The Effect of Ultrasonic Waves on the Conductivity of Salt Solutions

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An adiabatic compression raises the conductivity of an aqueous salt solution because of direct pressure influence and of temperature increase. An ultrasonic wave therefore modifies periodically the conductivity of the solution. If a filament of current passes normal to the wave propagation, the wave train produces under proper conditions an alternating potential, which can be picked up. This gives per atmosphere pressure and volt applied about 220 microvolts in $CuSO_4$. (At wave-lengths shorter than the cross dimension of the current, the effect decreases.) This permits the construction of a convenient receiver indicating absolute intensity for the investigation of ultrasonic fields in water, provided the frequency is not above 1.5 megacycles.

I. INTRODUCTION

TO measure the behavior of shock waves in water one uses at present crystal gauges, usually tourmaline[1] or quartz.

This performance of these gauges rests on the elastic and piezoelectric properties of the crystal and is therefore directly connected with the existence of a characteristic frequency of the gauge.

It seemed desirable to have, for the purpose of checking the results of the usual gauges, one based on an entirely different principle; if possible, this gauge should not have a characteristic frequency and its acoustic impedance should be as close as possible to that of water, to avoid reflection of the wave.

It occurred to us that the resistance of an electrolytic solution should vary with the pressure and therefore the measurements of this resistance provide a means of investigating shock waves with the help of a medium which has elastic properties very close to that of the surrounding water.

It was decided first to investigate the physics of the phenomenon, using ultrasonic waves instead of shock waves. A contract N171-s-57120 was made between the David Taylor Model Basin, Bureau of Ships, Navy Department, and the Catholic University of America, and a report on the work was submitted on September 26, 1943. The present paper contains the main results of this report.

It was decided to investigate a monovalent and a divalent salt solution. For the first NaCl solution with a composition near that of sea water was chosen, for the second $CuSO_4$, because it was easy to avoid here polarization at the electrode. Accordingly, the following two solutions were investigated:

(A) One containing about 59 g of $CuSO_4 \cdot 5H_2O$ per liter (about 0.47 equivalent),

(B) One containing 35 g of NaCl per liter (0.6 normal).

II. THE EXPECTED RESULTS

(1) Pressure in a Sound Wave

If ρ is the density of the liquid, V the sound velocity, κ the compressibility, p the effective pressure, all in absolute units, and F the energy flow per square cm in watts, one has

$$F = V2 \cdot \tfrac{1}{2}\kappa p^2 \times 10^{-7} \qquad (1)$$

or eliminating κ by

$$V^2 = 1/\rho\kappa,$$

$$F = (1/\rho V)p^2 \times 10^{-7}, \qquad (2)$$

$$p = (\rho V F \cdot 10^7)^{\frac{1}{2}}.$$

If we change to atmospheres, use $\rho = 1.02$, $V = 1.538 \times 10^5$ cm/sec., and neglect, between 0° and 30°C, the variation of these quantities with the temperature, one finds

$$p_{\text{atmos}} = 1.24(F_{\text{watts}})^{\frac{1}{2}}. \qquad (3)$$

(2) The Expected Magnitude of the Effect

Static measurements of the effect of pressure on the conductivity of salt solutions have been

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[1] M. F. M. Osborne and S. D. Hart, J. Acous. Soc. Am. 17, 1 (1945).

made by Koerber.[2] In the range of pressures below 1000 atmospheres, he finds an increase in conductivity proportional to the pressure. A maximum occurs near 1200–1500 atmospheres.

Koerber has found that for simple, monovalent salts like sodium and potassium chloride, the effect is nearly independent of the concentration of the solution and alike for the two salts.

At 19.18°C, the change of resistance between 0 and 500 atmospheres is found to be 3.7 percent, which means that the relative conductivity increase is 74×10^{-6} per atmosphere. This value agrees well with a calculation founded on the assumption that two effects contribute to the increase of conductivity: The decrease of viscosity with pressure,[3] and the increase of the concentration, due to compression.

At 0.01°C, only measurements for 0.01 n-potassium chloride solutions have been made. They give a relative increase in conductivity of 98×10^{-6} per atmosphere.

Koerber did not measure copper sulphate, but he measured zinc sulphate, which behaves electrolytically very similarly to copper sulphate. Therefore, it seems reasonable to assume that the dependence of conductivity on pressure will be similar for the two salts.

For $ZnSO_4$ the effect is found to be dependent on the concentration, contrary to the behavior of sodium chloride, and to be larger than in the former case.

This can be explained on the basis that the apparent dissociation of divalent salts like copper sulphate and zinc sulphate is not complete and is increased by pressure.

The increase of conductivity for 0.2 normal zinc sulphate is, at 19.18°C, 10 percent for 500 atmospheres, or 200×10^{-6} atmosphere.[4] No measurements exist near 0°C.

These measurements are for a static application of pressure. In a sound wave, or shock wave, pressure rise is accompanied above 4°C by a temperature rise, which diminishes the viscosity and increases the conductivity.

The increase ΔT in temperature is connected with the increase Δp in pressure by the following

formula.[5]

$$\left(\frac{\Delta T}{\Delta p}\right)_s = \frac{T}{C_p}\left(\frac{\partial V}{\partial T}\right)_p = \frac{TV}{C_p}\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p. \quad (4)$$

Here, $(\partial V/V\partial T)_p$ is the usual coefficient of heat expansion, and C_p/V the specific heat of unit volume.

One finds in Table I results for the temperature increase per atmosphere.

TABLE I. Calculated increase of temperature in water per atmosphere adiabatic pressure increase, and relative increase in conductivity.

$t =$	0	2.5°	4°	5.5°	10.5°	15.5°	20.5°
$\Delta t°C \times 10^3$	−0.38	−0.16	0	0.16	0.64	1.1	1.5
relative conductivity increase $\times 10^6$	−13	−5	0	5	18	28	37

These values are applicable to both sodium chloride and copper sulphate. One should therefore expect, at 20°C, an increase of conductivity per atmosphere, adiabatically applied, of $(74+37) \times 10^{-6} = 110 \times 10^{-6}$ for sodium chloride, and of $(200+37) \times 10^{-6} = 235 \times 10^{-6}$ for 0.2 normal copper sulphate. At 0°C, one should expect $(94-13) \times 10^{-6}$ or 81×10^{-6} for 0.01 n-sodium chloride solution. It cannot be decided whether the small dependence on concentration would materially affect the dependence on temperature.

III. THEORY OF THE METHOD

(1) The Principle of the Method

The method uses a "detector." The "detector" consists of an arrangement in which current flows through the electrolytic solution in a direction parallel to the wave fronts. If a sound wave passes through the solution, the conductivity changes periodically. Call f_0 the frequency of the sound waves, R_0 the normal resistance of the solution, R the momentary resistance, then

$$R = \frac{R_0}{1 - Sp \cos 2\pi f_0 t}, \quad (5)$$

where S is the relative increase of conductance per atmosphere pressure. If one has a high impedance source for an a.c. voltage E_{eff} of low frequency f_1, there flows in the absence of sound

[2] F. Koerber, Zeits. f. physik. Chemie 67, 212 (1909).

[3] Water is an exception in so far as the viscosity decreases with increasing pressure. For oils, the viscosity increases for increasing pressure.

[4] This value is probably only accurate to 10 percent.

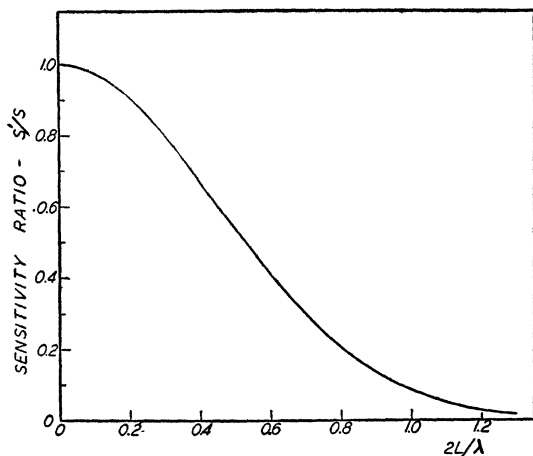[5] See, e.g., J. K. Roberts, Heat and Thermodynamics (Blackie and Son, London, 1933), p. 450.

FIG. 1. The effect of current distribution on sensitivity. The ordinate S'/S is the fraction of the maximum, which is found if the current density is proportional to $\exp[-(x/L)^2]$, the maximum sensitivity occurring for a current filament. The abscissa is the length $2L$, which measures the current distribution, over the wave-length λ.

waves a current through the detector

$$R_0\sqrt{2}E_{\mathrm{eff}} \cos 2\pi f_1 t.$$

If the sound wave is applied, the current remains constant because of the high impedance of the source, and the voltage at the electrodes of the detector is

$$\frac{\sqrt{2}E_{\mathrm{eff}} \cos 2\pi f_1 t}{(1 - Sp \cos 2\pi f_0 t)} = \sqrt{2}E_{\mathrm{eff}}\{\cos 2\pi f_1 t$$

$$+ \tfrac{1}{2}Sp \cos 2\pi (f_0 + f_1)t + \tfrac{1}{2}Sp \cos 2\pi (f_0 - f_1)t\}. \quad (6)$$

That means that two side bands with frequency $(f_0 - f_1)$ and $(f_0 + f_1)$ have been produced (the potential of the a.c. has been modulated by the sound wave) with effective voltage $\tfrac{1}{2}E_{\mathrm{eff}}Sp$.

One should therefore expect, for NaCl solution at 20°C, 55 microvolts per applied volt and atmosphere for each side band, for a CuSO$_4$ solution 115 microvolts per applied volt and atmosphere for each side band.

(2) The Expected Dependence on Frequency

In formula (5), the pressure $p \cos 2\pi f_0 t$ is the average pressure in the region traversed by the current. This can be identified with the pressure at a point in the sound wave only if the region through which current flows extends only over a distance small compared to the wave-length in the direction of wave propagation. Otherwise, the

average pressure amplitude is always smaller than the local pressure amplitude and this diminishes the sensitivity. Assume that the current flows in a direction normal to x, the direction of wave propagation, and that the current density is

$$j_0 \exp[-(x/L)^2], \quad (7)$$

that is, is distributed according to a Gaussian distribution of width $2L$. Then the space average pressure over the current is given by

$$\bar{p} \cos 2\pi f_0 t = p \exp[-(\pi L/\lambda)^2] \cos 2\pi f_0 t,$$

and the sensitivity S is replaced by S'

$$S' = S \exp[-(\pi L/\lambda)^2]. \quad (8)$$

Figure 1 shows the sensitivity ratio S'/S, against $2L/\lambda = 2Lf/V$. It is remarkable how closely the curve resembles a straight line in the middle region. However, one would get a result about 11 percent too high if this straight line were extrapolated to zero frequency. Another reason for a variation of sensitivity with frequency lies in the fact that the ions form space charges at the electrodes, depending on the potential applied. One can see that this can be formally treated by considering the electrodes as having capacity[6] (possibly depending on the electrode potential), which capacity is in parallel with the resistance of the solution.

For the method used here to standardize the electrical circuit for different frequencies—namely the insertion of a known high frequency voltage into the circuit, including the solution (see Section IV 9)—the electrode capacity does not affect the results of the measurements reported below. The effect of the electrode capacity appears only as a decrease of the over-all electrical sensitivity of the measuring circuit with increasing frequency. This change of sensitivity is determined, besides the electrode capacity, by the cable losses and the dependence of the sensitivity of the receiver on frequency.

IV. THE EXPERIMENTAL ARRANGEMENT

(1) Schematic Description (Fig. 2)

An electrical generator A excites a quartz crystal B, which is mounted in the side wall of a water tank C. The vibrating quartz crystal pro-

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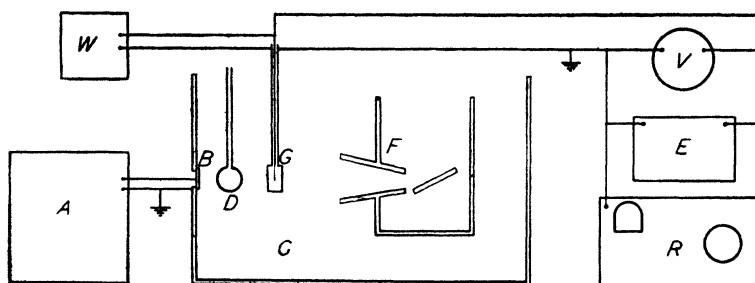[6] F. Krueger, Ann. d. Physik [IV] 21, 701 (1906).

FIG. 2. Schematic diagram of the experimental set-up: A radiofrequency oscillator, B quartz crystal, C tank, D acoustic shutter, E audiofrequency oscillator, F acoustic blackbody, G detector, R radiofrequency receiver, V a.c. voltmeter, W bridge.

duces ultrasonic waves which fall on the detector G. D is a hollow air-filled glass sphere which can be used as shutter for the sound. F is[7] a "black body," which prevents reflection of the sound beam from the back of the tank.

The detector is connected to a high impedance source E of low frequency a.c., a low frequency voltmeter V, the high frequency receiver R, and a Wheatstone bridge W. The sound beam modulates the low frequency a.c. produced by E, the high frequency voltage being measured by R.

(2) The Oscillator A

The oscillator used for the 0.5-, 1.5-, and 2.5-megacycle tests was constructed at the Catholic University of America. It is capable of delivering 300 watts at frequencies from 0.5 to 20 mc and about 50 watts up to 50 mc. A frequency stabilized oscillator having a tuning range of 0.5 to 2 mc furnished the master frequency control. The output of this was fed to a buffer amplifier, and this whole unit provided with a voltage-stabilized power supply. The output from the buffer amplifier was fed to a series of frequency-doubling tubes and a switch allowed one to feed the output of any doubler into a power amplifier. A separate power supply was provided for these doublers and a third for the last stage, in which the plate voltage could be regulated between 0 and 2500 volts by means of a Variac in the primary of the transformer.

In the arrangement used the output of the power amplifier was capacitively coupled to the quartz plate which served then as the source of the ultrasonic waves.

The oscillator used for the 213-kc tests, also constructed at the Catholic University of America, was of the Meissner type with a large L/C quotient to give high r-f voltages (up to 4000 v) for the excitation of the quartz plate. Two tubes were used in parallel with approximately 700-volt plate excitation.

The output from the oscillator was continuously monitored by a wave meter, for which the current was supplied by dry cells.

(3) The Crystal B

For the frequencies 0.5 mc, 1.5 mc, 2.5 mc, an X-cut crystal with fundamental frequency 0.5 mc was used, being driven at its fundamental mode at 0.5 mc, at its third and fifth harmonic for 1.5 mc and 2.5 mc. The fundamental frequency was checked against station WWV (National Bureau of Standards, 5 mc).

For 213 kc, an X-cut crystal was used at its fundamental frequency.

(4) The Tank

The water tank had the dimensions $92 \times 34 \times 32$ cm, the bottom and sides were of glass, held together by metal borders, the two small end faces of metal. One of the end faces had an opening for the crystal holder.

(5) Standardization of Sound Intensity

The sound intensity was determined by measuring the deflection of a sphere, slightly denser than water and of about $\frac{1}{2}$-inch diameter. The sphere hung on a bifilar suspension of 72-cm length and could be moved in the sound field so that the center of the deflected sphere was at the

[7] F. E. Fox and G. D. Rock, Phys. Rev. 54, 223 (1938).

place otherwise occupied by the detector. The deflection was observed through a microscope equipped with a micrometer screw.

The deflection caused by radiation pressure is proportional to the sound intensity F; for our bead 1.00 cm deflection corresponds[8] to

$$F = 0.035 \text{ watt/cm}^2,$$

i.e., to an effective pressure in the sound wave of 0.247 atmosphere. That was the sound intensity with which we usually worked.

The method encounters difficulties if the sound field is very inhomogeneous, so that the sound intensity varies strongly over distances smaller than the diameter of the bead. If one chose a smaller bead, the accuracy would be diminished too much. During measurements with the detector, the bead was behind the detector; relative measurements of sound intensity could be made and the strength of the sound beam monitored. The standardization was checked almost daily.

(6) The Detector

Figure 3 shows the detector. A brass ring of 55-mm inner diameter and 12-mm depth forms the casing for the volume containing the solution. It contains on top a hole with screw-threads, into which a brass tube can be screwed. This brass tube serves as holder for the detector, as holder for the electrodes in the detector, and as protection for the leads to the electrodes. The inside of the brass ring and the holder of the electrodes are coated with paraffin (for purposes of insulation).

The front and back windows of the detector are made of sheets of Pliofilm, which are pressed against the container by two brass rings, fastened by screws.

Pliofilm turned out to be superior to the other materials which were tried, for two reasons:

(a) It transmits the sound waves without measurable loss, as we found by using the bead behind the detector.

(b) It does not deteriorate and become permeable to water, whether the filled detector is taken out of the water tank and left in air, or is left in the water of the tank. Cellophane, on the other hand, becomes permeable if left in contact with water for several days, and the solution must be renewed.

The detector is supported on the tank by an arrangement of iron bars and clamps, which permits adjustment in all directions and also furnishes rigid clamping. An exact control of the position of the detector along the direction of the sound beam was provided in the following manner: The detector was clamped by its holder to a horizontal axis, located above the tank and across it. A micrometer screw moved the lower end of a long vertical arm, the upper end of which was clamped to the same axis. For small motions, this produces a measurable motion of the detector along the direction of sound travel.

The detector was moved up and down and sideways until a maximum effect was observed. For 0.5, 1.5, and 2.5 mc, the effect varied only slowly with the normal distance from the crystal, showing that the sound field is sufficiently uniform.

For 213 kc, however, the sound field was very irregular, showing maxima extending over only about 2 mm. All attempts to get a uniform field by shielding from reflected sound failed, so that we had to give up attempts to make absolute measurements at this frequency, and had to be content to measure the ratio of the sensitivity of salt water and copper sulphate (see V 4).
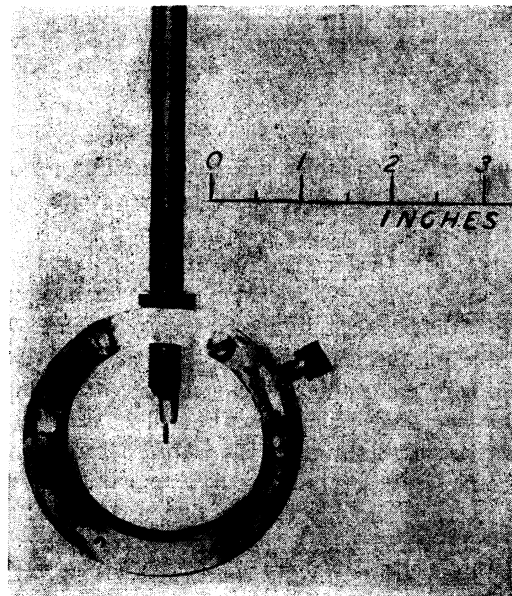
The electrodes were, for copper sulphate, two

FIG. 3. Detector.

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[8] F. E. Fox, J. Acous. Soc. Am. 12, 147 (1940).

copper wires about 3 mm long and about 1 mm apart, of 1-mm diameter for 0.5 mc, and of 0.5-mm diameter for all other frequencies. In spite of the use of alternating current, it was found that one electrode decreased in size and that trees of copper grew between the electrodes. When these came close to forming metallic bridges between the electrodes, the receiver showed great irregularity, and the loudspeaker connected to the receiver gave off loud crackling sounds. We attribute this to local heating.

For sea water, a direct current would develop hydrogen and oxygen on the two electrodes. If gas bubbles actually developed, they would influence the current (see later) and a shock wave, sweeping the bubbles away, would give unpredictable results.

With alternating current, hydrogen and oxygen are alternately developed in small quantities at *each* electrode. If these gases are united catalytically again to water with sufficient speed, no bubbles should appear. For this reason we chose as electrodes platinum wires covered with platinum black, which is a very good catalyst.

The platinum wires, of 0.3-mm diameter, were bent so that their ends were about 0.5-mm distant, forming almost a U, and all but about 0.5 mm at the end was covered with paraffin. Platinum black was then deposited.[9]

These electrodes gave very good results, particularly after some aging, if E was kept below about 2.3 volts (for an a.c. frequency of 3000–4000 cycles).

If the voltage E is run up too high, gas is developed and bubbles are formed. The sensitivity to sound waves (that is the high frequency voltage) then suddenly goes up by a factor of about 5. We did not follow up this effect, because, as previously mentioned, a shock wave would sweep the bubbles away in an uncontrollable manner, but it is possible that a useful detector of ultrasonic waves could be developed in this way. We think that the effect might be caused by oscillations of the bubbles set up by the sound waves, which oscillations would result in a periodic change of size of the bubbles at the electrodes, which change in turn would cause a variation in resistance.

[9] The following method, according to E. V. Angerer, *Handbuch der Experimentalphysik* (Leipzig, 1926), Vol. 1, p. 464, was used.

(7) The Bridge W and the Variation of Temperature

The alternating current passing through the detector heats the solution. To be able to follow this temperature change and also to measure the temperature dependence of the detector action, an a.c. Wheatstone bridge was permanently connected with the detector.

The impedance of the bridge is so high that no high frequency current can pass through it. The low temperature resistance of the sea water detector was around 125 ohms, that of the copper sulphate around 145 ohms. The resistances were measured to within 1 ohm.

The procedure to measure temperature dependence was as follows: The filled detector was put into ice water and left there until the resistance had become constant. This was taken as the resistance at 0°C.

A tin can $10 \times 10 \times 12$ cm, had two windows cut into opposite sides. These windows were covered by Pliofilm, the can filled with ice water and put into the water tank, so that the sound beam went in by one window and out by the other. The detector at 0°C was then put into the tin can and allowed to heat up by conduction from the water of the tank. During the warming up process measurements of the performance of the detector together with resistance measurements were made.

From the ratio of the resistance to the resistance at 0°C, the temperature was determined.[10,11]

In this manner, during the time of warming up from near 0°C to room temperature, a series of sensitivity measurements were made at different temperatures.

(8) The Low Frequency Oscillator E and the Low Frequency Voltmeter V

A Hewlett-Packard audio oscillator served as source E of the low frequency a.c. Its frequency range is continuous up to 20,000 cycles per second. We usually worked in the frequency range between 3000 and 4000. A choke coil was

[10] T. R. Lyle and R. Hosking, Phil. Mag. 3, 487 (1902) for sea water. *International Critical Tables*, Vol. VI, p. 235, 236 for copper sulphate.

[11] The reason for the use of the tin can was this: If the cold detector is put directly into the tank water at room temperature, it warms up too quickly to make sufficient measurements possible. If, on the other hand, the whole tank is filled with ice water, the process takes too long.

connected in series, to avoid short circuiting the high frequency through the audio oscillator. A switch was also in the same circuit.

In the first stages of this investigation, it had proved exceedingly difficult to keep the very selective receiver R (see IV 9) tuned to the side band. The results were also not very reproducible. Finally, the difficulties were traced to rather rapid variations of the line voltage of the power supply. The University is at considerable distance from the power station of the District of Columbia and there were found, particularly during the afternoon, voltage variations of 10–12 volts within a few minutes. These not only threw the audio oscillator off frequency but also changed the sensitivity of the receiver R. The difficulty was remedied by putting voltage regulators on all the power lines except those supplying the oscillator A (however, as described in 2, a voltage regulator was built into the power supply of the first unit of A).

The a.c. voltage E applied to the detector was measured in the following manner: A rectifier type a.c. voltmeter was first standardized at 4 volts with the help of a thermocouple and resistances, the thermocouple having been standardized with d.c. The rectifier type voltmeter was then used to standardize a cathode-ray oscillograph with the help of a potentiometer arrangement. The setting of the cathode-ray oscillograph was frequently checked with the rectifier type voltmeter. The actual effective low frequency voltage E was then measured and continuously followed with the cathode-ray oscillograph. This had the advantage that irregularities showed up immediately.

(9) The Receiver R and Its Standardization

The measuring instrument was a National high frequency receiver, which was used with rather high selectivity. Two improvements were made.

The receiver has a built-in logarithmic meter with rather wide divisions, the receiver not having been designed for exact quantitative work. We put a Triplett microammeter, with 41 ohms in series, in parallel with the built-in meter. This shunt diminishes the readings of the built-in meter by about 10 percent; the microammeter readings are about 10 times those

of the built-in meter, so that the accuracy is multiplied by about a factor 9, the improvement being particularly important for the low readings. In addition the microammeter has heavier damping and is therefore steadier than the built-in meter.

Secondly, the frequency drifts slowly and it is therefore necessary to retune the receiver all the time to maximum sensitivity. This is almost impossible with the original wave-length dial; it can, however, be done with the help of a vernier, which we clamped to it.

The procedure of tuning was as follows: First the receiver was tuned, with the help of its wave-length dial (or with the dial of the frequency converter where this was used) to the "main band," i.e., the frequency of oscillator A, which frequency is transferred by electromagnetic induction to the receiver circuit. (This main band is one of the reasons why we used a.c. of a frequency of about 4000 on the detector. Direct current or 60-cycle a.c. would have given a detector effect with a frequency at or so near to the main band, that this main band, electromagnetically transmitted, would have nearly covered the effect of the pressure variation. At first we had to measure the detector effect as a small difference.)

Then, the dial is turned to the upper side band, i.e., a frequency higher than the main band frequency by 4000. At the side band there is no direct electromagnetic induction, the current going to zero when the sound is cut off by the shutter D.

The vernier is now clamped on the dial and adjustment to maximum current made by either adjusting the receiver dial with the vernier or the sideband frequency with the audio oscillator frequency dial.

It is found that the maximum is reproducible in a series of frequency settings to about $\frac{1}{2}$ division on the microammeter, involving a setting to perhaps 20 cycles per second[12] (in 500,000).

The receiver was standardized with the help of a standard signal generator at the actual frequencies of the side bands used, after it had been ascertained that the sensitivity of the

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[12] It is believed that the performance of the receiver as an exact measuring instrument has been pushed considerably higher than the manufacturer contemplated.
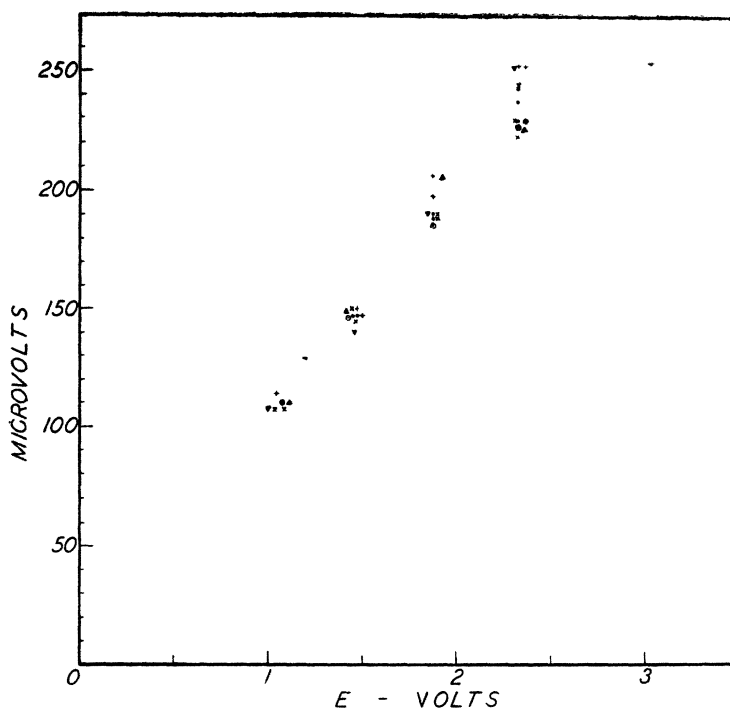
Fig. 4. Microvolts per atmosphere as function of applied audiofrequency voltage for sea water. The different marks refer to runs on different days. The ultrasonic frequency is 0.5 mc.

receiver did not vary appreciably with a frequency change of ±500 cycles/sec.

In the actual measurement of detector performance, there are, in the high frequency circuit, cables with appreciable capacity, the audio oscillator, etc. Therefore, not only was the receiver standardized by itself, but the following procedure was followed:

The standard signal generator was inserted in series with the detector into the actual circuit used in the measurement of detector performance, directly next to the detector, the sound being turned off. The receiver readings were then noted for the different standard signal generator settings. In this way, the voltage produced by the standard signal generator takes actually the place of the high frequency voltage produced at the detector by the sound waves, the circuit being the same (except for the negligible impedance of the standard signal generator).

We have, therefore, something like a substitution method and this standardization can be used directly for the detector measurements.

To give an idea of the actual sensitivity, Table II is inserted:

Table II. Microvolts needed to give a deflection of 30 divisions on microammeter.

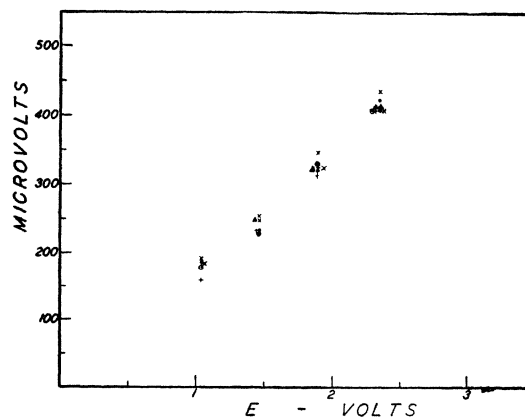| Frequency in mc	0.21	0.5	1.52	2.52
Microvolts applied directly to receiver	22	22	6.5	7.5
Microvolts applied to circuit	70	40	16	8.5

Fig. 5. Microvolts per atmosphere as function of applied audiofrequency voltage for copper sulphate. The different marks refer to runs made on different days. The ultrasonic frequency is 0.5 mc.
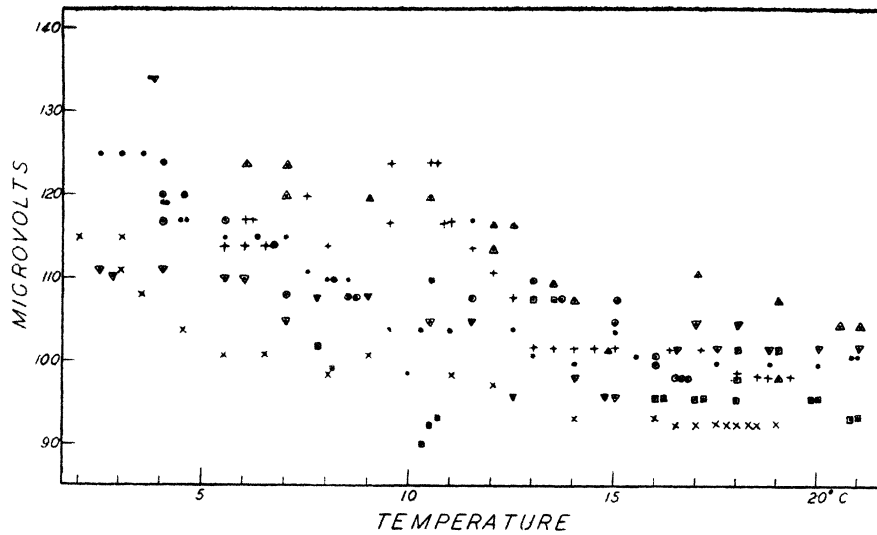
FIG. 6. Microvolts produced per atmosphere and volt in sea water as function of temperature. The different marks refer to different runs. The ultrasonic frequency is 0.5 mc.
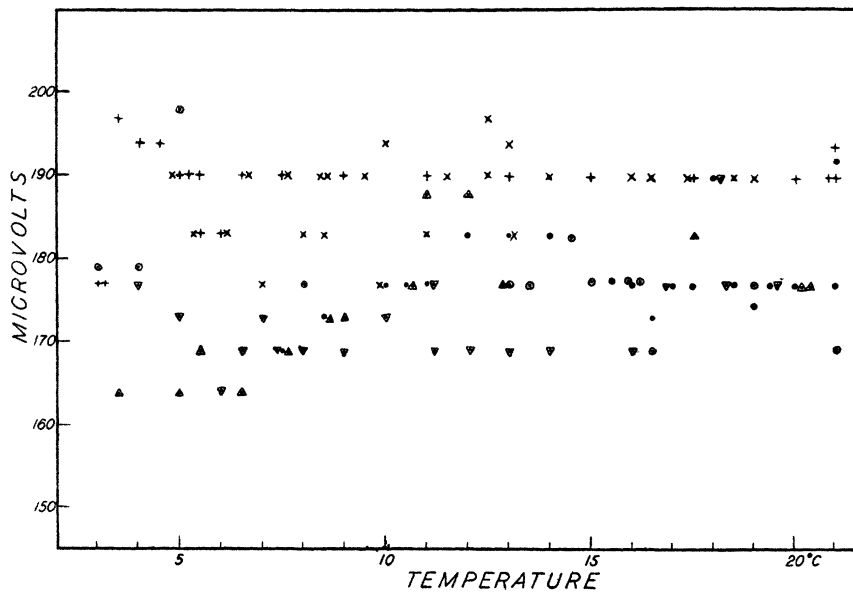

FIG. 7. Microvolts produced per atmosphere and volt in copper sulphate, as function of temperature. The different marks refer to different runs. The ultrasonic frequency is 0.5 mc.

V. RESULTS

(1) For 0.5 mc

For the reasons mentioned in IV-6, we have only a very few measurements at 213 kc; the discussion of that frequency will therefore be postponed to V-4, and we will start out with 0.5 mc, where most measurements were made.

The first question to be answered was whether the high frequency voltage is proportional to E, the a.c. voltage applied. This was tested for both sea water and copper sulphate. The results are given in Fig. 4 for sea water and Fig. 5 for copper sulphate. On the horizontal axis is plotted the applied low frequency alternating voltage E.

Most measurements were made at the four voltages, 1.04; 1.45; 1.82; 2.32 volts.

The ordinates were found so: From the standardization curve the number of microvolts was determined which would give the same deflection on the microammeter. This value was multiplied by 2, because only one sideband was measured, and by 4.04, to reduce the results to 1 atmosphere effective pressure, the effective pressure in the sound beam being $0.247 = 1/4.04$ atmosphere.

The different marks (dots, crosses, etc.) designate different series, usually made on different days. It is seen that in both cases the results can be well represented by straight lines going through the origin.

We are therefore justified in introducing the term "sensitivity" as the number of microvolts produced per atmosphere pressure and per volt applied to the detector (see formula (6) in III-1).

Next all the measurements (except those used in Fig. 4 and Fig. 5) are plotted against the temperature, those for sea water in Fig. 6, those for copper sulphate in Fig. 7. The vertical axis represents the sensitivity in microvolts, which is found as follows: From the standardization curve is found the number of microvolts in the circuit, which produces the same microammeter deflection. This value is multiplied with 8.08 (see above) and divided by E.

The different marks again signify different series (each series being made while the detector warms up, see IV-7).

Obviously, the sensitivity for copper sulphate is constant in the temperature range from 3°C to 21°C. For sea water, the sensitivity seems constant in the range from 2°C to 8°C, then drops slowly to a value 15 percent lower, which remains constant from 14° to 21°C (actually, the measurements have been made up to 26°C). This temperature variation is in the opposite direction from that expected according to an elementary theory (see II-2).

(2) For 1.5 mc

The measurements at the other frequencies were only made at room temperature. At 1.5 mc, a total of 71 measurements with sea water were made at different voltages E. The average for the sensitivity is 61 microvolts, or 55 percent of the theoretical low frequency sensitivity.

For copper sulphate, 8 measurements (for four different E) were made with electrodes of 1-mm diameter. They gave a sensitivity of 70 microvolts (or about 32 percent of the theoretical low frequency value). The electrodes were then replaced by copper wire of 0.5-mm diameter, and 38 measurements for the four values of E mentioned before were made. The resultant average sensitivity was 98 microvolts or 44 percent of the theoretical low frequency value.

It is understandable (see III-2) that electrodes of 1-mm diameter diminish the sensitivity, the wave-length in water being 1 mm for 1.5 mc. In fact, one might be astonished that the sensitivity is still as great. It seems possible that a decrease in the size of the electrodes below 0.5 mm (and perhaps a corresponding decrease in the case of sea water) might still further improve the sensitivity, but this was not tried for practical reasons.

(3) For 2.52 mc

Ten measurements gave for sea water a sensitivity of 16 microvolts, or 15 percent of the theoretical low frequency sensitivity. Eight measurements with copper sulphate (0.5-mm diameter of the electrodes) gave a sensitivity of 27 microvolts, or 12 percent of the theoretical low frequency value.

Different voltages E were used without changing the result, so that we are sure that the effect is proportional to the low frequency voltage E also at this frequency.

(4) For 213 kc

As explained in IV-6, it was found impossible to produce at this frequency a sound field sufficiently constant over a distance larger than 2 mm. As this distance is much smaller than the diameter of our bead, it was impossible to make measurements of the sound intensity. We could not secure a bead of 2-mm diameter with sufficiently exact spherical shape, and sufficiently small density.

Therefore, all that could be done was to measure the microvolts produced by a sound beam of unknown intensity for 1 volt low frequency a.c. in the sea water detector, located at a position with maximum sound intensity. Then, the electrodes (0.5-mm diameter) of the copper
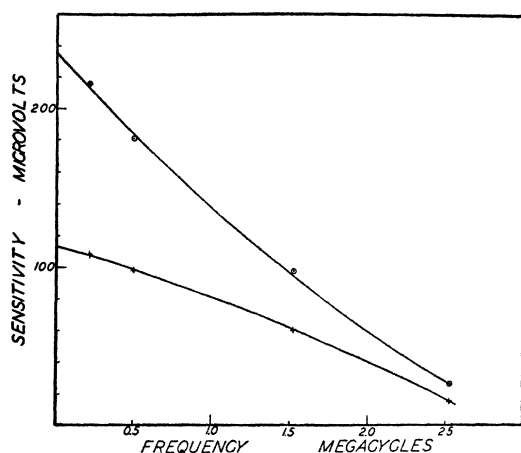
FIG. 8. Sensitivity in microvolts per atmosphere and volt as function of the ultrasonic frequency. The copper sulphate values are marked by dots, the sea water values by crosses. All measurements are at room temperature.

sulphate detector were clipped to 2-mm length and bent together to a distance of 1 mm so that the electrodes were localized within a region of 2×2×2 mm. The copper sulphate detector was then moved, by locating the maximum deflection, to the same place where the sea water detector had previously been, and the microvolts per volt low frequency a.c. determined. In that way the ratio of the sensitivity of the copper sulphate detector to the sea water detector was determined and found to be 2.

It seems reasonable to assume that the sea water detector would have, at 213 kc, almost the theoretical low frequency sensitivity of 110 microvolts, in view of the fact that its sensitivity is already 98 microvolts at 0.5 mc. We take 108 microvolts if that is assumed, then the sensitivity of the copper sulphate detector is 216 microvolts, which falls within the limits of the theoretically expected low frequency sensitivity of 217 to 237.

(5) Discussion of the Results

The frequency dependence of the sensitivity S' is shown in Fig. 8. Each point is an average of many measurements at room temperature, as discussed in the preceding section. The copper sulphate values are marked by a dot, those for sea water by a cross. We estimate that the results are accurate to five percent.

According to Fig. 8, the points lie nearly on a straight line between 213 kc and 2.5 mc. This is in agreement with the predictions of IV-2 and Fig. 1. The characteristic length $2L$ of Eqs. (7) and (8) is calculated from Fig. 8 as 0.6 mm for copper electrodes and 0.44 mm for the platinum electrodes, which values seem reasonable in view of the construction of the detector.

If one extrapolates the curves of Fig. 8 linearly, one finds as low frequency sensitivity for sea water 114 microvolts, for copper sulphate 236 microvolts. According to Section IV-2, these values are 11 percent too high, and therefore the true values are 103 microvolts for sea water and 211 for copper sulphate. These values lie lower then but sufficiently close to the theoretical prediction (Section II-2).

(6) Use as an Instrument for Measuring Sound Fields

For frequencies below 500 kc, the detector can be used as a convenient absolute instrument to measure inhomogeneous sound fields. Even at higher frequency, it is convenient for many purposes, although it must be standardized for absolute measurement and is not adapted to measure inhomogeneities of the order of a mm (no other instrument can do that either).
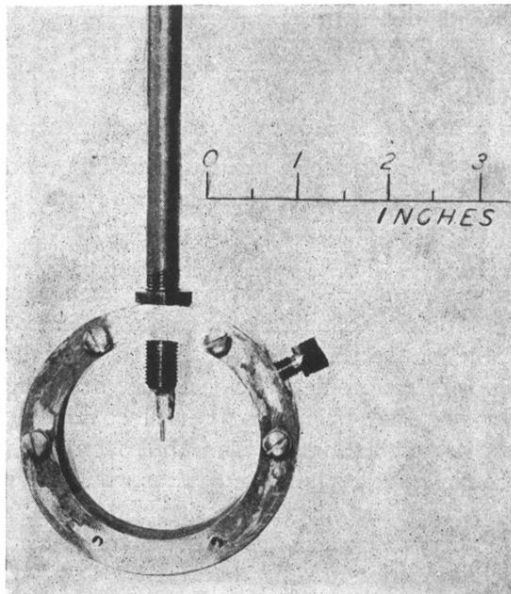
FIG. 3. Detector.