COMPARISON OF THE FALL OF A DROPLET IN A LIQUID AND IN A GAS.

By O. W. Silvey.

Introduction.

In his early work on the determination of the elementary electrical charge by the oil drop method, Professor Millikan[1] found that the values of the charge computed from droplets of different radii indicated a breakdown in Stokes's[2] formula by which the radii were determined. Since this formula was deduced on the assumption of a continuous medium and no slipping at the surface, a breakdown in the law was to be expected when these conditions were not fulfilled, and various attempts have been made to modify the formula so as to make it apply to cases where slipping occurs and where the medium becomes discontinuous. Among the formulæ making such corrections are those of Basset,[3] Cunningham,[4] and Hadamard.[5] Basset treated the phenomenon as a case of slipping at the surface; Cunningham made use of the kinetic theory point of view and assumed that the medium becomes discontinuous when the ratio of the mean free path of the molecules of the medium to the radius of the drop becomes appreciable, and that the departure from the law is further modified by the fraction of the whole number of impacts which are elastic. Hadamard deduced a correction for the case of liquid drops moving in a fluid, assuming a vortex motion of the liquid within the drop due to its motion through the fluid. He wrote his correction in terms of the coefficient of viscosity of the liquid and that of the fluid through which it moves.

To account for the apparent variation of the value e, Millikan wrote empirically the correction to Stokes's law as a function of the ratio of the mean free path to the radius of the drop, which when higher powers of $1/a$ than the first are neglected, reduces to the approximate formula:

$$v = 2/9\, g \frac{a^2(\sigma - \rho)}{u} (1 + A\ 1/a),$$

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[1] Millikan, Phys. Rev., Vol. 32, 1911, p. 350.

[2] G. G. Stokes, Mathematical and Physical Papers, III, p. 59.

[3] Basset, Hydrodynamics, Vol. II., p. 270.

[4] Cunningham, Proc. of the Royal Soc. of London, Series A, Vol. 83, 1909–1910, p. 357.

[5] Hadamard, Comp. Rendus, March, 1911.

where v is the velocity of fall, a the radius of the spherical droplet, σ the density of the falling substance, ρ the density of the surrounding medium, u its viscosity, l the mean free path of the molecules of the medium, and A an empirical constant.

Coincident with the determination of the absolute value of the elementary charge, by plotting the apparent value of $e^{2/3}$ as ordinates and $1/a$ as abscissas, he obtained the value of the constant A from the slope of the curve, the intercept of which on the axis of ordinates gave the absolute value of e. The value of A given in the original paper for oil drops in air was .874, which in accordance with Maxwell's analysis indicates that most, though not all, of the impacts are non-elastic. Assuming that the same value held for mercury, he computed the value of the charge on droplets of this metal which were found to be in good agreement with those computed from oil drops. Experiments with substances, other than oil and air, have given slightly different values. Lee[1] obtained for shellac drops in air, $A = 1.067$, while Millikan[2] obtained $A = .820$ for oil and very pure hydrogen.

Recently an attempt was made by Roux[3] to determine the constant A for mercury and sulphur drops by allowing them to fall in air, then in a liquid, and measuring their velocity in the two media. He assumed that Stokes's law holds for the fall in the liquid and that from the ratio of the velocities the value of A could be determined.

Roux's method appears capable of yielding good results but since the correction in the formula depends upon a 5.1 per cent. increase in the velocity in air for the smallest drop used by him (time of fall in air, 5.2 sec.), and this time of fall was measured by means of a stopwatch, which permitted an error of 0.2 second on both stop and start, he has attempted to measure a quantity which depends on 5.1 per cent. error in the time of fall by an instrument which permitted an error of 7.7 per cent. of the quantity measured.

I.

GENERAL METHOD.

It was, therefore, proposed to repeat the work of Roux under conditions which permitted a more accurate measurement. Instead of a stopwatch, the Hipp chronoscope, used by Millikan, was employed, by which time intervals could be measured to 1/1000th of a second. To aid in overcoming the personal error, an apparatus was so arranged that a charged droplet could be raised by an electrostatic field and allowed to

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[1] Lee, PHYS. REV., Vol. IV., Second Series, Nov., 1914, p. 420.

[2] Millikan, Barber and Ishida, PHYS. REV., Vol. V., Second Series, 1915, p. 334.

[3] Roux, Annales de Chimie, Vol. 29, p. 69. 1913.

fall a number of times. About 40 falls usually were measured for each droplet. It was then allowed to fall into the liquid and its fall measured through as many one or two millimeter intervals as was possible. From these measurements and the necessary constants the value of the radius a, and of the constant A, were calculated.

Choice of Liquid.

No better liquid was available than xylol, the one used by Roux. In order that droplets of the size required enter the liquid it was necessary that the liquid have little or no surface film; in order that the ratio of the two velocities be small, the viscosity should be low; and in order that there be as little evaporation as possible the boiling point should be high. Xylol has the first two of these properties, but in spite of its high boiling point it is quite volatile. To prevent changes of viscosity and density due to evaporation, the xylol was distilled, the middle third being reserved for use. This had a boiling point of about 140° C. and measurements of density and viscosity made a month apart on liquid that had stood in the apparatus during that interval showed no appreciable change.

Measurement of Density and Viscosity.

The density was measured by means of a pyknometer, and the viscosity by improved Ostwald viscosimeters, which had been used a number of years both here and at the Boston School of Technology. These[1] tubes were kindly loaned me by Professor Harkins, of the Kent Chemical Laboratory. Measurements were also made by a longer tube of the same kind which was made for the Ryerson laboratory. The measurements made by these three tubes (16.5, 21.5 and 25 cm. in length) agreed within 0.2 per cent. For comparison, water was used, the absolute value of the viscosity being taken as the mean of the various values given in the tables. A linear relation was assumed between the values at 20.°0 and 25.°0 C. for water and between 23° and 25° for xylol when determining the viscosity within these ranges. The values obtained for density and viscosity are

	23°.00.	25°.00.
Density	.8504	.8487
Viscosity	.006226	.006107

For the viscosity of air at 23° the value, .0001824, was used.

Deduction of the Value of A.

To deduce the formula used in solving for A, the corrected Stokes's equation was written for the velocity in air

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[1] A description of a similar type of tube is given by Washburn, Journal American Chemical Society, p. 737, 1913.

$$v_1 = 2/9\, ga^2 \frac{(\sigma - \rho_1)}{u_1}(1 + A\, l/a) \qquad (1)$$

and the ordinary equation for the fall in the liquid,

$$v_2 = 2/9\, ga^2 \frac{(\sigma - \rho_2)}{u_2}, \qquad (2)$$

where the subscripts 1 and 2 refer to the constants of the air and the liquid respectively.

Dividing (1) by (2) we get

$$\frac{v_1}{v_2} = \frac{\dfrac{\sigma - \rho_1}{u_1}(1 + Al/a)}{\dfrac{\sigma - \rho_2}{u_2}}.$$

Whence

$$A = \left\{ \frac{v_1 u_1 (\sigma - \rho_2)}{v_2 u_2 (\sigma - \rho_1)} - 1 \right\} \frac{a.}{l}$$

If now, v be replaced by $1/T$ and $u_1/u_2(\sigma - \rho_2)/(\sigma - \rho_1)$ by α, where T is the time required for the droplet to fall unit distance and α a constant for the particular temperature of the medium at the time of fall, we obtain

$$A = \left(\frac{T_2}{T_1}\alpha - 1\right)\frac{a}{l}.$$

The radius a was obtained by solving equation (2). For this we get

$$a = \sqrt{\frac{9}{2}\frac{u_2}{g(\sigma - \rho_2)T_2}} = \beta\sqrt{\frac{1}{T_2}},$$

where

$$\sqrt{\frac{9}{2}\frac{u_2}{g(\sigma - \rho_2)}} = \beta$$

and

$$T_2 = \frac{1}{v_2}.$$

The Apparatus.

The apparatus used (Fig. 1) consisted of a pair of condenser plates PP_1, of the Millikan type each 22 cm. in diameter. At the center of the upper plate were three holes about 0.7 mm. in diameter through which droplets of mercury might fall. Over the center of the plate was a sheet-iron disc D_1 which by means of a lever could be moved aside to allow the falling droplets to reach the holes. At one edge a small cup filled with mercury provided an electrical contact with the wire leading through the ebonite plug E. The lower plate, shown in horizontal section in Fig. 1*A* was made of sheet iron about 0.36 mm. thick. It was fastened by means of screws to a short wooden cylinder of the same diameter and about 5 cm. deep. On it rested three glass posts, *T*, *T*, *T*, 15.875 mm. high, all cut from the same piece of plate glass. These supported the upper plate. In the center of this thin plate was cut a circular hole 6.4 mm. in diameter and to the lower side was cemented a rectangular glass vessel *V*, the inner dimensions of which were 4.9 × 4.9 × 2.1 cm. The wooden cylinder was cut away at the center to receive the vessel *V*, and was furrowed along the diameter L_1L_2 *O*, as shown by the dotted lines to allow light to reach the vessel. A second furrow along the radius *CO*, allowed the illuminated vessel to be seen. The vessel, except on the upper side was, therefore, almost completely surrounded by wood. A brass cylindrical post screwed into the center of the lower side of the wooden cylinder supported the whole condenser firmly with its axis coincident with that of an enclosing cylindrical sheet

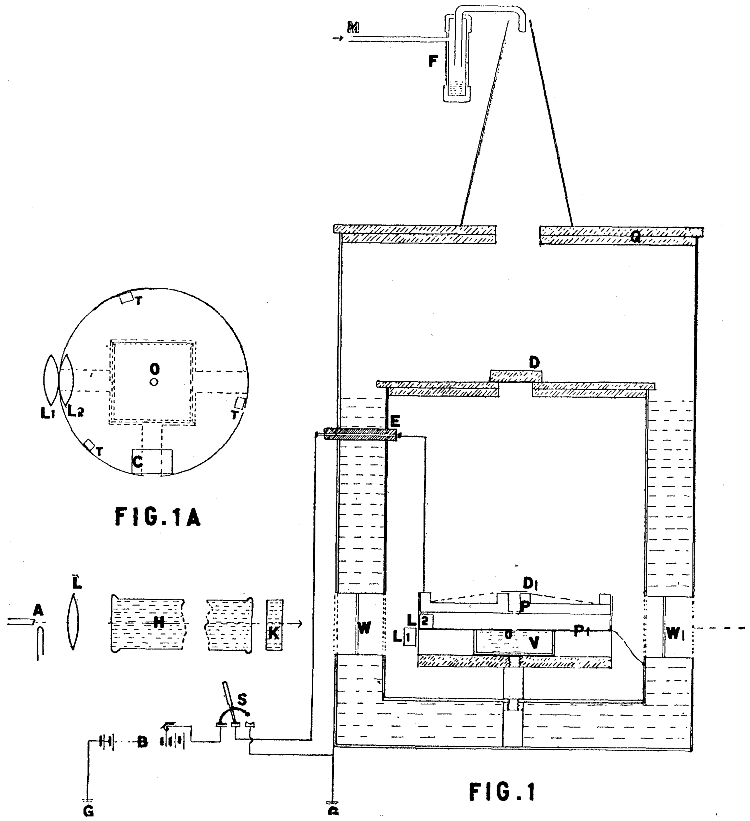

FIG. 1A

FIG. 1

iron tank, 30.5 cm. in diameter and 30.5 cm. high, which in turn was enclosed in a larger tank of the same material, 45.7 cm. in diameter and 67.5 cm. high. The two tanks were held rigidly in their relative position by the central brass post, which supported the condenser and by four pieces of telescope tubing. One of these, 2.5 cm. in diameter, held the insulating ebonite plug E, the other three were 7 cm. in diameter. Two of these in the position WW allowed light to pass through the vessel V and between the condenser plates, the other at right angles to the line WW_1 permitted the observer to see the vessel. Each of the 7 cm. tubes contained a glass window. The lower plate of the condenser was electrically connected to the inner tank and therefore, to earth by grounding the outer one. Both tanks were provided with wooden covers and to prevent rapid changes of temperature the outer was covered on top and sides with asbestos about 6 mm. thick. The space between was filled with oil.

In his original work on mercury drops, Millikan had formed his drops both by atomizing liquid mercury and by condensing the vapor rising from boiling mercury. Roux had used the latter method and it was followed here. The droplets were formed by blowing a sudden strong pulse of air through the iron vessel, F, which contained a small amount of boiling mercury. To prevent, as much as possible, the heat from the bunsen burner and the boiler F reaching the condenser, they were placed about 50 cm. above the top of the outer tank. The mercury vapor carried by the air blast condensed into droplets after leaving the boiler and fell freely to the condenser when the slide covering the inner tank and the one covering the holes in the condenser were held aside. A conical sheet iron shield supported by the cover of the outer tank prevented the mercury from being blown over the room.

The system was illuminated by a right-angle arc lamp, A, the crater of which was focused by lens L in the position of the window W, and was further focused by cylindrical lenses L_1 and L_2 into a vertical band of about 8 mm. wide and 3.5 cm. long at the axis of the condenser. Lens L_2 was of short focal length and it was necessary to place it between the condenser plates. Lens L_1 was of longer focal length and was placed just outside the wooden cylinder. To eliminate non-luminous radiation, the light was filtered through a 52.5 cm. column of water H and through K, a 12 per cent. solution of cupric chloride in alcohol 1 cm. thick. This has been shown by work at the Ryerson Laboratory more efficient than a solution of cupric chloride in water. It allows practically no radiation except that of the visible spectrum beyond the yellow to reach the condenser.

The upper plate was electrically connected to a series of storage cells *B* by closing switch *S*, and the two plates were short circuited and earthed by the opposite throw of the switch. The potential of the plate could be varied by using a greater or less number of cells, the maximum potential being about 4,000 volts.

To prevent air currents, due to irregularities in the density of the air in the tank, the condenser was surrounded by a transparent celluloid screen in which windows were cut at positions L_1L_2 and at *C* (Fig. 1*A*). Lenses L_1 and L_2 closed the window at this position.

The droplets which fell through the holes in the upper condenser plate were viewed in the direction *CO* through a short-focus telescope of constant length which had an objective 2.5 cm. in diameter and a magnifying power of about 23. A carefully ruled scale in the eyepiece provided a means of measuring in millimeters the distance traversed by the droplets. The telescope could be raised by rack and pinion so as to view the droplet in the air, then lowered to see it in the liquid which except for the curved meniscus at *O* completely filled the vessel *V*. To insure finding the droplet in the liquid a piece of optical glass *C* was placed between the condenser plates which served the double purpose of closing the window in the surrounding shield and of compensating for the change of optical path when the droplet entered the liquid. The thickness of xylol for which the glass *C* would compensate was determined by experiment before cementing *V* to the condenser plate.

Manipulation.

With the condenser plates earthed, slide *D* open, and D_1 closed, a cloud of droplets was blown. Slide D_1 was now opened, allowing them to fall through the small holes of the condenser, switch *S* was then closed and the droplets of the correct sign were drawn up by the electric field. One of these was chosen for observation, slide D_1 again closed to prevent any more entering the condenser, then by letting the chosen droplet rise and fall a number of times, all others were cleared from the field. This droplet was now drawn to the upper part of the field and allowed to fall under gravity through one centimeter distance, the time of fall being measured each time by the chronoscope. This was repeated a number of times, usually about 40. During this time all other droplets which had entered the liquid fell to the bottom. The droplet was now allowed to fall into the liquid, the telescope was quickly lowered and the droplet was seen to come through the meniscus and slowly fall. The telescope was now shifted until the droplet was accurately in focus and the time of fall measured through as many millimeter or two millimeter distances as was possible. The average of these millimeter readings multiplied by ten was taken as the time of fall of 1 cm. in the liquid. Irregularities in the speed at various parts of the path made it necessary to record the time of fall of separate millimeter distances rather than over the whole course.

The electrical field was not uniform at the center because of the central hole in the lower plate. This non-uniform field proved an advantage over a more uniform one obtained by means of a wire gauze employed in preliminary experiments, because, if the droplet was not originally well over the hole, it rose following along the lines of force but fell vertically. Therefore, after a few journeys it was well over the hole and within the band of concentrated light. Its chance of falling into the liquid instead of striking the plate was thereby assured.

Sources of Error.

Simple as the experimental arrangement seems, some difficulties were met in carrying out the measurements. These were of two types, one the faint illumination of the isolated droplet in the liquid, and the other the convection currents in both the air and the liquid. The first was met by using the cylindrical lenses previously discussed and by eliminating all possible reflecting surfaces of glass. This was the reason for closing the windows in the shield by the lenses and optical glass *C*. The part of the window at *C* below the lower condenser plate was left open. A further reason for eliminating these glass surfaces was that the xylol sometimes condensed on them, producing a hazy, or double image of the droplet.

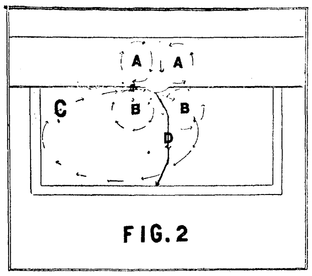

FIG. 2

The most difficult problem was the elimination of convection. It was found that nearly all irregularities in readings were finally traced to this cause and that a very weak convection current in either air or liquid caused an enormous error in the measurements. After all convection due to any mechanical disturbance had died away, which usually was within an hour, a set of currents gradually increasing in strength, shown by *AA* and *BB*, Fig. 2, appeared. These were due to the evaporation and consequent cooling of the xylol at the hole *O*. The mercury droplet, having a comparatively large inertia gave no information regarding these currents except that in air it appeared to fall with different speeds in the upper and in the lower part of the path. In the liquid if streams of particles were allowed to fall from all three holes, during the first five millimeters, the three streams, which were originally about 2 mm. apart, were drawn very nearly together, the droplets moving very rapidly. After falling this distance, the three streams again separated, fell vertically, and more slowly for a few millimeters, to be again slightly accelerated at the bottom of the vessel. The cause of this behavior was discovered by introducing dust particles which were sufficiently light to take on the motion of the medium. The complete path of the lighter particles was then seen to be as shown by *BB* in Fig. 2.

The acceleration at the bottom of the vessel which for a long time was thought to be only apparent and due to a defect in the optical system was caused by a weaker convection current which probably took the course indicated by *C*, Fig. 2. This drift was not discovered until *A* and *B* were eliminated and then only by the introduction of lighter particles. The particles introduced were obtained by leaving the holes in the upper plate uncovered when the cloud of droplets was blown. The shower of droplets which fell under these conditions was sometimes preceded by a cloud of slowly falling particles, especially if the mercury in *F* had been boiled very vigorously for about a minute before blowing. Some of these were found later to be extremely small mercury droplets, too small to be seen in the liquid, others were probably compounds of mercury, and others, no doubt, were dust particles of the air. When one of these particles entered the liquid, it was seen to drift out of the field of view in the direction of the light propagation at the top of the vessel. A six-second[1] mercury droplet drifted sometimes as much as two millimeters in the direction of the light during the first four or five millimeters of fall in the liquid, then fell vertically with a uniform velocity through from 5 to 10 millimeters, then drifted in the opposite direction with an accelerated speed during the last few millimeters of fall. The path of the droplet was as shown by *D*, Fig. 2. Mercury droplets less than four seconds showed the change of speed but no appreciable drift.

Since this type of convection current was found to increase with time of observation, and was in the direction of the light at the top and oppositely directed at the bottom of the vessel, it was probably due to a very slight heating of the liquid on the side of the incident light. The fact that no convection appeared at the center was probably due to the particular size of vessel used.

When taking observations it was necessary to know to what extent

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[1] It is convenient to refer to the size of the droplets in terms of the time required for them to fall one centimeter in air. Hence a six-second droplet is one that falls one centimeter in six seconds. The smallest droplet upon which measurements were made was a 6.19 second drop, and the largest was a 1.41 second drop.

these convection currents existed. The presence of continually visible, slowly falling particles was undesirable and was avoided as much as possible, but it was learned early in the investigation that a few minute, irregular particles were always present in xylol. These were invisible except when a reflecting side was turned to the light, but, due to Brownian movements, they would be turned to show scintillation for a second or two, which would fade away to sparkle again in approximately the same place a few seconds later if the liquid was perfectly at rest. This test for convection was used through all the latter part of the work, and the fact that the convection was not observed at the top and bottom was, as stated above, because the change of speed at this position was thought to be apparent and due to an optical defect, and only the middle half of the liquid was tested for convection. After it was proven that the change of speed was real and that no optical defect existed, a careful study of the scintillations plainly showed the drift.

Since the illumination could not be further decreased without diminishing the brightness of the droplet, no attempt was made to eliminate the convection of type *C* further than to keep the light screened from the liquid except during the time of observation of the droplet in it. With this precaution observations could be taken on five to eight droplets before it became too troublesome. In the computation, only the minimum speed was used, for it was with this minimum speed that the droplet was falling through this middle portion of the field where the liquid was at rest and Stokes's law was applicable.

The convection currents *AA*, *BB*, were eliminated by partially saturating the air in the tank with xylol vapor. The space could not be completely saturated because the xylol then condensed on the glass surfaces, thus producing a blurred image of the droplet and also condensed on the droplet itself, thus decreasing its speed in the air. To produce this saturation, the inner tank was lined at top and sides with cotton which was held in place by iron wire gauze. This cotton was saturated with xylol and about 200 c.c. of xylol was kept at the bottom of the tank. The tank was left closed for at least twelve hours before observations were taken, then if all other conditions were favorable, no evaporation took place at *O*, and no convection currents were present. Before beginning observations the tanks were uncovered and the saturated air removed by an electric fan. The cover of the inner tank was as a rule not replaced during observation and since the cover to the outer one was not tight-fitting, the air in the tank was not completely saturated. Under these conditions, a small droplet sometimes showed a gradual increase in time of fall, and in time of rise under the electric field, showing that they were slowly collecting liquid. In the computation, only the average of the first ten falls was used for droplets showing this condensation.

Mercury droplets as a rule evaporate in dry air, the evaporation being plainly shown by their gradual increase in time of fall and their gradual decrease in time of rise under the field. This has been observed here by all who have attempted to work with mercury droplets and has been observed and well described by A. Schidlof and A. Karpowicz.[1] However, with our experimental arrangement, not all droplets showed this evaporation, and it was found to decrease with time and finally cease. In the presence of xylol vapor even far below saturation, this evaporation did not occur or was so slight that it was not detected.

Besides the convection currents just described, even a small change of room temperature, extending over a period of as much as three hours, produced unequal heating in the liquid, thus setting up convection, which did not die away until the room temperature had been held constant several hours. Therefore, for the latter part of the work, a thermostat and an electrical-heating apparatus was set up, by which the temperature of the room was maintained constant within 0.°4 C. except during the time of observation. A Beckmann thermometer placed in the oil between the tanks showed no greater change than 0.°01 C. in the temperature when the arc was not in use.

During an observation period, the extra heating of the room due to the arc, and the blasts of hot air used in blowing the droplets into the chamber gradually caused convection currents which first began to be detectable after two or three hours of observing. Care was taken that the observations recorded below are free from error due to this source.

A further troublesome feature was a film forming on the surface of the xylol when it stood undisturbed for a few hours. As a result of this, the droplets which fell into it at the beginning of a period gave no agreement among themselves for the value of the constant A. It seemed that the film retained the smaller particles, and a droplet with sufficient weight to break the film carried with it part of the film together with the neighboring small particles. Very frequently a droplet would carry a sufficient amount with it to glow brighter in the liquid than in the air. To eliminate this difficulty, at the beginning of each period, the surface film was, therefore, destroyed by allowing several showers of large particles to fall through it.

It has already been stated that not all the particles which fell were

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[1] Schidlof and A. Karpowicz, Phys. Zeitschrift, 15, No. 4, Feb., 1915, p. 42. Also Comp. Rend. 158, p. 1992, 1914.

pure mercury. In the choice of a droplet, one was not always able to select pure mercury. The dust particles could be easily avoided because they either fell much more slowly, or were very much larger and brighter. Sometimes the droplet selected seemed to be a small dust particle upon which mercury had condensed. These on entering the liquid fell with a greater or less speed than if they had been pure mercury, depending upon whether they lost or retained the dust particle at the surface. A few of this type which were selected showed clearly a difference in brightness on rising with the field, from that shown when falling. After carrying out observations on the first one of these, any others which showed this effect were dropped as undesirable.

Observations.

A set of observations on a single drop are as follows:

April 14. Temperature: 25°.18.

Time of Fall, Sec. per Cm.	Time of Fall, Sec. per Cm.	Time of Fall, Sec. per Cm.	Time of Fall, Sec. per Cm.
2.940	2.995	2.861	2.865
2.953	2.901	2.919	2.853
2.977	2.899	2.919	2.924
2.909	2.928	2.897	2.865
2.897	2.879	2.880	2.845
2.927	2.884	2.821	2.872
2.978	2.881	2.922	2.898
2.920	2.886	2.855	2.851
2.839	2.912	2.916	2.890
2.864	2.902	2.910	2.911
		2.881	

Fall in Liquid.		Time of Fall, Sec. per Cm.
Distance.		
2 mm.	21699	108.49
2 mm.	21639	108.19
1.8 mm.	19442	108.01
2 mm.	19960	
2 mm.	20060	

Average time of fall in air, 1 cm. = 2.851 sec.
Average time of fall in liquid, 1 cm. = 108.23 sec.
$a = .000143.$
$A = 1.050.$

Since the chronoscope ran only seventy seconds without rewinding, no readings were taken, in general, until the droplet had fallen five millimeters in the liquid. In this case, three readings were taken before it ran down. This corresponded to a fall of 7 mm. Time required to rewind allowed the droplet to fall into a liquid which accelerated its speed.

Observations on a smaller drop are as follows:

April 21. Temperature: 25°.00.

Time of Fall in Air.	Sec. Per Cm.	Fall in Liquid, One and 2 Mm. Distances.
6.119	6.268	23.976
6.144	6.246	23.702
6.106	6.254	48.981
6.083	6.206	46.495
6.105	6.284	21.563
Average.................	6.181	

Average time of fall, 1 cm. in air = 6.181.
Average time of fall, 1 cm. in xylol = 241.6 sec.
a = .0000956.
A = 1.039.

In all, forty-three readings were taken on this droplet in air but the constant increase in time of rise under the field and fall under gravity showed clearly that xylol was condensing on it. It was so small that it drifted where the liquid was not at rest. It fell vertically while the first three readings were taken, which corresponded to about 5 mm. During the fourth reading it drifted about 1 mm. Therefore, only the first ten readings in air and the first three in the liquid were used in computation.

	Date of Observation.	No. of Falls in Air.	Temperature.	Average Time of Fall, 1 Cm. in Air, Sec.	Max. Time of Fall in Xylol, Sec. per Cm.	$\frac{T_1}{T_a}$	$a \times 10^4$	A
1	Jan. 29	44	22.82	1.421	54.78	38.5	2.036	1.07
2	Jan. 27	43	22.82	1.443	55.40	38.38	2.025	.95
3	Jan. 27	39	22.82	1.532	59.06	38.50	1.961	1.01
4	Jan. 29	43	22.82	1.642	62.88	38.8	1.900	.84
5	Jan. 29	42	22.82	1.721	66.32	38.5	1.850	.95
6	Jan. 29	42	22.82	1.782	68.00	38.2	1.828	.74
7	Jan. 27	43	22.82	1.792	68.52	38.3	1.821	.78
8	Jan. 29	42	22.82	1.862	72.38	38.8	1.772	1.07
9	Jan. 27	42	22.82	2.041	79.14	38.8	1.693	1.02
10	Jan. 27	39	22.82	2.308	89.55	38.8	1.589	.93
11	Feb. 27	43	22.82	2.439	94.57	39.2	1.549	.95
12	Jan. 27	43	22.82	2.548	100.7	39.5	1.502	1.11
13	Feb. 27	41	22.82	4.020	159.39	39.7	1.194	.96
14	Mar. 6	43	22.82	4.435	175.83	39.7	1.138	.92
								.95

For this temperature α = .027219.
and β = .0015072.

Before installing the heating system, regulated by the thermostat, an attempt was made to maintain the temperature constant at 23.°0 (calibration of the thermometer showed this to be 22.°82), and on some days conditions were sufficiently good to take observations. The temperature was not accurately known. It was measured by a thermometer placed beside the condenser and read through the window W_1. The temperature recorded was probably as much as 0.°2 C. in error in some cases. A summary of the readings taken under these conditions is as follows:

In the early spring the increase of the outside temperature and the fact that work was being carried on in an adjoining room at higher temperature than 23.°00 made it necessary to work at a higher temperature if the room remained constant. The remainder of the work was, therefore, done at 25.°00.

The following observations were made:

	Date of Observation.	No. of Falls in Air.	Temperature.	Average Time of Fall, 1 Cm. in Air, Sec.	Max. Time of Fall in Xylol Sec. Per Cm.	$\frac{T_1}{T_a}$	$a \times 10^4$	A
15	Apr. 14	43	25.2	1.668	62.49	37.4	1.878	1.13
16	Apr. 17	20	25.2	1.707	63.59	37.4	1.740	.95
17	Apr. 16	43	25.3	1.987	74.40	37.4	1.720	1.05
18	Apr. 19	43	25.0	2.059	77.52	37.7	1.692	1.06
19	Apr. 21	42	25.0	2.114	79.69	37.8	1.665	1.07
20	Apr. 21	43	25.0	2.145	80.69	37.7	1.732	1.03
21	Mar. 23	43	25.3	2.257	84.95	37.7	1.610	1.08
22	Apr. 16	43	25.3	2.866	108.56	38.0	1.424	1.06
23	Apr. 14	42	25.2	2.849	108.23	38.0	1.427	1.05
24	Apr. 21	43	25.0	2.924	110.94	37.8	1.415	1.01
25	Apr. 19	43	25.0	3.074	116.33	38.0	1.377	.98
26	Mar. 22	43	25.2	3.754	143.30	38.2	1.241	.99
27	Apr. 14	42	25.2	4.852	185.00	38.2	1.091	.86
28	Apr. 14	43	25.2	5.189	200.59	38.75	1.048	1.00
29	Apr. 21	10	25.0	5.343	208.44	38.75	1.029	1.06
30	Apr. 21	10	25.0	6.182	241.64	39.10	0.9563	1.04
31	Apr. 3	40	25.0	6.190	244.00	39.5	0.952	1.08
								1.03

Discussion of Results.

The temperatures recorded here were taken by placing the bulb of a thermometer in the vessel of xylol after completing the observations on the last droplet of the period. Where heating occurred during the period, the use in the formula of the coefficient of viscosity at the final temperature would make the computed value of A for early droplets of the period a little too high. Further, since only the maximum values of the time of fall in the liquid were used in the computation, the value of A can not be greater than the average of the results given, if the simple Stokes's formula holds for the fall in the liquid.

Comparison of the chronoscope with a standard laboratory clock showed it to have an error of 0.4 per cent. when the period was long enough that the personal error could be neglected. With the short periods used here, the personal error was the greater. Comparisons with the clock on different days showed this to be sometimes in one direction, sometimes in the other, the greatest error being 1.4 per cent. on a one-second interval. An error of 1.4 per cent. in the fall of the largest drop of the second series, 1.668 sec., would produce 28 per cent. in the computed value of A. For intervals greater than 4 seconds, the maximum error was 0.7 per cent., which would produce an error of at least 6 per cent. on the smallest drop of the second series.

The application of the correction of Hadamard to the fall in the liquid yields a value of A about 140 per cent. too high. With this correction, the modified Stokes's formula for the velocity in the liquid becomes

$$v_2 = 2/9\, ga^2 \frac{\sigma - \rho_2}{u_2} \left\{ \frac{u^1 + u_2}{u^1 + \frac{2}{3}u_2} \right\},$$

where u^1 is the viscosity of mercury.

The viscosity of mercury given in the tables for 25° is .0124; that of xylol at 25° is .00628. Then the correction becomes

$$\frac{.0124 + .00628}{.0124 + .00419} = 1.125.$$

The equation for the radius of the drop becomes

$$a = \beta \sqrt{\frac{1.125}{T_2}} = 1.0606\beta \sqrt{\frac{1}{T_2}}$$

and that for A becomes

$$A = \left\{ 1.125\alpha \frac{T_2}{T_1} - 1 \right\} \frac{a}{1}.$$

Solving for a and A in these modified formulæ, we get for the 6.19 second drop

$$a = 1.00979 \times 10^4 \text{ instead of } 0.952 \times 10^4.$$

and

$$A = 2.635 \qquad \text{instead of } 1.0808.$$

It was pointed out by Nordlund[1] that the Hadamard correction did not apply for mercury droplets in water, and the above data indicates that it does not apply to mercury droplets in xylol.

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[1] Nordlund, Zeitschrift für Physikalische Chemie, 87, 1914, page 40, also Science abstracts, 180, Vol. 17, 1914.

II.

DETERMINATION OF A FROM THE APPARENT VALUE OF THE ELEMENTARY CHARGE.

The lower plate and wooden cylinder were now replaced by an iron condenser plate of the same diameter and an attempt was made to determine roughly the value of A from the apparent value of the elementary charge. With the installation of an X-ray outfit, the apparatus thus altered became the same as that originally used by Millikan for measuring e.

With the apparatus two experimental conditions rendered an accurate measurement impossible. (1) The evaporation of the droplet made the work progress very slowly and produced great uncertainty when it did not entirely cease. Occasionally a droplet could be selected which did not evaporate, whose elementary charge determination showed that it was mercury. As a rule, however, the more slowly falling particles indicating no evaporation, gave values for the ultimate charge much lower than if they had been pure mercury droplets with the same speed under gravity, although the change in charge was an exact multiple of the charge originally on the droplet. These were regarded as impure mercury particles, or dust particles. It was found better to select a droplet which showed evaporation and retain it until the evaporation either ceased, or took place very slowly. In general it was necessary to hold a droplet about an hour before any readings could be taken, and in many cases it faded from view before the evaporation ceased. Why it should cease in any case was not learned. When the plates were coated with oil to catch dust particles it seemed to cease more quickly, although the vapor pressure of the oil used was very low. If the pressure of the oil vapor caused the evaporation to cease it must have been due to a change of surface of the droplet, and we should expect the value of A to agree with the value for oil drops. (2) A second source of error in the measurement was the piling up of the mercury on the lower plate. Unlike the oil which spread over the surface, the mercury accumulated in small globules which rendered the electrical field non-uniform.

The first fall of this droplet was in twenty seconds. It was held until the evaporation ceased, before the above readings were taken. The time required to obtain the above data was fifty minutes.

The values found for $e_1^{\frac{2}{3}}$ and $1/a$ are plotted as ordinates and abscissæ respectively, in order to obtain the value of A.

The curve was drawn to give the greatest weight to the droplets whose fall under gravity was greater than 20 seconds per centimeter, because measurements on droplets of this size were more reliable. The

The following are the observations on one of the smaller droplets:

Drop No. 13.

T_g	T_f	$\frac{1}{T}$	n'	$\frac{1}{n'}\left(\frac{1}{T_f'}-\frac{1}{T_f}\right)$	n	$\frac{1}{n}\left(\frac{1}{T_g}+\frac{1}{T_f}\right)$
55.6	43.8	.02304			1	.04083
56.6	43.0					
55.8	9.635	.10446				
56.8	9.640		2	.04083	3	.04075
55.8	9.443					
56.2	9.572					
56.2	42.4	.02353	2	.04046		
56.4	42.6				1	.04131
	15.464	.06437	1	.04084		
	15.487				2	.04108
	15.651		1	.04078		
	42.4	.02359			1	.04137
56.4	9.427	.10514	2	.04078		
	9.594				3	.04098
	43.0	.02320	2	.04097		
	43.2				1	.04099
	15.674	.06377	1	.04047		
	15.423				2	.04073
	15.994		1	.04008		
56.4	42.4	.02359			1	.04137
56.22				.040684		.041023

$V_1 = 3410.$ $V_f = 3385.$

$t = 24.8$ C. $a = .00003001.$

$$\frac{1}{a} = 3203,$$

$$e_1 = 6.585 \times 10^{-10},$$

$$e_1^{\frac{2}{3}} = 75.69 \times 10^{-8}.$$

Observations and computed values on other droplets are recorded in the following table:

Drop.	Tg.	$a \times 10^5$	$e_1 \times 10^{10}$	$1/a \times 10^2$	$e_1^{2/3} \times 10^8$
1	8.066	8.469	5.143	11.36	64.19
2	8.327	8.463	4.967	11.23	63.74
3	11.054	7.175	5.350	13.24	66.11
4	13.668	6.447	5.345	14.72	65.86
5	19.340	5.332	5.572	17.82	67.72
6	23.261	4.812	5.762	19.74	69.24
7	25.910	4.536	5.906	20.95	70.39
8	26.437	4.533	5.720	20.96	68.91
9	27.970	4.442	5.979	21.88	70.97
10	36.150	3.761	6.305	25.26	73.53
11	40.140	3.575	6.228	26.57	73.49
12	45.000	3.399	6.266	27.95	73.22
13	56.22	3.001	6.585	32.03	75.69
14	62.15	2.791	6.645	34.04	76.14

slope of this curve gives .886 for the value of A.[1] The low value of the intercept (59.3 instead of 61.05) shows the imperfection of the apparatus.

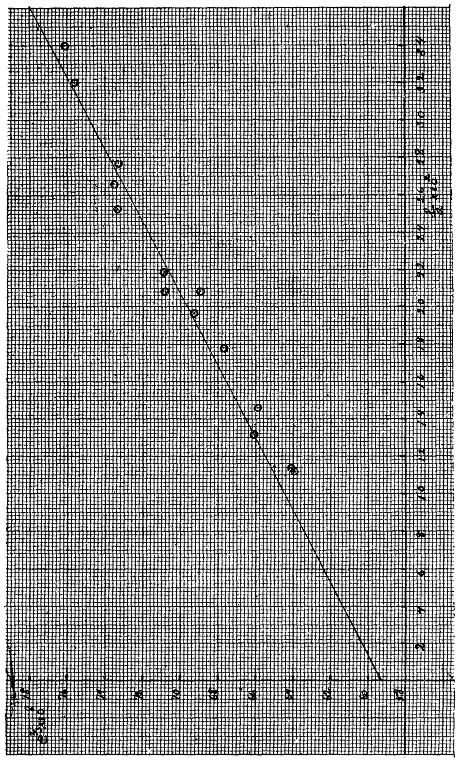

Fig. 3.

SUMMARY.

I. By a direct measurement of the velocity of fall of a droplet of mercury in air and in xylol, and assuming that Stokes's law is applicable to the motion in a liquid, the value 1.03 was found for the empirical constant A of the modified formula for the fall in air.

II. These measurements show that the Hadamard correction does not hold for mercury droplets in xylol.

III. A rough measurement of A from the apparent value of the elementary electrical charge on mercury droplets gave a value in the neighborhood of that obtained by use of oil drops.

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[1] Strictly Millikan's value of the intercept should be used in the calculation of A. If his value, 61.05, is used the value of A is found to be .824. This low value of the slope of the curve is in accordance with the assumption that oil vapor condensed on the droplets for a film on a smaller droplet would have a greater effect than one on a large droplet thus decreasing the slope of the curve.

I wish to express my thanks to Professor R. A. Millikan, at whose suggestion the above problem was undertaken, and whose helpful suggestion and kindly counsel have been an inspiration throughout the investigation. My thanks are also due to Messrs. R. S. Buddenburg, P. I. Pierson, S. A. Rowland, and J. B. Derieux, who, in turn, assisted me in making the observations.

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