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Theory of the Dispersion of Magnetic Permeability in Ferromagnetic Materials at Microwave Frequencies*

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The transition in the value of the initial magnetic permeability of iron and nickel from the d.c. permeability ~100 to the infra-red permeability ~1 is known to occur principally in the microwave frequency range. An explanation of the experimental facts is proposed by considering the equations of motion of a domain boundary in an applied magnetic field for frequencies such that the skin depth of the magnetic field is smaller than the thickness of a domain. An analytic solution of Maxwell's equations is found for the magnetization of a layer one domain thick. The definition of the permeability at high frequencies is considered carefully, and it is shown that the natural definition leads to complex values for the permeability. In experiments two different types of permeabilities are found. The relationship of the complex μ to the μ_R determined from resistive losses in a circuit element and to the μ_L determined from reactance changes is developed. A criticism is given of theories of ferromagnetic resonance. The status of Becker's theory of eddy current damping is considered. Several suggestions are made for further experiments.

I. INTRODUCTION

THE initial magnetic permeability of iron and nickel is known to decrease abruptly in the microwave frequency range from the d.c. permeability of the order of 100, which obtains up to frequencies near 100 mc/sec., to the infra-red permeability of unity found by Hagen and Rubens[1] at 10^7 mc/sec.

At frequencies higher than 100 mc/sec. the skin depth for the penetration of the electromagnetic field into the specimen is smaller generally than the dimensions of the elementary ferromagnetic domains. Domain dimensions are estimated to be of the order of 10^{-3}–10^{-4} cm. It is plausible to expect at such frequencies that the usual concepts of the domain mechanisms leading to microwave magnetization will break down, since the applied magnetic field is no longer effective over the entire volume of a domain. The present paper represents an attempt to construct a theory of domain dynamics applicable to such a situation. The theory is found to account satisfactorily for the general trends of the existing experimental data on magnetic dispersion in ferromagnetic materials at microwave frequencies.

Ferromagnetic materials are composed of small regions called domains.[2] In general the extent of a domain is smaller than that of the microcrystalline structure of the material. The atomic spins in a domain are nearly all lined up in the same direction, but the principal direction of the domain spins varies from one domain to its neighbors. In the demagnetized state the domains are oriented at random, giving zero macroscopic magnetization.

The process of macroscopic magnetization occurs in three stages in ordinary ferromagnetic materials:

(a) In *weak fields* (~1 gauss) the boundary walls between domains are displaced reversibly in such directions that domains oriented in the direction of the applied field increase in volume at the expense of unfavorably oriented domains. The initial permeability of the material is determined by the average curvature of the local potential energy curve for boundary displacements, at the undisplaced position of the domain boundaries.

(b) In *medium fields* (~10 gauss) irreversible displacements of the boundary walls occur, and domains with unfavorable orientation to the

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[1] E. Hagen and H. Rubens, Ann. d. Physik [4] **11**, 873 (1903).

[2] For an account of modern theories of ferromagnetism see F. Bitter, *Introduction to Ferromagnetism* (McGraw-Hill Book Company, Inc., New York, 1937), and R. Becker and W. Doering, *Ferromagnetismus* (Edwards Brothers, Ann Arbor, Michigan, 1943).

applied field may disappear completely. The irreversible displacements are known as Barkhausen discontinuities.

(c) In *strong fields* ($\sim$500 gauss) the spins of domains as a whole are turned into orientations parallel to the applied field. These field strengths may be compared with the magnitude of the important spin-dependent interactions in iron. The spin-orbit forces responsible for anisotropy are equivalent to $\sim$1000 gauss and the exchange forces are equivalent to $\sim 10^7$ gauss.

Each of the three stages of magnetization will be restricted at high frequencies by various physical processes:

(a) In weak fields it appears that incomplete penetration of surface domains by the applied field is the predominant effect.

(b) In medium fields the Barkhausen jumps are inhibited by local microscopic eddy currents, as suggested by Sixtus and Tonks[3]; and Becker.[4]

(c) In strong fields the domain rotations are damped by microscopic eddy currents. In a paper which neglects eddy current damping, Landau and Lifschitz[5] suggest the possibility of ferromagnetic resonance effects in domain rotation.

The interpretation of experimental data regarding dispersion in weak fields is assisted by the fact that the medium field strength processes (up to $\sim$25 gauss) are extinguished at frequencies above 100 mc/sec. This fact means that the measured values of the permeability in ordinary ferromagnetic materials may not be particularly dependent upon the magnetic field strength in measurements made with medium power microwave transmitters; for weak field processes the susceptibility is nearly constant. Experiments in support of this statement are reported by Arkadiew[6] and Kreielsheimer.[7] The peak value of the magnetic field in a cavity is of the order of $(QP/A)^{\frac{1}{2}}$ gauss; where Q is the "Q" of the cavity, P the peak power input in kilowatts, and A the area of the internal surfaces in sq. cm. For $Q=1000$, $P=10$ kw, and $A=100$ sq. cm this expression gives $H\sim 10$ gauss.

A large d.c. magnetic field superposed on the small alternating microwave field may have an effect on the permeability by shifting the central point of the microwave cycle up the permeability curve. This may change the magnitude and nature of the magnetization processes involved in the microwave cycle. Such effects have been observed for iron powder cores[7a] composed of iron particles about one or two microns in diameter. Medium field processes which are strongly damped in ordinary specimens may still play a role in the case of very small particles.

II. EXPERIMENTAL RESULTS

A comprehensive review of the experimental data on the permeability of ferromagnetic materials (including alloys and dusts) at frequencies between 100 kc/sec. and 10,000 mc/sec. has been published by Allanson.[8] It is proposed here for the purpose of comparison with the theory developed below to summarize briefly the results of measurements on iron, nickel, and cobalt at frequencies above 100 mc/sec., including some data not available to Allanson.

A. Iron

Experimental values for iron and steel are plotted in Fig. 1, as determined by Arkadiew,[6] Hoag and Jones,[9] Potapenko and Sänger,[10] Lindman,[11] Hoag and Gottlieb,[12] Glathart,[13] and E. Maxwell.[14] The curve labeled μ_R is drawn through the points for the experimental permeabilities deduced from *resistive losses* in a circuit element containing the ferromagnetic specimen, while the curve labeled μ_L is deduced from the *reactance* of the circuit element. The relationship

[3] K. J. Sixtus and L. Tonks, Phys. Rev. **37**, 930 (1931); **39**, 357 (1932); **42**, 419 (1932).
[4] R. Becker, Physik. Zeits. **39**, 856 (1938); Zeits. f. tech. Physik **19**, 542 (1938); Ann. d. Physik [5] **36**, 340 (1939).
[5] L. Landau and E. Lifschitz, Physik. Zeits. Sowjetunion **8**, 153 (1935).
[6] W. Arkadiew, Ann. d. Physik [4] **58**, 105 (1919).
[7] K. Kreielsheimer, Ann. d. Physik [5] **17**, 293 (1933); cf. P. D. Zottu, Inter. Sci. Rad. Union **5**, Pt. 1, 190 (1938).
[7a] M.I.T. Radiation Laboratory Report 906.
[8] J. T. Allanson, J. Inst. Elec. Eng. **92**, Part III, 247 (1945); see also, W. Arkadiew, J. Phys. U.S.S.R. **9**, 373 (1945).
[9] J. B. Hoag and H. Jones, Phys. Rev. **42**, 571 (1932).
[10] G. Potapenko and R. Sänger, Naturwiss. **21**, 818 (1933); Zeits. f. Physik **104**, 779 (1937).
[11] K. F. Lindman, Zeits. f. tech. Physik **19**, 159 (1938).
[12] J. B. Hoag and N. Gottlieb, Phys. Rev. **55**, 410L (1939).
[13] J. L. Glathart, Phys. Rev. **55**, 833 (1939).
[14] E. Maxwell, M.I.T. Radiation Laboratory Report 854 (1946), unclassified. The values of the permeabilities credited to Maxwell were calculated from his values for the attenuation of 1.25-cm radiation in rectangular wave guides.

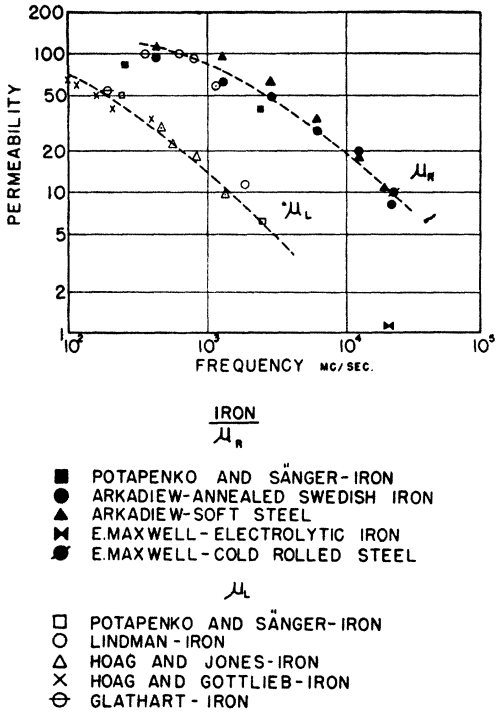

FIG. 1. Permeability measurements for iron.

between μ_L and μ_R is discussed in the next section.

The measurements of Lindman have not been taken into account in drawing the μ_L curve, since his values are far out of line with those of Hoag and Jones, Hoag and Gottlieb, Glathart, and Potapenko and Sänger. The apparent discrepancy here may be caused by real differences in the dimensions of surface domains or in the electrical conductivity of the surface layer of the specimen. Maxwell has studied the effect of surface finish on microwave attenuation in wave guides and finds considerable variation between samples prepared by different methods. This result may be accounted for, perhaps, by the increased resistance offered by a rough surface to the eddy currents, which flow at these frequencies in a layer less than 10^{-4} cm in thickness. The measurement by Maxwell on electrolytic iron at 24,000 mc/sec. has not been taken into account in drawing the μ_R curve, although this value may be of more fundamental significance than his measurement on a cold-rolled steel specimen which had only a machined surface; the latter value, however, agrees with two measurements by Arkadiew at the same frequency.

The frequencies at which the permeabilities of iron are down to *one-half of the d.c. initial permeability* are given by:

$\sim$3000 mc/sec. for μ_R,
$\sim$200 mc/sec. for μ_L.

B. Nickel

Experimental values for nickel are plotted in Fig. 2, as determined by Arkadiew, Lindman,[15] Hoag and Gottlieb, Glathart, and E. Maxwell. Not shown in the figure are values from unpublished measurements of Potapenko and Sänger kindly communicated by Professor Potapenko: they find $\mu_R=10.2$; 9.4; 8.6; 6.7; and 6.3 at frequencies of 250; 341; 615; 1190; and 1700 mc/sec., respectively.

There are fewer measurements on nickel than on iron, and the conclusions to be drawn from the data are less certain. Lindman's values, as in the case of iron, are out of line with the results of other observers.

The frequencies at which the permeabilities of nickel are down to *one-half of the d.c. initial permeability* are given by:

$\sim$1500 mc/sec. for μ_R,
$\sim$170 mc/sec. for μ_L.

These frequencies are somewhat lower than the corresponding frequencies for the case of iron.*

C. Cobalt

There is some indication from magnetic powder patterns that the size of domains in cobalt is considerably larger than for iron and nickel. This suggests that a comparison of magnetic dispersion in these metals would be of particular interest. Unfortunately cobalt is difficult to work, and there is little information on its magnetic properties at high frequencies. Potapenko and Sänger[10] find $\mu_R\sim 5$ at 250 mc/sec.; in subsequent unpublished work they find $\mu_R=7.0$ at 250 mc/

[15] K. F. Lindman, Zeits. f. tech. Physik **19**, 323 (1938).

* Note added in proof: A preliminary account of recent Czech measurements of the transmission of microwaves through thin films has appeared recently [I. Simon, Nature **157**, 735 (1946)]. Simon finds that μ_R for a nickel film 1000 to 2000A thick falls from the value 10 at 1500 mc/sec. to $\sim$1 at $\sim$20,000 mc/sec.

sec. and $\mu_R=6.6$ at 341 mc/sec. Strutt[16] gives $\mu_R=11.0$ at 4.7 mc/sec. and $\mu_R=12.1$ at 7.5 mc/sec. These data are not adequate to decide whether or not the dispersive region in cobalt occurs at lower frequencies than in iron and nickel.

D. Infra-Red Measurements

The classical series of experiments by Hagen and Rubens[1] on the reflectivity and emissivity of metal surfaces in the infra-red region furnishes convincing evidence that the value of μ_R for nickel and steel is approximately unity at frequencies $\sim 10^7$ mc/sec.

III. DEFINITIONS OF PERMEABILITY AT HIGH FREQUENCIES

There is a great deal of confusion in the literature with regard to the connection between the permeability μ_R determined from resistive losses (as by measuring the "Q" of a cavity or the attenuation of energy along a coaxial line) and the permeability μ_L determined from reactance measurements (as by measuring the resonant frequency of a cavity or the wave-length of standing waves along a coaxial line). It does not appear to have been realized in the literature that the two types of measurements inherently reveal different aspects of the same fundamental physical phenomena. In this section the fundamental philosophy will be developed which underlies the interpretation, in terms of permeability, of r-f measurements in a dispersive region. The ideas were stimulated by a paper of Rayleigh's[17] in which the concept of an out-of-phase component of magnetization is introduced in connection with hysteresis losses.

The usual definition of initial permeability as the ratio of B to H for weak fields does not correspond at high frequencies to the quantities actually observed in experiments. In a dispersive region the value of the ratio B/H may vary from point to point in the radiation field within the specimen in both amplitude and phase. Detailed knowledge of this "permeability field," supposing it could be determined, would be less useful (for most purposes) than a convenient summing up of the magnetic behavior of the material in terms of an *effective permeability*.

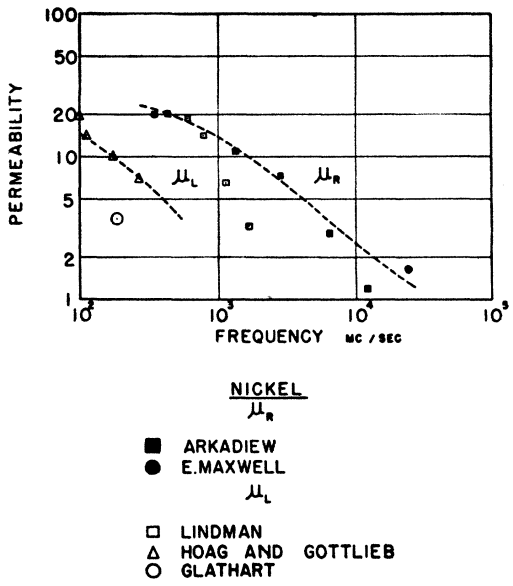

FIG. 2. Permeability measurements for nickel.

The natural and logical method of defining the effective permeability is as follows: The impedance

$$Z_{\text{calc}}(\mu, \omega)=R_{\text{calc}}(\mu, \omega)+jX_{\text{calc}}(\mu, \omega) \qquad (1)$$

of a circuit element containing the ferromagnetic material can in principle be calculated from Maxwell's equations in the usual form, given μ and ω. This calculation can actually be carried out in closed form for the important experimental geometries, such as a rectangular wave guide or coaxial line resonator. Suppose that the result of a series of measurements on the circuit element gives us experimental values

$$Z_{\text{exp}}(\omega)=R_{\text{exp}}(\omega)+jX_{\text{exp}}(\omega). \qquad (2)$$

The *effective permeability* μ is then defined to be that value of μ which, when substituted in Eq. (1), makes

$$Z_{\text{calc}}(\mu, \omega)=Z_{\text{exp}}(\omega). \qquad (3)$$

In general μ defined in this way will be a function of the frequency ω and will be complex.

It will be seen that, in order to determine the effective permeability, measurements of both the resistive and reactive parts of the impedance of the circuit element are required. Now in most

[16] M. J. O. Strutt, Zeits. f. Physik **68**, 632 (1931).
[17] Lord Rayleigh, Phil. Mag. **23**, 225 (1887); *Scientific Papers* (Cambridge University Press, Cambridge), Vol. 2, p. 579; cf. W. Arkadiew, Zeits. f. Physik **27**, 37 (1924); and E. Hinze, Ann. d. Physik [5] **19**, 143 (1934).

of the experiments performed in the past either the resistive part *or* the reactive part was measured, but not both parts of the impedance. This is a serious deficiency in the measurements. If, however, we know either $R(\omega)$ or $X(\omega)$ for all frequencies, we can calculate the other using Kramer's theorem.

If we have only $R_{\text{exp}}(\omega)$ the permeability is usually taken to make

$$R_{\text{calc}}(\mu_R, \omega) = R_{\text{exp}}(\omega). \tag{4}$$

This relation determines the real function $\mu_R(\omega)$. Similarly, if we have only $X_{\text{exp}}(\omega)$, the permeability is taken to make

$$X_{\text{calc}}(\mu_L, \omega) = X_{\text{exp}}(\omega), \tag{5}$$

thus determining the real function $\mu_L(\omega)$. There is no simple and direct connection between μ_R, μ_L, and the complex μ.

The preceding reasoning, which underlies the introduction of a complex permeability, is somewhat more involved than with the introduction of a complex dielectric constant in the dielectric case. There are two reasons for this difference: because of the skin effect the magnetic field distribution in the material is a complicated function of the space coordinates; and because of the domain or coarse-grained structure of ferromagnetic material it is possible to have a microscopic permeability which varies from point to point in the material.

In the literature μ_R is sometimes called the "outer permeability" and may be denoted also by μ_r or μ_k; μ_L is sometimes called the "inner permeability" and may be denoted by μ_i or μ_n.

IV. THEORY OF DOMAIN DYNAMICS

In the microwave region the skin depth for field penetration is comparable to or smaller than the dimensions of a domain. For iron with $\mu = 100$ we have $\delta \cong 1.6 \times 10^{-3}/\sqrt{f}$, where δ is the skin depth in cm and f is the frequency in mc/sec. This gives the following values:

f (mc/sec.)	10^2	10^4	10^6,
δ (cm)	1.6×10^{-4}	1.6×10^{-5}	1.6×10^{-6},

whereas domain dimensions are estimated at 10^{-3} to 10^{-4} cm. It is, therefore, necessary to reconsider the conventional application of Maxwell's equations to the skin effect problem.

In the limiting case in which the surface energy of the domain boundaries is greater than the magnetization energy, the domain wall shifts as a whole in the direction of the applied field even if, because of the skin effect, the magnetic field only penetrates a short distance into the domain. If H is constant across the domain boundary (low frequency case) the average macroscopic magnetization M for weak fields is given by

$$M = \chi H \tag{6}$$

by the usual definition of the initial susceptibility χ. If now H varies across the boundary we suppose that the effective magnetization is given by the susceptibility times the average value of H across the boundary:

$$M = \chi(1/2d) \int_{-d}^{d} H(y)dy. \tag{7}$$

This assumption is based on the physical concept that the boundary pressure $2M_sH$ is integrated over the area of the boundary to give a force which shifts the boundary in the direction of the applied field up to the point where this force is balanced by the force exerted on the boundary by the internal stresses in the material. Here M_s is the saturation magnetization obtaining in a domain. The distance the wall is shifted is related to the macroscopic magnetization M observed in an experiment. At very high frequencies the following happens: the skin depth is a small fraction of the domain thickness, so that the applied magnetic field only penetrates a little way down the domain boundary. The force on this boundary associated with the surface value of this field strength is correspondingly less than the force obtaining at low frequencies, so that the boundary is shifted by a reduced amount. To the field this reduced shift looks like a reduced permeability.

It is of interest to consider as a simplified model a film one domain in thickness (Fig. 3), since in this case Maxwell's equations can be solved to give what is essentially the equation of motion of a domain boundary. The longitudinal extent of a domain along the surface of the film is supposed to be small in comparison with the thickness of the film. The domains are considered for simplicity to have only "180°" walls—that

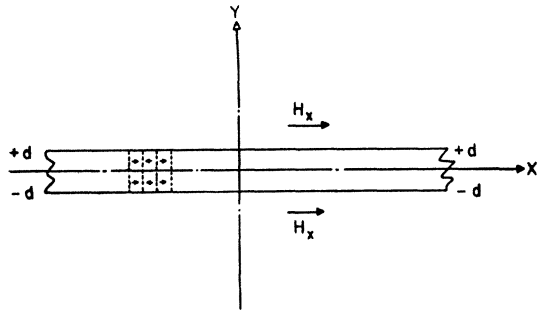

FIG. 3. Model of film one domain in thickness for theoretical calculation of permeability.

is, the domains are either magnetized in the direction of the applied field or in the opposite direction. The applied field is parallel to the surface of the film and is symmetric about the central plane of the film.

From Maxwell's equations

$$\text{curl } \mathbf{H}=4\pi\sigma\mathbf{E}/c, \tag{8}$$

$$\text{curl } \mathbf{E}=-(\dot{\mathbf{H}}+4\pi\dot{\mathbf{M}})/c, \tag{9}$$

we get

$$\text{curl curl } \mathbf{H}=-j(4\pi\omega\sigma/c^2)(\mathbf{H}+4\pi\mathbf{M}) \tag{10}$$

for time dependence of the form $\exp[j\omega t]$. By the symmetry of the problem we have

$$\text{curl curl } \mathbf{H}=-\mathbf{i}(\partial^2 H_x/\partial y^2), \tag{11}$$

so that

$$\partial^2 H_x/\partial y^2=j(4\pi\sigma\omega/c^2)(H_x+4\pi M_x). \tag{12}$$

Since $4\pi M_x$ is constant with respect to y we can add this quantity to H_x on the left side without altering the value of the derivative there. Now by definition $B_x=H_x+4\pi M_x$, giving

$$\partial^2 B_x/\partial y^2=j(4\pi\omega\sigma/c^2)B_x, \tag{13}$$

which can be compared with the usual skin effect equation:

$$\partial^2 B_x/\partial y^2=j(4\pi\omega\sigma\mu/c^2)B_x. \tag{14}$$

For convenience in working with Eq. (13) we shall hereafter omit the subscript x on B_x, M_x, and H_x. We write

$$p^2=j(4\pi\sigma\omega/c^2)=2j/D^2, \tag{15}$$

where D is the skin depth for permeability unity. Equation (13) becomes

$$\partial^2 B/\partial y^2=p^2 B. \tag{13a}$$

A symmetric solution of this equation is

$$B=C\cosh py, \tag{16}$$

where C is a constant to be determined in terms of $H(d)$, the magnetic field at the surface of the film. From the definition of B we have $H=B-4\pi M$; or using Eq. (7),

$$H(y)=B(y)-(4\pi/2d)\chi\int_{-d}^{d}H(y)dy; \tag{17}$$

here $B(y)$ is given by Eq. (16). The solution of this equation is found to be

$$H(y)=C[\cosh py-(4\pi\chi/\mu_0 pd)\sinh pd], \tag{18}$$

where μ_0 is defined as $1+4\pi\chi$.

Thus the constant C is given in terms of $H(d)$ by

$$C=H(d)[\cosh pd-(1-1/\mu_0)(\sinh pd/pd)]^{-1}. \tag{19}$$

The solution of the ordinary eddy current Eq. (14) above for permeability μ is

$$B(y)=B(d)\cosh qy/\cosh qd=\mu H(y), \tag{20}$$

where

$$q^2=j(4\pi\sigma\omega\mu/c^2). \tag{21}$$

Now that we have the formal solutions of both the ordinary eddy current equation and the domain eddy current equation, we can go on to calculate μ, μ_R, and μ_L by following the procedure outlined in Section III. In calculating the impedances it is not necessary to specialize the calculation for a particular cavity or line; we can work with the *intrinsic surface impedance* of the film, which is defined by[18]

$$Z=E_{\text{tang}}/H_{\text{tang}}=E_z/H_x. \tag{22}$$

From Eq. (8),

$$E_z=-(c/4\pi\sigma)\partial H_x/\partial y, \tag{23}$$

so that

$$Z=-(c/4\pi\sigma)(\partial H_x/H_x\partial y)_{y=d}. \tag{22a}$$

[18] S. A. Schelkunoff, *Electromagnetic Waves* (D. Van Nostrand Company, New York, 1943); J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1941), p. 282; J. C. Slater, *Microwave Transmission* (McGraw-Hill Book Company, Inc., New York, 1942), p. 95.

Using Eq. (18) we have for the domain model

$$Z_{\text{dom}} = -(c/4\pi\sigma)\frac{p \tanh pd}{1-(1-1/\mu_0)(\tanh pd/pd)}. \quad (24)$$

For the ordinary eddy current equation we have, using Eq. (20):

$$Z_{\text{ord}} = -(c/4\pi\sigma)q \tanh qd. \quad (25)$$

The two expressions for Z are equal if q is chosen so that

$$qd \tanh qd = pd\frac{\tanh pd}{1-(1-1/\mu_0)(\tanh pd/pd)}; \quad (26)$$

that is, if the *effective permeability* μ is chosen so that

$$\mu^{\frac{1}{2}} \tanh pd\mu^{\frac{1}{2}} = \frac{\tanh pd}{1-(1-1/\mu_0)(\tanh pd/pd)}. \quad (27)$$

Since p involves $j^{\frac{1}{2}}$ the value of μ satisfying this equation will be complex.

Let us consider limiting cases of Eq. (27):

(*a*) *Low frequency.*—Here $|pd| \ll 1$, so that we can replace $\tanh \phi/\phi$ by unity. This gives $\mu = \mu_0$, the correct low frequency value.

(*b*) *Very high frequency.*—Here $|pd| \gg 1$, so that $\tanh \phi/\phi$ approaches zero. This gives $\mu = 1$ in agreement with the measurements of Hagen and Rubens.

Values of μ satisfying Eq. (27) for various values of pd are given in Table I. These values were calculated by cut-and-try methods with assistance from Kennelly's tables[19] and the tables prepared by Lowan, Morse, Feshbach, and Haurwitz.[20]

Theoretical values of μ_R and μ_L are also given in Table I according to the definitions, Eqs. (4) and (5), where we identify

$$R_{\text{exp}} \leftrightarrow R_{\text{dom}}; \quad X_{\text{exp}} \leftrightarrow X_{\text{dom}};$$
$$R_{\text{calc}} \leftrightarrow R_{\text{ord}}; \quad X_{\text{calc}} \leftrightarrow X_{\text{ord}};$$

so that μ_R is the real number which satisfies the real part of Eq. (27):

$$\text{Re}\{\mu_R^{\frac{1}{2}} \tanh pd\, \mu_R^{\frac{1}{2}}\} = \text{Re}\left\{\frac{\tanh pd}{1-(1-1/\mu_0)(\tanh pd/pd)}\right\} \quad (27a)$$

and μ_L is the real number which satisfies the imaginary part:

$$\text{Im}\{\mu_L^{\frac{1}{2}} \tanh pd\, \mu_L^{\frac{1}{2}}\} = \text{Im}\left\{\frac{\tanh pd}{1-(1-1/\mu_0)(\tanh pd/pd)}\right\}. \quad (27b)$$

In Fig. 4 μ_R and μ_L are plotted together with the smoothed experimental curves for iron taken from Fig. 1. The arbitrary constant $2d$, which is the thickness of the domain film model, has been taken to be 2.48×10^{-4} cm; this value was chosen to make the half-value points on the experimental and theoretical curves coincide.

It is seen that the theory predicts the order of magnitude of the spacing between the μ_R and μ_L curves correctly. The general nature of the theoretical permeability change is in accordance with the experimental data, but the slopes of the theoretical curves are steeper than the experimental. The thickness of the film is within the limits of reasonable estimates of domain dimensions, although the thickness is somewhat on the small side. In this connection it should be noted that Elmore[20a] has found evidence of a fine-scale layer magnetization 2×10^{-4} cm thick localized near the surface; this gives way to coarser layers within the specimens.

The discrepancy in the slopes is most likely to be accounted for by local variations in domain dimensions and d.c. permeability, since these variations will act to smear out the dispersive region. The absence of the hump predicted for the μ_R curve on the low frequency side may be attributed in part to these causes and in part to the over-simplification of the present model.

TABLE I. Theoretical permeability vs. frequency. Film thickness 2.5×10^{-4} cm.

Frequency mc/sec.	Parameter $(d/D)\sqrt{2}$	Permeability μ Amplitude	Phase	μ_R	μ_L
83	0.1	104	0°	102	100
334	0.2	127	−18°	157	58
750	0.3	89	−53°	160	11.5
2080	0.5	35	−78°	70	1.7
4670	0.75	16	−79°	32	0.5
8300	1.0	9	−80°	18	0.3
33000	2.0	2.6	−62°	4.5	0.2
75000	3.0	1.6	−38°	2.6	0.6
210000	5.0	1.0	−18°	1.7	0.9

[19] A. E. Kennelly, *Tables of Complex Hyperbolic and Circular Functions* (Harvard University Press, Cambridge, 1914).

[20] A. N. Lowan, P. M. Morse, H. Feshbach, and E. Haurwitz, *Tables for Solutions of the Wave Equation for Rectangular Boundaries Having Finite Impedance*, Applied Mathematics Panel Note No. 18; Section No. 6.1-sr1046-2043 (June, 1945); unclassified.

[20a] W. C. Elmore, Phys. Rev. 62, 486 (1942).

In the ordinary microwave radio range of frequencies from 3000 to 30,000 mc/sec. the permeability μ is chiefly imaginary, according to Table I.

V. FERROMAGNETIC RESONANCE

Several predictions have been made that resonance effects or peaks in the permeability *vs.* frequency curve would be found at high frequencies; see for example the paper of Landau and Lifschitz[21] in which magnetic resonance is predicted in nickel at ~ 2500 mc/sec.

Such effects have not been found experimentally, and it is possible to see one of the reasons why from the argument of the preceding section. The predictions have all neglected completely the effect of skin depth and eddy currents, yet in the frequency range considered we have shown that such effects are of predominant importance.

It is possible, however, that magnetic resonance effects may be detected in the magnetic oxides and sulfides of iron.[22] These are ferromagnetic but have low electrical conductivity, so that the skin depth will be much greater than in the ferromagnetic metals. The skin depth in magnetite (Fe_3O_4) is $\sim 5\times 10^{-3}$ cm at 10^4 mc/sec., as compared with 1.6×10^{-5} cm in iron at the same frequency. The d.c. initial permeability[23] of magnetite is ~ 17. Measurements on films of ferromagnetic materials should also be pertinent when the film thickness is less than the calculated skin depth.

The resonance phenomenon may be understood as occurring when the frequency of the applied field is equal to the Larmor frequency of the atomic spins in the internal anisotropy field. This is the field caused by spin-orbit interactions and distinguishes energetically different directions of magnetization in the crystal lattice. Since the anisotropy field is of the order of 1000 gauss, the corresponding Larmor frequency is in the microwave range.

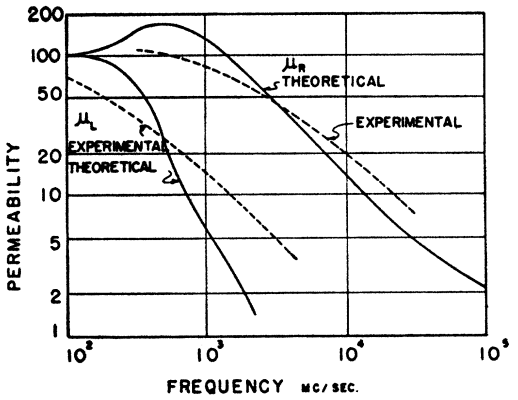

FIG. 4. Comparison of smoothed experimental values for iron with theoretical calculations using $2d=2.5\times 10^{-4}$ cm.

It is interesting to consider a classical model in which the atoms are replaced by *non-gyroscopic* bar magnets pivoted at the lattice points of the crystal. With zero applied field each magnet is attracted in a definite direction relative to the lattice by means of individual coiled springs representing the spin-orbit interaction, and the magnets will oscillate in an applied field with a component perpendicular to the rest position of the magnets. The sine of the angle of oscillation is proportional to the macroscopic magnetization. Resonance occurs when the applied frequency is equal to the free period of a magnet+spring unit.

The bar magnet analogy supposes that the relaxation time of the spins is sufficiently short so that gyroscopic effects may be neglected. It is not usually recognized that this assumption is being made. If this assumption is not true, the spins will precess about the field direction without lining up. It is indeed a prerequisite for any type of magnetization that the magnetic moments have time in which to line up in the instantaneous local field to which they are subjected. The calculation of the relevant relaxation time is a problem in the kinetics of thermodynamic equilibrium.

The time-dependent processes can be described by assuming the existence of a relaxation time, as was done by Gorter and Kronig for paramagnetic relaxation, and by Landau and Lifschitz for the ferromagnetic case. The quantum-mechanical calculation of the relaxation times, starting from the detailed interactions of spins with the lattice, is extremely difficult and

[21] Reference 5; see also R. Gans and R. G. Loyarte, Ann. d. Physik [4] **64**, 209 (1921); L. Page, Phys. Rev. **21**, 456 (1923); J. Dorfmann, Zeits. f. Physik **17**, 98 (1923); K. Kartschagin, Ann. d. Physik [4] **67**, 325 (1922). The experiments by Kartschagin and others in which resonance phenomena were reported are now discredited. Cf. G. R. Wait, Phys. Rev. **29**, 566 (1927).

[22] The interesting possibilities of the ferromagnetic semiconductors were pointed out to the writer by Prof. A. von Hippel, who is planning to investigate them experimentally.

[23] *International Critical Tables*, Vol. VI, p. 374.

uncertain. Calculations for the paramagnetic case have been made by Waller[24] and others. No calculations have been carried out for the ferromagnetic case, so far as the author is aware.* It seems plausible to suppose that the strong spin-dependent coupling in ferromagnetic materials will assure that the relaxation frequency will occur above the microwave range. This question should be looked into more closely.

It should be noted in passing that the collision frequency of the lattice phonons at room temperature is $\sim 10^{12}$ collisions per second, as estimated from values of the thermal conductivity of nonmetallic crystals. This figure determines an approximate upper limit to the order of magnitude of the spin relaxation frequency. The actual spin relaxation frequency may be lower depending on the strength of the coupling between the spins and the lattice. In metals the relaxation frequency of the lattice phonons is estimated to be of the order of 10^{13} sec.$^{-1}$ at room temperature, based on electrical conductivities.

VI. DISCUSSION

Arkadiew[25] first suggested that eddy current effects might be important in ferromagnetic dispersion. This approach was developed further by Becker,[4] who pointed out that the local microscopic eddy currents associated with the movement of domain boundaries and the rotation of domains set up a magnetic field which opposes the applied field. This back field adds a term to the equations of motion which is proportional to the velocity of boundary movement or spin rotation; that is, the eddy currents behave like a viscous force. Becker's treatment gives a good qualitative account of the damping of irreversible displacements characterizing magnetization in *medium* fields, at frequencies *below* the microwave range, although an apparent difficulty in reconciling these results with the measurements of Sixtus and Tonks on the velocity of boundary propagation has been suggested by Miss van Leeuwen.[26] It should be pointed out that the local eddy current effects considered by Becker have no direct connection with the use made of the eddy current equation in the present paper, according to which the incomplete penetration of the surface domains by the applied field is the major cause of dispersion.

Becker also has given a calculation for the dispersion of the initial permeability, and this calculation leads to results in some respects similar to those of the present paper. The "back field" is calculated as in the medium field strength case just mentioned. The basis of Becker's theory supposes that the skin depth is *greater* than the domain dimensions, so that the calculation is not applicable to the microwave range, where the skin depth is less than the domain dimensions. At 3×10^3 mc/sec. the skin depth is only ~ 0.1 of the domain thickness.

The present theory probably could be improved by working with a more complicated model than that of a film one domain thick. If the film is backed on one side by a mass of ferromagnetic material the motion of the domain boundaries in the film will induce eddy currents in the backing material. The currents will produce a back field giving rise to additional damping. However, the correction to the permeabilities calculated in this paper is expected to be small, since on the low frequency side of the dispersive region the eddy current damping is small anyway, while on the high frequency side the failure of the applied field to penetrate the film leads to greatly restricted domain movement and hence the additional back field is unimportant.

It does not seem worth while at this time to attempt to calculate the permeability with a more elaborate model. The present model gives results in reasonable agreement with experiment, and the dispersive mechanism proposed here appears to correspond to the physics of the situation. The most important direction in which the model should be extended would seem to be in a treatment of the case in which the surface energy of the domain wall is small in comparison with the magnetization energy, so that the domain wall yields locally to the field. The dispersion would be owing to the magnetic inhomogeneity of the material. The model treated in the present paper supposes that the domain wall moves rigidly under the influence of the applied

[24] See, for example, I. Waller, Zeits. f. Physik **79**, 370 (1932). The writer is indebted to Professor L. Tisza for several discussions of the paramagnetic relaxation problem.

* Note added in proof: A paper bearing on this point has just appeared: A. Akhieser, J. Phys. U.S.S.R. **10**, 217 (1946).

[25] W. Arkadiew, Comptes rendus, Acad. Sci. U.R.S.S. (Doklady) **2**, 204 (1935); see also reference 3.

[26] H. J. van Leeuwen, Physica **11**, 35 (1944).

field. There are reasons for believing that both cases may occur in different actual materials.

It should be pointed out that the information regarding domain behavior obtained from dispersion measurements on metals pertain only to the domains in the surface layers of the material. With this qualification, dispersion measurements may prove to be an important method for studying domain mechanisms.

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APPENDIX A

Relation of Intrinsic Surface Impedance to Resistive Losses and Inductance of Film

It can be shown that the resistive losses in the film considered in Section IV and the contribution of the film to the inductance of a circuit element are related directly to the intrinsic surface impedance which is defined according to Eq. (22) by $Z=E_z/H_x$, evaluated at $y=d$.

The average rate of energy loss per unit area normal to the y direction is given by the average value of the Poynting vector

$$S=-\mathrm{Re}[(c/4\pi)E_z(d)H_x^*(d)],$$

when it is recalled that the film has two surfaces. Now $E_z=ZH_x$, so that

$$S=-(c/4\pi)H_x(d)H_x^*(d)\mathrm{Re}[Z],$$

a well-known result.

The contribution of the film to the inductance of the magnetizing circuit is given by the quotient of the magnetic flux through the film by the current in the magnetizing circuit:

$$L=\mathrm{Re}\left[\mu\int_{-d}^{d}H_x dy/J\right].$$

Now $J=(c/4\pi)H_x(d)$ and $\mu H_x=+j(c/\omega)(\partial E_z/\partial y)$, so that

$$\omega L=8\pi\mathrm{Re}[jZ]=-8\pi\mathrm{Im}[Z].$$

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Decay in Brightness of Infra-Red Sensitive Phosphors*

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An equation is derived for the decay in brightness of infra-red (IR) sensitive phosphors under constant IR stimulation. The derivation is based on a second-order process, but takes into account the absorption of the stimulating IR and the emitted visible light. The theoretical curves obtained differ markedly from the simple second-order decay curve, especially when the IR absorption is greater than the visible absorption. The equation involves three arbitrary constants which depend on the quantum efficiency, the number of active centers, and the absorption coefficients of the phosphor for visible and infra-red radiation. The equation is shown to fit the decay curve of a strontium selenide phosphor for an effective decrease in brightness of 10^5. It is shown that theories involving a distribution of electron traps, with or without interaction, lead to conclusions that do not agree with the experimental results. Some experimental evidence is offered indicating that there is no change in the trap distribution as this phosphor is exhausted. The apparent deepening of the traps as the decay process continues can be attributed to the absorption of the infra-red.

THERE have been developed recently by Urbach, Ward, Fonda, and others[1] a group of phosphors that show a remarkable increase in brightness when irradiated with infra-red (IR) radiation. This effect can be made to occur after the normal phosphorescence has decreased to a negligible value. Although a good deal of progress has been made in the technique of making such phosphors, there is still uncertainty regarding the mechanism. An analysis of the decay of the

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[1] Proceedings of the March 1946, meeting of the Optical Society of America, J. Opt. Soc. Am. **36**, 351–353 (1946).