Introducing polar coordinates we get in the usual way:

$$\psi = P_l^m(\vartheta, x)\cdot\varphi(r)/r, \tag{5}$$

where $\varphi(r)$ has to be solved from the following equation (restricting ourselves to S-states):

$$\frac{d^2\varphi}{dr^2}+\frac{2\mu}{\hbar^2}(E-U(r))\varphi=0. \tag{6}$$

We look for a solution of (6) satisfying the following boundary conditions:

$$\varphi=0 \quad \text{for} \quad r=0, \tag{7a}$$

$$\varphi=0 \quad \text{for} \quad r=\infty. \tag{7b}$$

(At this point Morse introduces a different boundary condition: $\varphi=0$ for $r=-\infty$ instead of (7a); since r is a polar coordinate it will, however, never take negative values. Furthermore, Morse remarks himself that φ should be equal to zero for $r=0$ but he adds that for the eigenvalues of E, which he finds, $\varphi(0)$ is very small. This last fact corresponds just to the outcome that Eq. (2) is such a good approximation for the energy levels.)

Introducing expression (1) for $U(r)$ into Eq. (6) and applying condition (7b) we get the following solution for φ:

$$\varphi(r)=N\cdot z^{Ap}\cdot e^{-z}\cdot M(Ap+\tfrac{1}{2}-A\sqrt{D}, 2Ap+1; 2z), \tag{8}$$

where

$$z=A\sqrt{D}\cdot\exp\left[-a(r-r_0)\right], \tag{9}$$

and $p=(-E)^{\frac{1}{2}}$, N a normalizing factor while $M(\alpha,\beta;x)$ is the confluent hypergeometric series, satisfying):[3]

$$x\frac{d^2M}{dx^2}+(\beta-x)\frac{dM}{dx}-\alpha M=0. \tag{10}$$

Applying now condition (7a) and using (9) we find the energy levels of the closed stationary states from the equation:

$$M(Ap+\tfrac{1}{2}-A\sqrt{D}, 2Ap+1; 2A\sqrt{D}\exp(ar_0)). \tag{11}$$

In all applications to diatomic molecules $2A\sqrt{D}\cdot\exp(ar_0)$ is large so that we can use the asymptotic expression for M:[3]

$$M(\alpha,\beta;x)\sim\frac{\Gamma(\beta)}{\Gamma(\beta-\alpha)}(-x)^{-\alpha}\left(1-\alpha\cdot\frac{\alpha-\beta+1}{x}+\cdots\right)+\frac{\Gamma(\beta)}{\Gamma(\alpha)}e^x x^{\alpha-\beta}\left(1+\frac{(1-\alpha)(\beta-\alpha)}{x}+\cdots\right) \tag{12}$$

and even:

$$M(\alpha,\beta;x)\sim\frac{\Gamma(\beta)}{\Gamma(\alpha)}e^x x^{\alpha-\beta}, \tag{13}$$

which gives the following zero points:

$$\Gamma(\alpha)=\infty \quad \text{or} \quad \alpha=-n, \quad n=0,1,2,\cdots. \tag{14}$$

Comparing this with (11) we get:

$$p=\sqrt{D}-\frac{n+\frac{1}{2}}{A}, \tag{15}$$

which gives with $p=(-E)^{\frac{1}{2}}$ just Eq. (2). We see, however, that this is an *approximation* and not the rigorous solution. The deviations are, however, so small as to be negligible in every case of diatomic molecules. From (12) it is easily seen that these deviations are only beginning to be appreciable if:

$$\left.\begin{array}{l} A\sqrt{D}\cdot(\exp(ar_0)-ar_0-1)\lesssim 1 \quad (r_0\neq 0) \\ \text{or} \\ A\sqrt{D}\lesssim 1 \quad (r_0=0). \end{array}\right\} \tag{16}$$

Morse, Fisk, and Schiff[4] have used the potential (1) for the interaction between two nucleons and in this case (16) is satisfied and they use indeed Eq. (11) to determine the relation between E and D; they do not, however, refer to Morse's original paper[1] and they do not point out the fact that for all cases of diatomic molecules Eq. (2) is the solution of Eq. (11). In Table I we have collected for different values of r_0 and $A(-E)^{\frac{1}{2}}$ the solution of $A\sqrt{D}$ from (11) according to Morse, Fisk, and Schiff[4] and the solution of $A\sqrt{D}$ from (2) (between brackets). As should be expected, the deviations decrease for increasing $A\sqrt{D}$ and ar_0.

TABLE I. Values of $A\sqrt{D}$.

ar_0 \ $A\sqrt{-E}$	0	0.5	1.0
0	0.90 (0.50)	1.48 (1.00)	2.03 (1.50)
0.5	0.73 (0.50)	1.23 (1.00)	1.71 (1.50)
1.0	0.61 (0.50)	1.07 (1.00)	1.54 (1.50)

Finally we may add that a similar point arises for the potential introduced by Rosen and Morse.[5] The boundary conditions in this case should be (using their notation):

$F=0$ for $u\rightarrow 1$ and for $u=\frac{1}{2}$ (and not for $u=0$).

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[1] P. M. Morse, Phys. Rev. **34**, 57 (1929).
[2] E.g., H. Hellmann, *Einführung in die Quantenchemie* (Leipzig-Wien, 1937), p. 294; G. Herzberg, *Molecular Spectra and Molecular Structure* (New York, 1939), Vol. I, p. 109. (Herzberg gives Eq. (2) and adds: *without* any higher powers of $n+\frac{1}{2}$ (italics of Herzberg).)
[3] See e.g., E. Jahnke and F. Emde, *Tables of Functions* (New York, 1945), Chap. X.
[4] P. M. Morse, J. B. Fisk, and L. I. Schiff, Phys. Rev. **50**, 748 (1936).
[5] N. Rosen and P. M. Morse, Phys. Rev. **42**, 210 (1932).

Cosmic Radiation Above 40 Miles

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WE have obtained cosmic-ray data above the earth's atmosphere by means of an apparatus contained in a German V-2 rocket. The rocket was fired by the Ordnance Department, United States Army on June 28, 1946 in connection with a series of tests being made by the Army at its White Sands, New Mexico, proving grounds.

Data were transmitted back to a receiving station on the ground by means of a multi-channel radio equipment. Difficulties which developed in this and accompanying electronic circuits prevented satisfactory records below 200,000 feet. Forty-one seconds of data were obtainable after this time, all of it on ascent. Maximum altitude obtained was 350,000 feet.

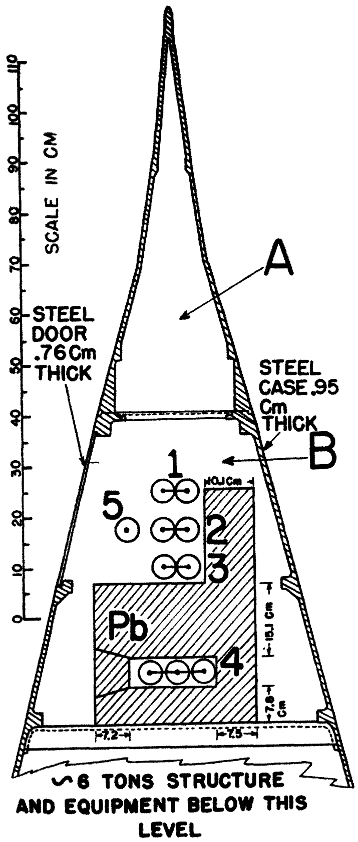

FIG. 1. Arrangement of cosmic-ray equipment as mounted in the warhead.

Figure 1 is a schematic drawing of the cosmic-ray equipment as mounted in the warhead. Chamber A above the cosmic-ray chamber had a total weight, including contents, of 100 pounds, which was almost entirely steel. If this is considered spread uniformly across the top of chamber B, it is equivalent to about 7.8 centimeters of iron. Single counts in counters 1, 5, and 4 were transmitted, as well as coincidences (1, 2, 3), (2, 3, 4), (3, 5), and (1, 3, 5). In addition, coincidence between each of these various data could be read off the record on the ground. Coincidence resolving times in the rocket were 20×10^{-6} sec., while the resolving time for inter-channel coincidences on the ground was 5×10^{-3} sec. The cosmic-ray counters were fastened in a light aluminum rack which could be removed from the lead for solid angle calibration. A series of calibration runs was made both at Washington and at White Sands, (altitude 4000 feet, geomagnetic latitude 42°N).

Considering first the data from the single counters, counters 1 showed an increase in rate above 200,000 feet over the rate on the ground of 21.3 ± 1.0 times. Counter 5 showed an increase of 20.7 ± 1.2 times. Counters 4 gave 34.9 ± 1.7 for this ratio. In a separate experiment on the ground at White Sands with a vertical telescope, it was determined that the ratio of hard count to total count was 0.651 ± 0.024. If the primary rays are all hard, then the shielded counters (4) should have a ratio of counting rate in flight to ground rate higher than that for the unshielded counters by the ratio 1 to 0.651. Reduction of the shielded counting rate by the reciprocal of this factor gives 22.7 ±1.4, which agrees with the ratios for the other counters within probable error. Probable errors are determined from statistics only. Counting rates in flight were 36.2/sec., 22.0/sec., and 39.2/sec. for counters 1, 5, and 4, respectively.

The data from the coincidence channels were as follows: (1, 2, 3) increased by a factor of 56 over the ground rate, (3, 5) by a factor of 150, and (1, 3, 5) by a factor of 420. Channel (2, 3, 4) developed an electronic defect and furnished no usable data. Of the 61 counts observed in 28.6 sec. in the shower channel (1, 3, 5) 49, or 80 percent, accompanied coincidences (1, 2, 3). The latter channel in this time had 103 counts. Thus, 49/103 or 48 percent of the counts in (1, 2, 3) were accompanied by showers. This presumably accounts for the higher increase in counting rate than the single counter results indicate.

The effect of the warhead structure as determined in the ground calibration was to increase the soft part of the count in (1, 2, 3) by a factor of 2.2 over the rate without warhead. At the same time, a shower in (1, 3, 5) was recorded for each 6.6 soft counts recorded in (1, 2, 3) with the warhead in place. The high shower to total count ratio in flight probably indicates therefore, that showers of many particles are produced at high altitudes in the structure adjacent to the counters.

Further experimental work is being undertaken for future flights. The present data are perhaps best regarded as provisional pending subsequent corroboration. In particular, whatever effect the high shower count may have had on the single counters has not been completely determined.

The writers are indebted to their colleagues in the Rocket Sonde Section, Naval Research Laboratory, and to M. Schein for suggestions concerning the problem.

Single Scattering and Annihilation of Positrons

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I

IN the process of collisions between electrons, it is impossible to distinguish, after the collision, between the recoil electron track and that of the incident β-particle. One cannot, therefore, separate the cases of strong energy exchange (large angle of scattering, θ) from those of weak energy exchange ($\sim\frac{1}{2}\pi-\theta$). But in the case of collisions between positrons and electrons, if one investigates them in a cloud chamber with a magnetic field, one can distinguish the particles easily according to the sense of their curvatures, which permits one to study the problem of single scattering more in detail. But until now, as we know, such kind of collision has not yet been reported.