938). E. T. Wheatcroft and T. G. Hammerton, Phil. Mag. 26, 684 (1938).

[2] A. E. Harrison, Trans. A.I.E.E. 59, 747 (1940).

[3] L. B. Snoddy, Physics 4, 366 (1933).

[4] K. J. Germeshausen, private communication. The reports on the hydrogen thyratron results are available in the combined Radiation Laboratory Reports.

[5] If the power supply voltage is E_b and a series resistor R_p is used in the plate circuit of the thyratron, then it is evident that the voltage drop, V_t, across the thyratron is related to the anode current, i, of the thyratron by the equation: $V_t=E_b-iR_p$.

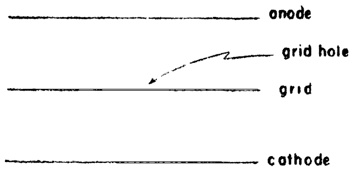

FIG. 2. Model of thyratron assumed for derivation of the equation describing the initiation of discharge. The anode, grid, and cathode are assumed to be parallel plane electrodes.

time lag or waiting time. Then between t_1 and t_2 the current builds up very rapidly. The time t_2-t_1 is called the breakdown time.[6] For small and moderate grid overvoltages, the time lag (t_1) is usually much greater than the breakdown time (t_2-t_1). For large grid-overvoltages the time lag becomes approximately equal to, or even less than, the breakdown time.

2. Ionization times varying from a few hundredths of a microsecond up to several hundred microseconds have been observed.

3. The time lag (t_1 in Fig. 1) is inversely proportional to the grid overvoltage.

4. The breakdown time (t_2-t_1) is decreased by increasing the anode voltage. Since for small and moderate grid overvoltage the time lag (which is not a function of anode voltage) makes up most of the total ionization time, the total ionization time is not greatly dependent upon the anode voltage except when large grid overvoltages are used.

5. The ionization time is decreased as the gas pressure is increased. The pressure was varied within rather narrow limits, and no quantitative study of ionization time as a function of gas pressure has been made.

6. The ionization time seems to be decreased by decreasing the dimensions of the tube.

III. MECHANISM OF THE INITIATION OF THE DISCHARGE

It will be assumed that the electrodes of the thyratron may be treated as parallel planes as shown in Fig. 2. If the thyratron is of the positive grid type, it will be assumed that the anode potential produces very little grid-cathode field. This assumption is valid because of the good shielding in these tubes. If the thyratron is of the negative grid type (requiring a negative grid bias to prevent firing when the anode potential is applied), it will be assumed that the grid-cathode field, plotted along a line passing from cathode to anode through a grid hole, caused by the application of the anode potential and the negative grid bias potential, is zero when the grid bias is adjusted to the cut-off value (the cut-off grid voltage is the minimum bias voltage which prevents firing for a given anode voltage).

With these assumptions the potential distribution (plotted along a line joining cathode and anode and passing through a grid hole) across the tube at the instant a grid overvoltage V_g is applied is of the form shown in Fig. 3. The potential minimum, V_m, which occurs at a distance X_m from the cathode, is due to electron space charge.[7]

The applied grid overvoltage, V_g, causes an initial electron current i_0 to flow across the tube. This current is practically the same as that which would be set up in a vacuum tube under the same conditions. The electrons ionize the gas atoms in the grid-anode region. The resulting positive ions move back toward the cathode region. A certain fraction of these ions will suffer collisions with gas atoms in or near the potential well centered at X_m. These ions are "trapped" in the potential well and neutralize some of the electron space charge; this reduces the height of

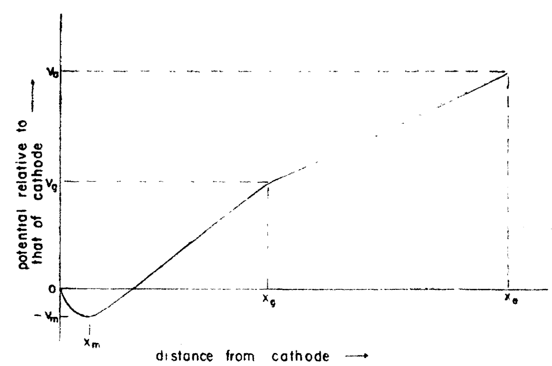

FIG. 3. Potential distribution (along a line joining cathode and anode and passing through a grid hole) across tube immediately after the application of a grid overvoltage, V_g. X_g and X_a are the distances of grid and anode from the cathode, respectively; V_a is the anode voltage. The potential minimum, $-V_m$, occurring at X_m, is caused by electron space charge.

[6] Excellent photographs of the decay of voltage across thyratrons are given, for example, by Harrison, reference 2.

[7] Cf., for example, K. K. Darrow, *Electrical Phenomena in Gases* (Williams and Wilkins, Baltimore, 1932), p. 323.

the potential barrier surrounding the cathode and thereby permits a greater electron current to flow from the cathode. The time required for the initiation of the discharge is the time required for the positive ions to accumulate in the "well" in sufficient numbers to completely remove the potential barrier. Klemperer[8] has calculated the ionization time on the assumption that the entire ionization process takes place in a single cycle, i.e., that the initial electron space charge cloud which is pulled into the grid-anode region when the grid over-voltage is applied produces sufficient positive ions to annihilate the potential barrier. The resulting ionization times agree with those observed experimentally when very large grid overvoltages are used. Thus it would seem that the initiation of the discharge may be completed in a single ion-cycle if very large grid overvoltages are used. However, for moderate and small grid overvoltages, many cycles are required for the discharge to develop. Engel and Steenbeck[9] have given a discussion of the initiation of the discharge when more than one cycle is required; however, these workers do not give results showing the rate at which current builds up or the dependence of ionization time on grid overvoltage.

IV. DERIVATION OF THE EQUATION DESCRIBING THE INIATION OF THE DISCHARGE

Suppose that at time t the positive ions have reduced the depth of the potential well from its original depth V_m by an amount V_+. Then the electron current density is given by

$$i = i_s \exp\left[-\frac{e}{kT}(V_m - V_+)\right], \tag{1}$$

where i_s is the saturation current density. (k = Boltzmann constant.)

The dependence of V_+ upon the number of trapped positive ions may be determined as follows: the positive ions are trapped at a distance X_m from the cathode and may be assumed to be distributed over an area approximately equal to the area, A, of the grid hole through

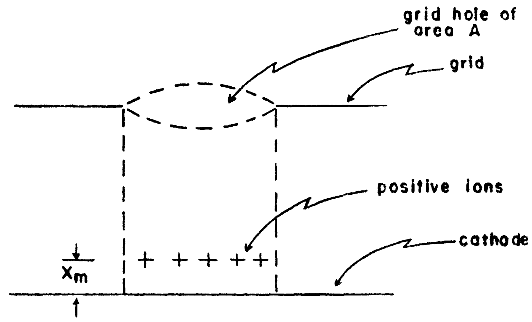

FIG. 4. The layer of positive ions and the cathode form a small parallel plate condenser of area A and thickness X_m.

which they enter the grid cathode region (Fig. 4).[10] Thus a small parallel plate condenser of area A and thickness X_m is formed. If C is the capacitance of this condenser, and Q is the total positive ion charge trapped in the potential well, then

$$V_+ = Q/C, \tag{2}$$

and

$$i = i_s \exp\left[-\frac{e}{kT}(V_m - Q/C)\right]. \tag{3}$$

The value of X_m is slightly dependent upon V_g; however, the dependence is rather small. For grid voltages in the range 0–25 volts the average value of X_m is about 0.5 mm.[11] Using this value of X_m and taking the area of the grid holes as about 1 cm^2

$$C = \frac{A}{4\pi X_m} \cdot \frac{1}{9\times 10^{11}} = 1.8\times 10^{-12}\ \text{farad}. \tag{4}$$

The value of Q may be calculated as follows: the electron current, i, traversing the grid-anode region sets up a positive ion current flowing back toward the cathode. The electron mean free path is of the order of the tube dimensions. If t_+ is the mean time for positive ions to return to the cathode, then the positive ion current in the cathode region is

$$i_+(t) = \alpha i(t - t_+), \tag{5}$$

where α is a proportionality constant which is

[8] Hans Klemperer, Archiv f. Electrotechnik, 27, 322 (1933).

[9] A. V. Engel and M. Steenbeck, *Elektrische Gasentladungen* (Verlagsbuchhandlung Julius Springer, Berlin, 1934), Vol. 2, pp. 190–195.

[10] Since the thermal velocities of the "trapped" ions are relatively small, they may be regarded as being at rest.

[11] Values of X_m as a function of voltage have been calculated by T. C. Fry, Phys. Rev. 17, 441 (1921). A table of some of the results obtained by Fry is given by K. K. Darrow, reference 7, p. 329.

proportional to the fraction of electrons which produce positive ions which get back into the grid-cathode region. If a fraction, β, of these positive ions are scattered into the potential well centered at X_m, then

$$Q=\beta\int_0^t i_+dt, \tag{6}$$

and

$$i=i_s \exp\left[-\frac{e}{kT}\left(V_m-\beta/C\int_0^t i_+dt\right)\right];$$

or

$$\ln i/i_s=-\frac{eV_m}{kT}+\frac{e\beta}{kTC}\int_0^t i_+dt. \tag{7}$$

Differentiating,

$$\frac{1}{i}\frac{di}{dt}=\frac{e\beta}{kTC}i_+. \tag{8}$$

But from (5)

$$i_+=\alpha i(t-t_+)=\alpha i(t)-\alpha t_+di/dt. \tag{9}$$

Therefore

$$\frac{1}{i}\frac{di}{dt}=\frac{e}{kT}\frac{\alpha\beta}{C}i-\frac{e}{kT}\frac{\alpha\beta}{C}t_+\frac{di}{dt}, \tag{10}$$

or

$$\frac{kTC}{e\alpha\beta}\frac{di}{i^2}+t_+\frac{di}{i}=dt.$$

Integrating,

$$\frac{kTC}{e\alpha\beta}\left(\frac{1}{i_0}-\frac{1}{i}\right)+t_+\ln\frac{i}{i_0}=t. \tag{11}$$

For *very small* grid overvoltages $i_0 \propto Vg^{\frac{3}{2}}$; however, for grid overvoltages of practical interest $i_0 \propto V_g$. Thus, $i_0=aV_g$, where the value of a should be approximately equal to the value of the constant occurring in the law for space charge limited emission; thus

$$i_0\cong 2.5\times 10^{-6}V_g \text{ amp. cm}^{-2}. \tag{12}$$

Thus Eq. (11) may be re-written in the form

$$t=\frac{kTC}{e\alpha\beta}\left(\frac{1}{aV_g}-\frac{1}{i}\right)+t_+\ln\frac{i}{i_0},$$

or

$$t=\frac{kTC}{e\alpha\beta}\frac{1}{aV_g}\left(1-\frac{1}{i/i_0}\right)+t_+\ln\frac{i}{i_0}. \tag{13}$$

Equation (13) describes the initiation of the discharge.[12] It will be assumed that the cathode has an emitting surface of about 1 cm². Then i in Eq. (13) represents the anode current.

Inspection of Eq. (13) leads to the following conclusions:

1. The current increases very slowly with time until

$$t=(kTC/e\alpha\beta)(1/aV_g);$$

thereafter, the current increases exponentially with a time constant t_+. Thus Eq. (13) can be used to explain the observed shape (Fig. 1) of the current *versus* time curves.

2. The time-lag (t_1 of Fig. 1) is given by

$$t_1=(kTC/e\alpha\beta)(1/aV_g)$$

and is thus inversely proportional to the grid overvoltage.

3. Since the factor α increases with increasing pressure, the ionization time should decrease as the pressure is increased.

4. The quantity t_+ (mean time for ions to return to cathode) is inversely proportional to the square root of the anode voltage. Thus the breakdown time (t_2-t_1 in Fig. 1) should be decreased by increasing the anode voltage.

All these conclusions are in agreement with experiment.

V. NUMERICAL CALCULATIONS

Equation (13) may be used to estimate the ionization times and the rate of current build up for thyratrons. The values of some of the quantities occurring in the equation have been given in the above sections. These are

$$C=1.8\times 10^{-12} \text{ farad},$$
$$a=2.5\times 10^{-6} \text{ amp. volt}^{-1}.$$

The value of β (fraction of positive ions moving into grid-cathode region which are scattered into the potential well) should be given approxi-

[12] It may be noted that according to Eq. (13) $i\rightarrow\infty$ as $t\rightarrow\infty$. This situation arises from the fact that Eq. (13) describes only the *initiation of the discharge* and not, of course, the *equilibrium state* of the discharge. Certain additional processes involved in the equilibrium state become of importance as the current approaches the steady state value. These processes, which have not been considered here, prevent the current from increasing indefinitely with time.

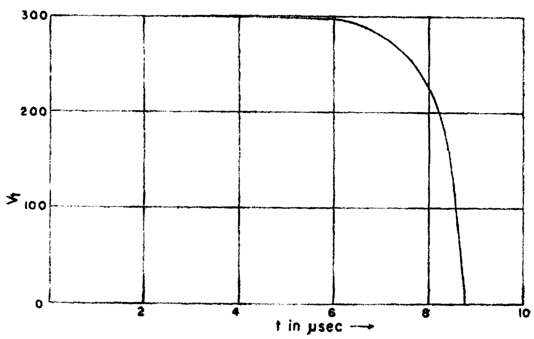

FIG. 5. Voltage across thyratron as a function of time as plotted from Eqs. (16) and (17) with $V_g=10$ volts, $E_b=300$ volts, and $R_p=10{,}000$ ohms. The grid overvoltage is applied at $t=0$.

mately by

$$\beta=\frac{\text{length of potential well}}{\text{grid cathode spacing}}\cong\frac{2X_m}{1}=0.1,$$

and taking $\alpha=1/100$, $T=800°\mathrm{K}$

$$t=\frac{50\times 10^{-6}}{V_g}\left(1-\frac{1}{i/i_0}\right)+t_+\ln\frac{i}{i_0},\qquad(14)$$

with V_g in volts, i in amperes, and t in seconds. t_+ is the mean time for ions to return to the cathode; hence

$$t_+\cong\frac{\lambda_-+l}{\bar{v}_+},$$

where λ_- is the electron mean free path, l is the grid-cathode distance, and $\bar{v}_+$ is the average positive ion velocity. Thus

$$t_+\cong\frac{1}{\bar{v}_+}=\frac{1}{\frac{1}{2}\left(\frac{2eV_a}{m_+}\frac{\lambda_+}{d}\right)^{\frac{1}{2}}},\qquad(15)$$

where

λ_+ = mean free path for positive ions,
m_+ = atomic weight of ion (grams),
V_a = anode voltage,
d = anode-grid spacing,

taking

$$\frac{\lambda_+}{d}\cong 1,\quad t_+\cong 1\bigg/\left(\frac{10^{12}}{2A}V_a\right)^{\frac{1}{2}}$$

where A = mass number of ion, and V_a = anode voltage in volts. For argon, $A=40$, and taking $V_a=300$ volts,

$$t_+\cong 0.5\times 10^{-6}\text{ seconds.}$$

$$t=\frac{50\times 10^{-6}}{V_g}\left(1-\frac{1}{i/i_0}\right)+0.5\times 10^{-6}\ln\frac{i}{i_0}\text{ seconds,}$$

or

$$t=\frac{50}{V_g}\left(1-\frac{1}{i/i_0}\right)+0.5\ln\frac{i}{i_0}\text{ microseconds.}\qquad(16)$$

In the experiments, the oscilloscope measurements do not give the thyratron current directly. Rather, the decay of the voltage drop across the thyratron is observed. The thyratron current, i, and the voltage drop across the thyratron, V_t, are related by the equation,[5]

$$V_t=E_b-iR_p.\qquad(17)$$

The value of i may be obtained from Eq. (16). The decay of the voltage across the thyratron may thus be obtained from Eqs. (16) and (17). The voltage *versus* time curve obtained from these equations with $V_g=10$ volts, and R_p = 10,000 ohms, is plotted in Fig. 5.