# The Disintegration of $Na^{24}$ and $P^{32}$

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The disintegration of $Na^{24}$ and $P^{32}$ has been studied by means of a $\beta$-spectrometer of the lens type. The $\gamma$-radiation of $Na^{24}$ consists of only two $\gamma$-lines incascade, the energies of which have been determined as 1.380 and 2.758 Mev. $E\beta_{max}$ for $Na^{24}$ is 1.390 Mev and for $P^{32}$ 1.712 Mev. In order to obtain $\beta$-spectra without secondary electrons, exceedingly thin foils and very small quantities of active materials have been used. With these precautions it appears that the electron distributions follow the allowed form of $\beta$-spectra. However, $Na^{24}$ and $P^{32}$ are empirically forbidden spectra of the first and second order, respectively. According to the theory of forbidden spectra developed by Uhlenbeck and Konopinski, the distribution for at least $P^{32}$ would differ from the allowed one.

## INTRODUCTION

$Na^{24}$ and $P^{32}$ are among the artificially active elements that, for various reasons, have been most frequently studied. The most important reasons are naturally the high activity and the convenient half-life periods obtained by bombardment of Na and P with deuterons. Furthermore, the problem of the disintegration of $Na^{24}$ and $P^{32}$ has a special relevancy to the $\beta$-theory, as the study of their energy-distribution curves in the $\beta$-disintegrations, which are forbidden of the 1st and 2nd order, respectively, may yield important information regarding the nature of the matrix elements occurring in the expressions for the disintegrations.

So far no results have been obtained that agree sufficiently well to permit a theoretical interpretation with any degree of certainty. In the case of $Na^{24}$, further complications are introduced by the $\gamma$-radiation accompanying the $\beta$-disintegration.

## $\gamma$-LINES OF $Na^{24}$

Opinions still seem to differ regarding both the number of $\gamma$-lines and their energies. The first determination of $Na^{24}$ $\gamma$-lines was carried out by Richardson and Kurie[1] by studying the Compton effect from Al in a Wilson chamber. They obtained three well-defined lines at 0.95, 1.93, and 3.08 Mev. This result was confirmed by a later investigation by Richardson[2] who found, using the same method, 1.01, 2.04, and 3.00 Mev. Later investigations with the spectrographic method, however, did not give concordant results. Japanese workers[3] first obtained the energy values 0.80, 1.49, and 2.97 Mev. Later investigations indicated[4] that only two lines existed, namely at 1.38 and 2.76 Mev. Since this result differs remarkably from the Wilson chamber determinations, the $\gamma$-lines of $Na^{24}$ have been subjected to further investigations by other methods. M. Goldhaber, G. S. Klaiber, and G. Scharff-Goldhaber[5] have determined with the aid of the photo-neutron method, using the processes $Be^9(\gamma, n)Be^8$ and $D^2(\gamma, n)H^1$ (applicable only for $\gamma$-radiation of energy higher than the threshold values for the reactions concerned, i.e., 1.62 and 2.18 Mev), the upper $\gamma$-energy value as 2.87 Mev.

P. G. Kruger and W. E. Ogle,[6] using another

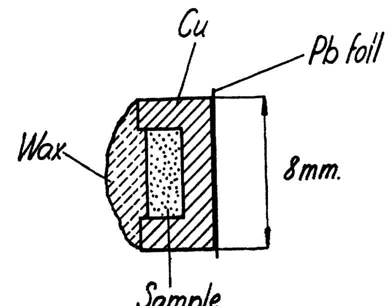


FIG. 1. The radiator for $\gamma$-ray studies.

[1] J. R. Richardson and F. N. D. Kurie, Phys. Rev. 50, 999 (1936).
[2] J. R. Richardson, Phys. Rev. 53, 124 (1938).
[3] S. Kichuchi, Y. Watase, J. Itch, E. Takeda, and S. Yamaguchi, Proc. Phys. Math. Soc. Japan 21, 260, 381 (1939).
[4] L. G. Elliott, M. Deutsch, and A. Roberts, Phys. Rev. 63, 386 (1943).
[5] M. Goldhaber, G. S. Klaiber, and G. Scharff-Goldhaber, Phys. Rev. 65, 61 (1944).
[6] P. G. Kruger and W. E. Ogle, Phys. Rev. 67, 273 (1945).

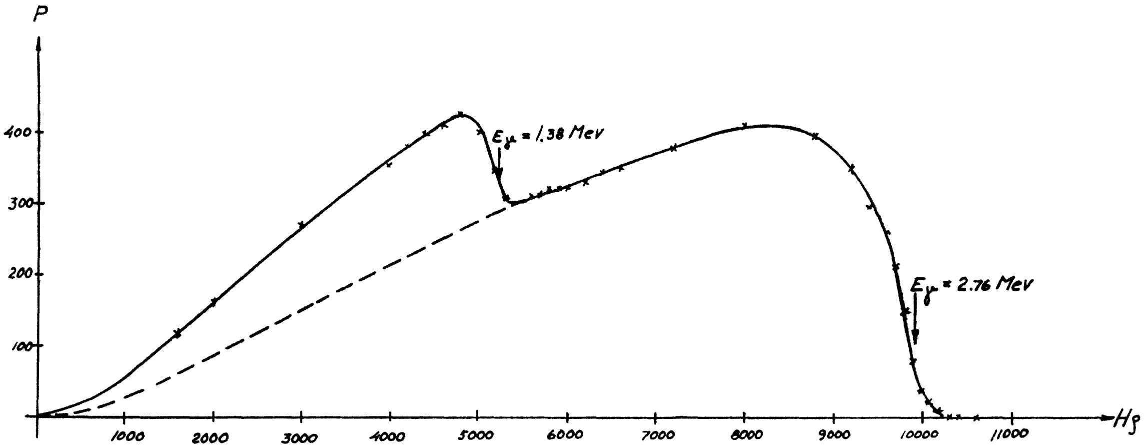


FIG. 2. Secondary electron spectrum expelled from a Cu radiator by the $\gamma$-rays of $Na^{24}$. The curve is in the undivided form.

method, have recently obtained a result differing significantly from those of earlier determinations. They studied the pair formation in a Wilson chamber. Out of 200 pairs, 56 were accepted for measurement. No less than seven $\gamma$-lines were reported, namely at 3.24, 2.89, 2.76, 2.68, 2.56, 1.38, and 1.26 Mev. As an explanation of these, the $\beta$-spectrum is considered to be complex, consisting of three components.

In order to investigate the $\gamma$-radiation from $Na^{24}$, the present author has employed a technique permitting accurate energy determinations on the $\gamma$-radiation from samples with $\gamma$-activities as low as a few $\mu$C per $\gamma$-line.[7] A similar technique has been described by Deutsch, Elliott, and Evans.[8] The active sample is placed in a small, cylindrical radiator of the form shown in Fig. 1. The wall is so thick that all $\beta$-radiation is absorbed. The thickness may be calculated with the aid of Feather's formula for the range of the $\beta$-radiation, $R=0.543E-0.160$ g/cm$^2$, where $E$ is the energy of the $\gamma$-radiation in Mev. The Compton electrons formed in the radiator by the $\gamma$-radiation are then studied in the $\beta$-lens spectrograph of high light intensity, described earlier by the present author.[9] In order to limit the size of the radiator, copper is used, which has a high density but a low atomic number. The photo-effect will thus appear only at very low energies. After the Compton spectrum has been recorded, a thin lead foil $(0.02-0.2$ mm$)$ is placed in front of the radiator, and the distribution of the photo- and Compton electrons is registered. By subtracting one of the two curves from the other, the photo-effect in the thin lead foil is obtained as the essential part of the remainder.

Figure 2 shows a recording of the Compton distribution in the copper radiator. The numbers of impulses counted per unit time ($P$) are here

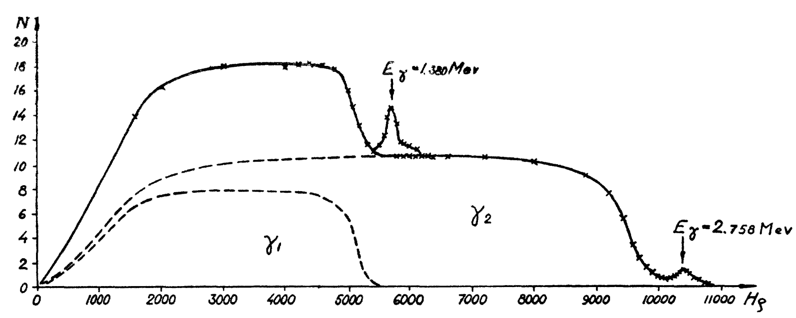


FIG. 3. Secondary electron spectrum expelled from a Cu radiator covered with a 0.02-mm Pb-foil by the $\gamma$-rays of $Na^{24}$.

[7] K. Siegbahn, Phil. Mag. (1946), in print.
[8] M. Deutsch, L. G. Elliott, and R. D. Evans, Rev. Sci. Inst. 15, 178 (1944).
[9] K. Siegbahn, Arkiv. f. mat., astr. o. fysik 28A, No. 17 (1944).

plotted as ordinates, and not the intensities over constant $H\rho$-intervals. In order to obtain these latter values, we must divide $P$ by the corresponding value of $H\rho$. Figure 2 clearly shows that there are two well-defined $\gamma$-lines. It is easy enough, using this method of plotting, to extrapolate the Compton distribution of the highest energy to zero energy (the broken line in the figure).

Figure 3 shows the distribution obtained with a 0.02-mm thick Pb foil placed in front of the radiator. The intensities over constant $H\rho$-intervals have here been chosen as ordinates. It is here also clear that only two $\gamma$-lines exist. Quite a small possibility for the existence of more than two $\gamma$-lines naturally remains, if the two $\gamma$-lines should happen to be closer together than what the spectrograph would resolve. The resolving power of the spectrograph amounted, on this occasion, to somewhat more than 3 percent, which, however, is in good concordance with the half-breadth value of the two lines, determined as $\sim$3.5 percent. It therefore seems rather unlikely that any of the photo-lines is complex.

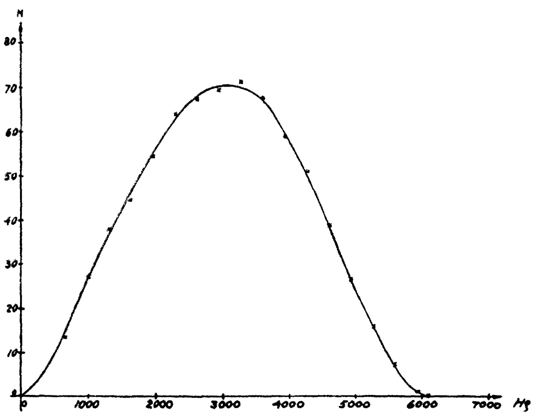


FIG. 4. The $\beta$-spectrum of $Na^{24}$.

The energies of the $\gamma$-lines are obtained with great accuracy from the corresponding $K$-photo-lines as 1.380 and 2.758 Mev, in perfect concordance with the above-quoted work of Elliott, Deutsch, and Roberts.[4]

The energies may obviously be obtained direct from the Compton distributions. It has been proved, in investigations of several other $\gamma$-lines employing this method, that $\gamma$-energy values may be obtained that very well agree with the results from the photo-lines, if measurements are made on the inflection points at the edge of the undivided Compton distribution (Fig. 2). When this procedure is employed, the $\gamma$-energies are

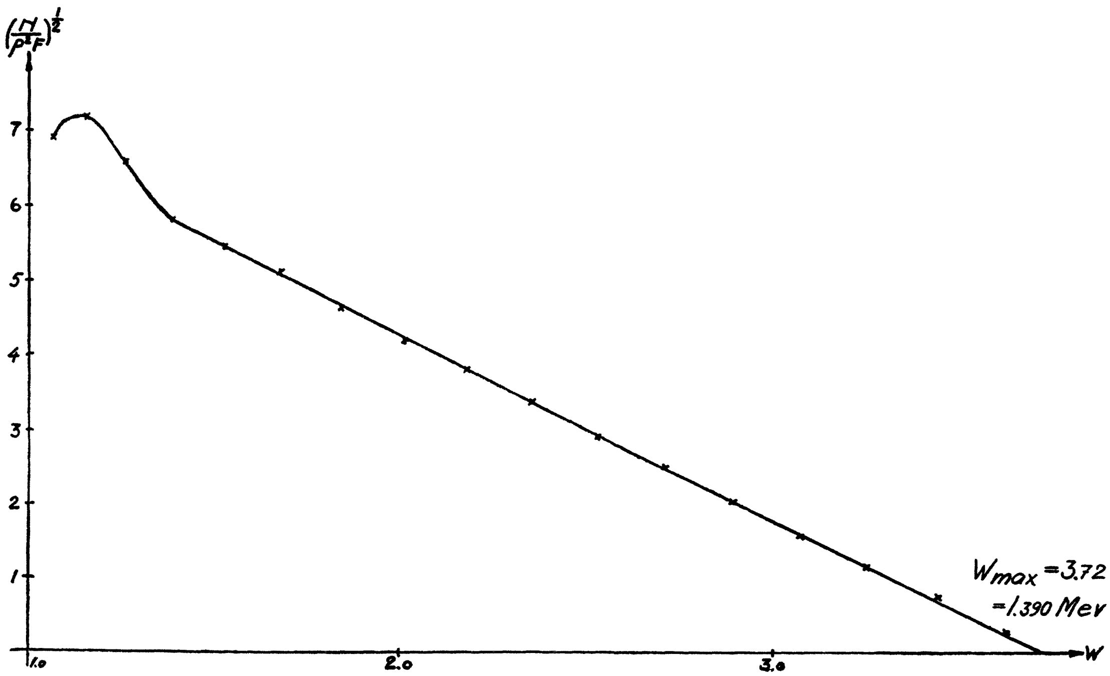


FIG. 5. The Fermi diagram for $Na^{24}$.

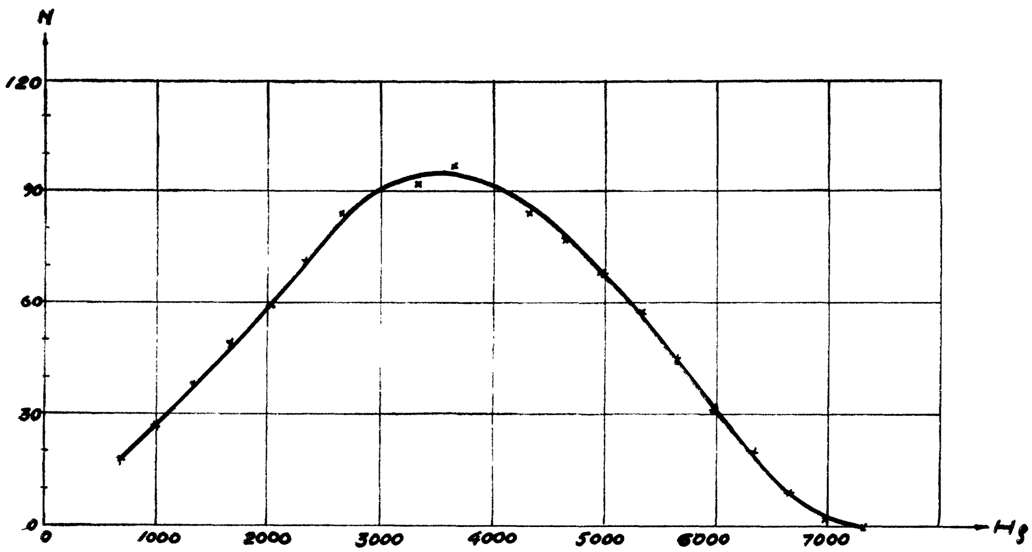


FIG. 6. The $\gamma$-spectrum of $P^{32}$.

determined as 1.38 and 2.76 Mev. Kruger and Ogle report, among other lines, a weak $\gamma$-radiation of 3.24 Mev (originating from one electron-positron pair). Though searching carefully for possible weak $\gamma$-lines above 2.76 Mev, the present author could not establish the presence of such.

### THE $\beta$-SPECTRUM OF $Na^{24}$

The $\beta$-spectrum of $Na^{24}$ has been studied by several authors with the aid of the $\beta$-spectrograph. Of these, Lawson's measurement is probably the most accurate.[10] Lawson used 3.5 mg substance per $cm^2$ on a substratum of 7.4 $mg/cm^2$. According to the experience of the present author, even such small quantities may cause a deformation in the spectrum, owing to secondary effects in the sample itself and in the substratum. Accordingly, the irradiation was performed in the 32″ cyclotron of this institute, so that the activity of the sodium sample became very high. A highly diluted drop of the sample was then allowed to evaporate on a backing foil, about 0.1 $\mu$ thick and suspended on a small ebonite ring of 2 cm internal diameter. Owing to the high light intensity of the spectrograph, the quantity of dry substance, in the order $\sim$0.1 mg, was sufficient to give good intensity in the registration of the $\beta$-spectrum. The result of an experimental series is shown in Fig. 4. A Fermi analysis of the spectrum (Fig. 5) gives as the upper limit $W_{max} = 1.390 \pm 0.005$ Mev.

In spite of the fact that $Na^{24}$, according to its place in the Sargent diagram, is a forbidden transition of the 1st order, the spectrum is of the allowed type, as is seen from the diagram. As a matter of fact, the Fermi curve is absolutely straight as far down as an energy $<0.2$ Mev, where a small peak appears. It is interesting to observe here that the more carefully the secondary radiation is eliminated, the nearer the curve approaches a straight line. The curve in Lawson's diagram deviates from a straight line at about 0.6 Mev. Judging from these two measurements, an extrapolation down to a negligible secondary

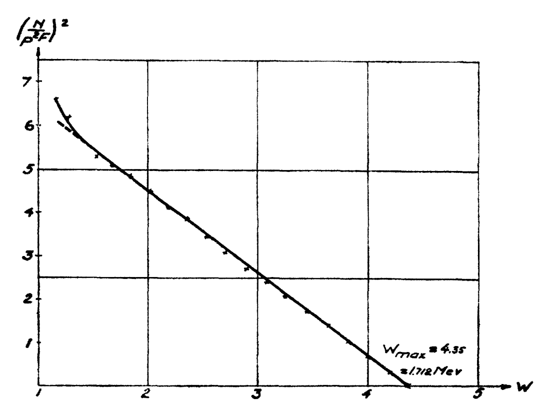


FIG. 7. The Fermi diagram for $P^{32}$.

[10] J. L. Lawson, Phys. Rev. 56, 131 (1939).

radiation would very probably give a straight line throughout the energy range.

### THE β-SPECTRUM OF P$^{32}$

The same impression is obtained by studying the $\beta$-spectrum of P$^{32}$. Here also, the optimum conditions are realized in respect of high specific activity. Furthermore, no $\gamma$-radiation, which might contribute to a secondary radiation, is present. Figure 6 shows the recording of the $\beta$-spectrum, the weight of the substance and substratum being approximately the same as for Na$^{24}$. Figure 7 is the corresponding Fermi diagram, from which the upper limit can be determined as $1.712 \pm 0.008$ Mev. The Fermi diagram consists in this case of a straight line as far down as an energy $<0.1$ Mev. The recording of the $\beta$-spectrum of phosphorus by Lawson, who used a sample weighing about 3 mg/cm$^2$, deviates from the straight line at about 0.8 Mev. It seems therefore probable that, under ideal conditions, the curve will follow a straight line along its whole course.

### DISCUSSION

#### The Term Scheme of Na$^{24}$

The Fermi analysis of the Na$^{24}$ $\beta$-spectrum shows quite clearly that the transition is simple and not complex. This interpretation is also supported by the experiments concerning the number of $\beta$-$\gamma$-coincidences per registered number of $\beta$-particles as a function of the $\beta$-energy, carried out by Langer, Mitchell, and McDaniel[11] and also by Feather and Dunworth,[12] who find the coincidence number to be independent of the $\beta$ energy. Konopinski[13] has suggested, in order to explain the deviation from the straight line in the Fermi analysis of Lawson's recording,[10] the possibility of internal conversion lines. Despite the fact that Mg$^{24}$, the end-product of the $\beta$-emission, is a light nucleus, the possibility for internal conversion is not quite excluded, provided that the $\gamma$-radiation has a high multipole character. The difference in spin between the excited levels and the ground state in Mg$^{24}$ must

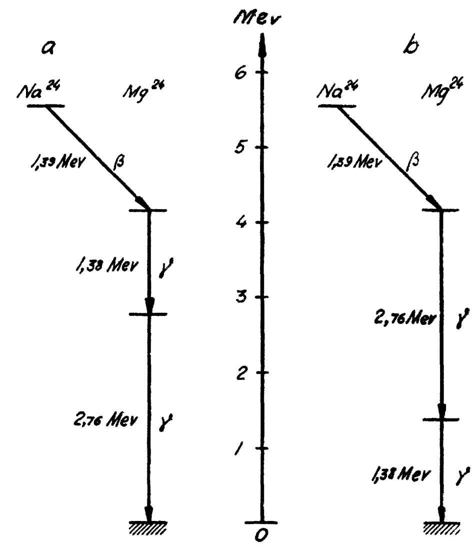


FIG. 8. Nuclear level scheme for the disintegration of Na$^{24}$.

actually be fairly large, for the direct $\beta$-transition from Na$^{24}$ to the ground state of Mg$^{24}$ is so strictly forbidden that no $\beta$-radiation with this energy can be observed, in spite of the fact that the energy release should be considerably greater. This possibility, however, must be rejected, judging from the present investigation, since the deviation from the straight line in the Fermi diagram has a tendency, when the secondary radiation is weaker, to disappear completely, and since no $\gamma$-line below 1.37 Mev has actually been encountered.

As the $\beta$-spectrum is simple and only two $\gamma$-lines are emitted, the term scheme should have a simpler appearance than that suggested by Kruger and Ogle.[6] The two alternatives possible are shown in Fig. 8. The order of sequence of the two $\gamma$-lines in cascade is, of course, impossible to determine from the present experimental material.

The mass of Na$^{24}$ may be obtained from the term scheme and the spectroscopically determined mass of Mg$^{24}$. If the latter be assumed to be $23.99300 \pm 0.00038$, the mass of Na$^{24}$ is obtained as $23.99300 + 1.074.10^{-3} \times 5.53 = 23.99893 \pm 0.00045$.

According to the term scheme, the two $\gamma$-lines must have the same intensity. The ratio of the "$\gamma$-efficiencies" of the radiator for the two $\gamma$-energies may thus be obtained directly by dividing the areas of the two Compton distribu-

[11] L. M. Langer, A. C. Mitchell, and P. W. McDaniel, Phys. Rev. **56**, 962 (1939).
[12] N. Feather and J. V. Dunworth, Proc. Camb. Phil. Soc. **34**, 442 (1938).
[13] E. J. Konopinski, Rev. Mod. Phys. **15**, 209 (1943).

tions. It does not appear to be excluded that an efficiency curve for copper, applicable to $\gamma$-GM tubes in, for instance, coincidence measurements, can be constructed in this way with the aid of additional $\gamma$-energies. At low energies, the contribution from the photo-effect ($<0.5$ Mev) should naturally be taken into account. The $\gamma$-sensitivities of the radiator at 2.76 and 1.38 Mev are in the ratio $\sim 2.65:1$, according to Fig. 3. This value is larger than would be expected from the efficiency curves for copper $\gamma$-tubes reported in the literature. No definite conclusion, however, can be drawn in this matter until the influence of the geometrical factors on the $\gamma$-efficiency of the radiator has been more closely investigated.

## THE FORM OF THE $\beta$-SPECTRA AND $\beta$-THEORY

The $\beta$-theory of forbidden transitions has been developed by E. J. Konopinski and G. E. Uhlenbeck[14] (cf. also E. J. Konopinski[15]). In contradistinction to the theory of allowed transitions, that of forbidden transitions gives different expressions for the energy distribution, depending on which of the five forms of interaction (scalar, polar, tensor, axial, pseudoscalar) is supposed to apply. Thus a study of $Na^{24}$ (empirically first forbidden) and $P^{32}$ (empirically second forbidden) might decide in principle which form of interaction is the correct one. Uhlenbeck and Konopinski give their result in the form of an energy-dependent factor $C$, which, when multiplied by the allowed Fermi function, will give the energy distribution for forbidden transitions. This factor is dependent on which assumption is made regarding the above-mentioned forms of interaction, and also on whether the forbidden transition is of the first or second order. It is interesting to consider whether the rather surprising result, that $Na^{24}$ and $P^{32}$ are spectra of the allowed type, is compatible with the above-mentioned theory. The condition for this is that the factor $C$ may be rendered energy-independent by suitable assumptions regarding the relative magnitudes of the nuclear matrix elements in $C$. Starting with $Na^{24}$ (following the notation of Uhlenbeck and Konopinski), the factors $C_{1S}$ and $C_{1P}$ can never be independent of energy and are hence rejected. $C_{1V}$, $C_{1T}$, and $C_{1A}$ are energy-independent, if the matrix elements $|\int \alpha|^2$, $|\int \alpha|^2$, and $|\int \gamma_5|^2$, respectively, are considerably larger than other matrix elements, such as $|\int r|^2$, $|\int \sigma \times r|^2$, etc. According to U. and K., such a relation between the relevant matrix elements is conceivable.

It is considerably more difficult, however, to combine the allowed type of the $P^{32}$ spectrum with an energy-independent factor $C_2$. Actually none of the various forms of interaction gives a factor $C_2$ that is energy-independent, as each matrix element occurs multiplied by energy functions. The possibility certainly remains that the different energy functions of each factor may together, by choosing matrix elements of suitable magnitudes, be made approximately energy-independent along a large interval of energy. A numerical investigation* shows, however, that $C_2$ cannot, even approximately, be made energy-independent for all choices of relative magnitudes for the matrix elements. Moreover, it seems unlikely that various special assumptions need be made in order to obtain, from rather complicated expressions, the simple form of energy distribution found in the experiments with both $Na^{24}$ and $P^{32}$. A possibility, not to be excluded, is that both $Na^{24}$ and $P^{32}$ actually are allowed as regards both spin and parity but forbidden by other rules of selection. It is also possible that further forms of interaction are to be taken into consideration.

[14] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).

[15] E. J. Konopinski, Rev. Mod. Phys. **15**, 209 (1943).

* The author wants to thank Dr. Svartholm for valuable discussion on this point.