CERTAIN CASES OF THE VARIATION OF SOUND INTENSITY WITH DISTANCE.

BY G. W. STEWART.

IT is not possible to secure a source of sound without the presence of reflecting surfaces. In quantitative considerations one desires to know the variation of intensity with distance from the source and is thus led to inquire as to the deviations from the inverse square law caused by the reflectors. A case of practical interest is that in which a small vibrating area is located on a rigid sphere, for it is possible to construct a source which conforms very closely to these theoretical conditions.[1] An additional interest arises from the fact that an investigation of such a source leads to an estimate of the deviations from the inverse square law in the case of a person speaking or singing. Furthermore, by utilizing the results of the same theoretical investigation, we can obtain the relative intensities on a sphere when the distance of the sphere from a simple source is varied. These values give an estimate of the deviations from the inverse square law in the case of a person listening. The deviations in the case of a speaker are of interest in architectural acoustics: those occurring with the listener are of importance in the psychological laboratory.

With a small vibrating area located on a rigid sphere, the geometrical figure of the reflector makes a mathematical investigation possible. Lord Rayleigh[2] was the first to obtain an expression for the sound intensity in the various directions at a great distance from such a sphere. Subsequently the writer[3] extended the investigation in order to obtain similar results for distances that are not great.

A brief statement of the theory will doubtless prove helpful to the reader. Let the source be confined to a small area on the surface of the sphere within which $P_n(\mu)$ of Legendre's series approximates unity. Assume that the velocity of this area is represented by Ue^{ikat}, and that it has the same magnitude at all points. Assume that

ψ is the velocity potential,

a is the velocity of sound,

[1] Stewart and Stiles, PHYS. REV., N. S., Vol. I., No. 4, 1913, p. 309, and PHYS. REV., N. S., Vol. III., No. 4, 1914, p. 256.

[2] Rayleigh, Theory of Sound, Vol. II., p. 254.

[3] Stewart, PHYS. REV., XXXIII., No. 6, p. 467, December, 1911.

r is the distance from the center of the sphere,
c is the radius of the sphere,
dS is an element of surface of the sphere,
k is 2π/wave length,
γ is $k(at - r + c)$,
F is $\Sigma \frac{2n\ 1+}{2} P_n(\mu) \frac{\alpha\alpha' + \beta\beta'}{\alpha^2 + \beta^2}$,
and that G is $\Sigma \frac{2n + 1}{2} P_n(\mu) \frac{\alpha\beta' - \alpha'\beta}{\alpha^2 + \beta^2}$,
where $f_n{}^1(ikr) = \alpha' + i\beta'$
and $F_n{}^1(ikc) = \alpha + i\beta$.

Then $\dot{\psi} = ka/2\pi r(F \sin \gamma + G \cos \gamma)\int\int Uds$. The mean potential energy, which is the "intensity" we desire, proves to be

$$\tfrac{1}{2}\rho_0 \frac{\dot{\psi}^2}{a^2} = \tfrac{1}{2}\rho_0(F^2 + G^2)\left(\frac{k}{2\pi r}\int\int UdS\right)^2.$$

In this formula $\left(\int\int UdS\right)^2$ measures the intensity of the source. For a constant source, a constant sound velocity and a fixed wave-length, the relative intensities are proportional to

$$\frac{(F^2 + G^2)}{r^2}. \qquad (1)$$

F and G are functions of k, r, c, and $\cos \theta$ or μ. The accompanying Fig. 1 will make clear the meaning of r, c and $\cos \theta$. The source is located at the point $(c, \theta = 0°)$. P is the point at which the intensity is desired and is located at a distance r and in the direction θ. Obviously P is any point on a circle whose radius is $r \sin \theta$ and whose circumference is everywhere at a distance r from the center of the sphere.

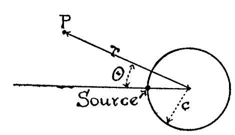

Fig. 1.

In view of (1), the deviations of the intensities from the inverse square law are indicated by relative values of $F^2 + G^2$. The computations involved in securing the numerical values of $F^2 + G^2$ are very laborious. Fortunately, certain numerical results are available, these having been obtained in previous investigations. As already indicated, the values of F and G depend upon those of ikr and ikc,[2] as well as upon μ.

[1] f_n (ikr) and F_n (ikc) are defined in Rayleigh's Theory of Sound, Vol. II., p. 238, and by Stewart, loc. cit.

[2] For the terms from which F and G can be computed in the two cases $kc = 0.5$, $kr = 25$ and $kc = 1.0$, $kr = 50$, see Phys. Rev., XXXIII., No. 6 (1911), Table I., p. 473, and Table II., p. 475.

Table I. indicates the variation in the relative values of $F^2 + G^2$ with r and with θ, the former being expressed in terms of c, the radius of the sphere. In order to show the percentage deviations in $F^2 + G^2$ which represent the percentage deviations from the inverse square law, the value of this sum is assumed to be unity at a distance of 50 c. Table I. utilizes the values of F and G when $k \times c$ is 1 and when $k \times r$ has the values 2, 3, 4 and 50. The values enclosed in parentheses are computed by substituting in each case for the distance r in (1) the distance from P the source on the sphere. In other words, the values in parentheses indicate the deviations from the inverse square law when the distances are measured from the actual source rather than from the center of the sphere.

TABLE I.

Relative Values of ($F^2 + G^2$), those at 50 c being assumed Unity.

Radius of sphere, $c = \frac{\text{wave length}}{2\pi}$.

Distance.	$\theta = 0°$.	30°.	60°.	90°.	120°.	150°.	180°.
2 c	4.24 (1.10)	2.58 (.98)	1.14	.622	.396	.329	.270
3 c	2.26 (1.05)	1.81 (.98)	1.18	.728	.564	.446	.420
4 c	1.75 (1.03)	1.53 (.98)	1.15	.862	.661	.544	.520
50 c	1.00 (1.00)	1.00 (1.00)	1.00	1.00	1.00	1.00	1.00

TABLE II.

Relative Values of ($F^2 + G^2$), Those of 200 c Being Assumed Unity.

Radius of sphere $= 0.5 \times \frac{\text{wave-length}}{2\pi}$.

Distance.	$\theta = 0°$.	30°.	60°.	90°.	120°.	150°.	180°.
2 c	6.28 (1.59)	3.54 (1.30)	1.28	.652	.655	.765	.797
50 c	1.04 (1.01)	1.03 (1.01)	1.02	1.00	.978	.968	.957
200 c	1.00 (1.00)	1.00 (1.00)	1.00	1.00	1.00	1.00	1.00

Table II. contains the values obtained when $k \times c$ is 0.5 and when $k \times r$ has the values 1, 25 and 100.

The tabulated results show that when distances are measured from the center of the sphere the direction of minimum variation from the inverse square law is found to depend upon the wave-length. With the longer wave there is less deviation in the rear of the sphere and with the

For values of F and G for $kc = 1$ and $kr = 2, 3, 4$ and 50 see PHYS. REV., N. S., IV., No. 3, Table I., p. 255, and Table II. (60 cm.), p. 256.

The values of $F^2 + G^2$ for $kc = 0.5$ and $kr = 1.0$ and $kr = 100$ appear only in the form of curves, Fig. 2, PHYS. REV., XXXIII., No. 6, December, 1911.

shorter, less deviation in front. Both of these facts are in accord with anticipation based upon elementary considerations. If the distances in the directions 0° and 30° are measured from the source on the sphere there is less deviation from the inverse square law, but it is yet large. Whether distances be measured from the source or from the center of the sphere, the differences obtained by changing wave-length are very marked. This suggests that, in any practical case, a closer approximation to the inverse square law in front of the source can be secured by increasing the frequency of the tone. It also suggests the well-known fact that the relative intensities of the components of any sound will change with distance and direction from the source.

In order to utilize these numerical values for an estimate of the deviation from the inverse square law in the case of a speaker, it is necessary to assume that the head acts as a rigid sphere and that the source occupies but a small area on that sphere. If we choose as a circumference, 60 centimeters, the results in Table I. and Table II. refer to wave-lengths of 60 and 120 centimeters respectively. Although the lack of conformity to the theoretical conditions is obvious, yet we can consider that the above results furnish a fairly satisfactory estimate of the deviations from the inverse square law in the case of a speaker or singer using tones approximately 60 and 120 centimeters in wave-length, or approximately 575 and 287 in frequency. With the sphere circumference 60 centimeters, the distances are approximately 19.1, 28.6, 38.2 and 477 centimeters in Table I., and 19.1, 477 and 1,910 centimeters in Table II. In the former table, with a frequency 575, the deviations from the inverse square law at the above distances are over 400 per cent. If the distance to the source of sound instead of to the center of the head is substituted in the inverse square formula, then the deviations are 10 per cent. or less. The deviations in various directions are readily read in the tabulation. Table II. should be used for a frequency approximating 287. Neither table gives the relations between intensities at points having a constant r and varying values of θ, for such a comparison has already been published.[1]

In order to apply the results to the case of hearing, we must have recourse to the reciprocal theorem of Helmholtz,[2] which permits us to interchange positions of the source and the points at distances r where the relative intensities are desired. Then we can consider a simple source at a distance r from the center of the head (or, using the parenthetical results, from the ear) and can obtain an estimate of the variation of intensity with distance from either ear. For the frequency 575, the maximum deviation

[1] Stewart, Phys. Rev., XXXIII., No. 6, December, 1911.

[2] Rayleigh, Theory of Sound, Vol. II., p. 294.

from the inverse square law with distances from the center of the head to the source of approximately 19.1 and 477 centimeters (9.5 to 468 centimeters from the ear) does not exceed 10 per cent. (see Table I., 0°). For the frequency 287 and the same distance from the ear, the deviation is almost 60 per cent. (see Table II., 0°). If with the latter frequency distances of 477 to 1,910 centimeters are selected, the deviation is only 1 per cent. There is a distinct advantage in using the higher frequency in cases where an estimate of relative intensities is obtained by using the inverse square law.

PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA.