THE CHARACTERISTICS OF TUNGSTEN FILAMENTS AS FUNCTIONS OF TEMPERATURE.

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THE tungsten filament offers the simplest means of producing very high temperatures under conditions suitable for accurate measurement. There have been, however, considerable difficulties in determining the true temperature of such filaments, so that the results published by different investigators have differed by several hundred degrees.

The characteristics of tungsten lamps have been investigated very extensively, but in nearly all instances only the relations between the variables—volts, amperes, and candle power, or functions of these—have been studied. These variables, in fact, are the only ones with which the illuminating engineer is directly concerned. Probably the most complete investigation of this kind is that of Middlekauff and Skogland[1] who give by equations and tables the changes in candle power and amperes for any given change in the voltage applied to tungsten lamps.

The physicist or chemist, however, is primarily interested in the relation between these variables (volts, amperes, and candle power) and the dimensions and temperature of the filaments. Pirani published some data on the relation between temperature and watts per candle and later gave data on the temperature coefficient of the resistance as a function of the temperature.

Several years ago a detailed study of the characteristics of tungsten filaments as functions of the dimensions and temperature, was undertaken in this laboratory. The data thus obtained have been in continual use ever since and have served as the basis for the calculation of temperatures in several investigations which have been published.[2]

More recently, in connection with a redetermination of the melting-point of tungsten,[3] an optical pyrometer was set up, suitable for measuring the true temperatures of filaments with considerable accuracy. With

[1] Bull. Bur. Stand., 11, 483 (1915), and Trans. Ill. Eng. Soc., *9*, 734 (1914).

[2] For example: "Convection of Heat in Gases," PHYS. REV., *34*, 401 (1912); "Vapor Pressure of Tungsten," PHYS. REV, *2*, 329 (1913); "Thermionic Currents," PHYS. REV., *2*, 450 (1913); "Dissociation of Hydrogen," Jour. Amer. Chem. Soc., *34*, 860 (1912), *36*, 1708 (1914), *37*, 417 (1915).

[3] PHYS. REV., *6*, 138 (1915).

this instrument a new study of the characteristics of filaments has been made and the results are now felt to be of sufficient accuracy to warrant their publication.

General Theoretical Considerations.

Any property of a filament which varies with the temperature may be used, after proper calibration, as a means of estimating the filament temperature. There are a large number of such properties and the choice of the one most suited to serve as the basis of a temperature scale, will depend entirely upon the experimental conditions. For example, the energy radiated per unit area might be adopted for a filament in vacuum, but would be very unsatisfactory if the filament were surrounded by a gas.

The various properties (or methods) that may be utilized to estimate the temperature of filaments may be divided into four groups according to the knowledge of filament dimensions required, as follows:[1]

1. Requiring no dimensions:
   - (*a*) Intrinsic brilliancy.
   - (*b*) Color of light emitted.
   - (*c*) Ratio of "hot" to "cold" resistance.
   - (*d*) Watts per candle (W/C).
2. Requiring both length and diameter:
   - (*e*) Resistance, $R' = \frac{Rd^2}{l}$.
   - (*f*) Power radiated $W' = \frac{W}{ld}$.
   - (*g*) Candle power $C' = \frac{C}{ld}$.
   - (*h*) Voltage $V' = \frac{V\sqrt{d}}{l} = \sqrt{W'R'}$.
   - (*i*) Electron emission $i' = \frac{i}{ld}$.
3. Requiring diameter only:
   - (*j*) Current $A' = \frac{A}{d^{\frac{3}{2}}} = \sqrt{\frac{W'}{R'}}$.

[1] l = length, d = diameter,
V = volts, A = amperes,
W = watts, R = resistance (ohms),
C = candle-power (measured perpendicular to axis of filament); C is thus equal to the total lumens divided by π^2 or to the mean spherical candle-power multiplied by $4/\pi$;
V', A', etc., specific properties defined below.

(*k*) $\frac{C}{Vd^{\frac{3}{2}}} = \frac{C'}{V'}$.

(*l*) $\frac{1}{d}\sqrt[3]{\frac{C}{R}} = \sqrt[3]{\frac{C'}{R'}}$.

(*m*) Candle power (using slit) $\frac{C}{d}$.

4. Requiring length only:

(*n*) $\frac{V\sqrt[3]{A}}{l} = \frac{\sqrt[3]{W^2R}}{l} = \sqrt[3]{(W')^2R'} =$

(*o*) $\frac{\sqrt[3]{CV^2}}{l} = \sqrt[3]{C'(V')^2}$.

(*p*) $\frac{C}{A^{\frac{3}{4}}l} = \frac{C'}{(A')^{\frac{3}{4}}}$.

(*q*) $\frac{\sqrt[3]{C^2R}}{l} = \sqrt[3]{(C')^2R}$.

(*r*) Thermal expansion $\frac{\Delta l}{l}$.

(*a*) *Intrinsic Brilliancy.*—This is the property utilized in the Holborn-Kurlbaum pyrometer, and most other optical pyrometers, for the measurement of temperature. The use of intrinsic brilliancy for this purpose has the particular advantages

1st. That measurement can be made at any point on a surface.

2d. Extremely rapid increase with temperature.

3d. Variation with temperature follows known laws.

Only relative measurements of intrinsic brilliancy are involved in the use of a pyrometer. Since it has been shown[1] that the emissivity of tungsten does not change appreciably with the temperature, the variation of the intrinsic brilliancy of tungsten with the temperature is given accurately by Wien's law ($C_2 = 1.4392$):

$$\log_{10}\frac{E}{E_0} = \frac{-.62503}{\lambda}\left(\frac{1}{T_0} - \frac{1}{T}\right). \tag{1}$$

(*b*) *Color of Light Emitted.*—The color of the light emitted by an incandescent body may be used to estimate its temperature. This method is a very old one but has only recently been used for accurate pyrometry. Coblentz[1] and Hyde[2] showed that by properly adjusting the temperature,

[1] PHYS. REV., *6*, 138 (1915).

[2] Bull. Bur. Stand., *5*, 359 (1909).

[3] Trans. Ill. Eng. Soc., 1909.

filaments of platinum, carbon, tungsten, or tantalum could be color-matched against a black body. This color match is made by setting up the filament on one side of a Lummer-Brodhun photometer, and a black body on the other side. By adjustment of the temperature of the filament and its distance from the photometer the color and intensity of the light from the two sources can be made identical within the limits of observation. Hyde[1] pointed out that when filaments of the above named materials were brought to a color match, the distribution of energy over the whole visible spectrum was the same for all filaments. Hyde states in fact that the color match method " is perhaps more sensitive than the spectrophotometric method and yields results in close agreement with the latter. It is conceivable, of course, that two lamps might have the same color, and yet show different spectral distributions. Such a case has not yet been observed. On the other hand it has been found that in certain cases, *e. g.*, with the osmium *vs.* the carbon lamp, it is impossible to obtain exact color matches, although the spectrophotometric curves differ by an amount so small that the differences might well be ascribed to experimental error."

Hyde showed that a filament could be set at definite temperature as accurately by the color-match method as the usual pyrometric methods.

The color-match method has been used for several years in this laboratory for estimating the temperatures of filaments.[2] It has proven to be of great practical value and appears to be of more universal applicability than any other pyrometric method.

The relation between the true temperature of a filament and that of a black body which matches it in color has only recently been the subject of study. Hyde considers that " there is much reason to believe that under this condition (of color-match) they are not operating at the same true temperature " and reasons that the temperature of the filament should in general be lower than that of the black body.

Lorenz,[3] by comparing the expansion of gases around similar filaments of platinum and tungsten which are alternately heated, concludes that filaments of these two metals are at the same true temperature when they are at a color-match.

Paterson and Dudding,[4] in an extended investigation with carbon and tungsten lamps, determine the relation between the watts per candle of these lamps and the temperature of a black body giving color-match. They also endeavor to find the difference in temperature between a

[1] Jour. Frank. Inst., *169*, 439 (1910); and ibid., *170*, 26 (1910).

[2] Langmuir & Orange, Trans. Amer. Inst. E. E., *32*, 1944 (1913.)

[3] Phys. Rev., *1*, 332 (1913).

[4] Proc. Phys. Soc. of London, *27*, 230 (1915).

platinum filament and that of a black body which color-matches it. They find as the average of about 30 determinations (varying from 1727° to 1789° C.) that the temperature of the black body which matches a platinum filament at its melting-point (1753° C.) is 1762° C. They also compare their data for the relation between watts per candle and temperature with published data (Forsyth, Pirani, Langmuir) on this relation. As a result they conclude that the differences in temperature between platinum or tungsten and a black body at color-match are small and probably do not exceed 1 or 2 per cent.

We shall see, from experiments to be described later, that there is a distinct difference in temperature between tungsten and a black body which matches it in color, and that this difference is in the direction predicted by Hyde. The difference however is small compared to the difference between true temperature and black body temperature.

(*c*) *Ratio of "Hot" to "Cold" Resistance.*—The ratio of the resistance at the temperature T to that at 0° C. or at room temperature has often been used for estimating temperature (v. Pirani, Corbino, Somerville).

This ratio affords the simplest and most convenient estimation of the temperature of filaments, and has the advantage that it can be used when the filament is surrounded by gas, or even when its *surface* is tarnished by oxidation. In practice, however, the method proves to be one of the least accurate methods of estimating temperatures for the following reasons:

1. The resistance increases relatively slowly with the temperature as compared with most other properties used for temperature estimation.

2. The resistance and its temperature coefficient are very sensitive to traces of impurities (carbon).

3. At room temperature the resistance of the filament is often so low that uncertainties in the lead and contact resistances are apt to play a large part.

(*d*) *Watts per Candle.*—This function has been used more than any other for the rating of tungsten lamps and is thus used as the basis of an arbitrary temperature scale.

Pirani has published[1] tables giving watts per candle as a function of the temperature.

Considerable confusion arises from the fact that the candle power may be measured in several ways: mean horizontal, maximum horizontal or mean spherical. For our purpose we shall measure the candle power of straight filaments in a direction perpendicular to their lengths. This corresponds most nearly to watts per mean horizontal candle power.

[1] Verh. d. D. Phys. Ges., *14*, 213 (1912), and ibid., *14*, 681 (1912).

The advantages in the use of watts per candle are:

1. It requires no knowledge of the dimensions of the filament.

2. The function increases much more rapidly with temperature than does resistance, and is not so greatly affected by impurities in the metal.

The disadvantages are:

1. Unless the mean spherical candle power is measured the results will depend on the geometrical configuration of the filament.

2. The watts per candle do not increase nearly as rapidly with the temperature as the intrinsic brilliancy or the candle power.

3. Watts per candle cannot be used for estimating temperature when the filament is surrounded by a gas.

(*e*) *Resistance.*—If ρ be the resitivity of tungsten at the temperature T, then the resistance of a filament heated electrically to T will be

(2) $$R = \frac{4\rho l}{\pi d^2}.$$

For convenience let us place

$$R' = \frac{4\rho}{\pi}.$$

Equation (2) thus becomes

(3) $$R' = \frac{Rd^2}{l},$$

where R' is a function of the temperature only and is numerically equal to the resistance of a filament of unit length and unit diameter.

(*f*) *Power Radiated.*—Since the energy radiated is proportional to the surface we may place for a filament in a vacuum

(4) $$W' = \frac{W}{ld},$$

where W' is numerically equal to the power (watts) radiated from a filament of unit diameter and unit length.

According to the Stefan-Boltzman law the power radiated per sq. cm. from a black body is σT^4 watts per cm.^{2}

From a tungsten surface the power radiated per cm.^{2} will be $E\sigma T^4$ where E, the "total emissivity," is also a function of the temperature. We may thus place

(5) $$W' = \pi E\sigma T^4.$$

(*g*) *Candle Power.*—If C is the candle-power of a filament measured in a direction perpendicular to the axis of the filament, then we may place

(6) $$C' = \frac{C}{ld},$$

where C' (a function of the temperature only) is the total intrinsic brilliancy of the filament. This property differs, however, from (a), the intrinsic brilliancy, in being the calculated (mean) intrinsic brilliancy for the entire filament rather than the observed brilliancy of a small part of the filament.

The quantity C' can be theoretically calculated from the Planck equation, visibility function, emissivity of tungsten, and mechanical equivalent of light.

The power radiated from a black body per sq. cm. is

$$J_\lambda = \frac{C_1\lambda^{-5}}{e^{\frac{c2}{\lambda T}} - 1} \tag{7}$$

If E_λ is the emissivity of tungsten for the wave-length λ then $E_\lambda J_\lambda$ will be the energy of wave-length λ, radiated from tungsten per sq. cm. The total light expressed in watts[1] is then

$$L_W = \int_0^\infty V_\lambda E_\lambda J_\lambda d\lambda. \tag{7a}$$

By dividing this by M, the mechanical equivalent of light (in watts per lumen) we obtain the total lumens per sq. cm. radiated by tungsten. To convert this to candles per sq. cm. the value in lumens per sq. cm. should be divided by π. Thus

$$C' = \frac{L_W}{\pi M}. \tag{8}$$

The function C' serves as a very convenient measure of temperature. It has the advantage over the measurement of intrinsic brilliancy by a pyrometer that it can be applied to filaments of very small diameter. The use of a photometer is often much more convenient than that of a pyrometer. In common with the pyrometer method it possesses the advantage of being applicable to filaments surrounded by gas.

(h) *Voltage.*—If we take the square root of the product of W' and R' we obtain for a filament in vacuum

$$V' = \sqrt{W'R'} = \frac{V\sqrt{d}}{l}. \tag{9}$$

Thus we see that the voltage drop per cm. along a heated filament in vacuum is inversely proportional to the square root of the diameter.

This relation is useful in determining what the length of a filament should be in order that it may be heated to a given temperature by a given applied voltage.

[1] Ives, Astrophys. Jour., *36*, 322 (1912).

(*i*) *Electron Emission.*—The use of electron emission to measure temperature was suggested by Richardson[1] but has apparently not been practically applied. Although under proper conditions[2] the electron emission from pure tungsten in a high vacuum is reproducible and could serve as a very accurate measure of temperature, it is found in practice that the electron emission is so enormously affected by minute traces of such substances as thorium in the filament or oxygen or water-vapor in the surrounding space, that this method is less reliable than almost any of the others considered in this paper.

(*j*) *Current.*—By taking the square root of the quotient of W' by R' we obtain

$$A' = \sqrt{\frac{W'}{R'}} = \frac{A}{d^{\frac{3}{2}}}. \tag{10}$$

In other words, the current necessary to heat a filament in vacuum to a given temperature varies with the 3/2 power of the diameter.

This function is especially convenient for estimating the temperature of filaments in experimental work. The diameter is readily measured (weight per unit length) and the length need not be known. This method proves in practice to be much more accurate than that based on resistance measurements (R') but less accurate than those based on optical measurements (C') or on total radiation (W'). Unfortunately it cannot be used when the filament is surrounded by gas.

(*k*) *and* (*l*) The two functions $C/(Vd^{\frac{3}{2}})$ and $\sqrt[3]{C/R}/d$ can be used to measure the temperature of filaments of unknown length but in most cases (*j*) will be found more convenient. However, when the filament is surrounded by a gas (*j*) is not applicable and in this case $\sqrt[3]{C/R}/d$ could be profitably used to measure temperature.

(*n*) $(V\sqrt[3]{A})/l$. This function proves very useful in estimating the temperature of filaments of varying diameter. For example, if it is desired to measure the rate of evaporation of a filament, or the rate of attack by a gas at very low pressure, the filament can be maintained at constant temperature by maintaining $V\sqrt[3]{A}$ constant, even when the diameter changes considerably. It also proves useful in estimating (by purely electrical measurements) the temperature of a filament in a sealed bulb. In this case the diameter often cannot be determined accurately but the length of the filament can be determined by cathetometer measurements.

This function is not applicable to filaments surrounded by gas.

(*o*) *and* (*p*) $(\sqrt[3]{CV^2})/l$, $C/(A^{\frac{2}{3}}l)$. These functions have as yet not found any application.

[1] Phys. Rev., *27*, 183 (1908).

[2] Langmuir, Physik. Zeitsch., *15*, 516 (1914).

(*q*) $(\sqrt[3]{C^2R})/l$. This function can be used in place of $(V\sqrt[3]{A})/l$ for filaments of varying diameter when surrounded by a gas at relatively high pressure. With sufficient gas pressure convection currents carry away the evaporated material to a part of the bulb where it does not interfere with candle-power measurements. It may also be used for measuring the temperature of filaments (not helically wound) of unknown diameter in gases.

(*r*) *Thermal Expansion.*—This function can be used as an approximate measure of temperature. The change of length of a single loop filament can be determined fairly accurately by a cathetometer, so that the temperature may be readily found within 30–50° at 2500°.

Experimental Measurement of Characteristics.

The following general plan was adopted in the experimental study of the characteristics of tungsten filaments.

Specially constructed lamps with carefully measured long single loop filaments were thoroughly aged by running the filaments at 2400° K. for 24 hours after which they were set up in front of a Holborn-Kurlbaum pyrometer. The volts and amperes were thus obtained as functions of the temperature. The lamps were then measured on the photometer, and the candle power per cm. of length was determined by photometering through a measured horizontal slit.

From these data V', A' and C' were calculated as functions of the temperature and from these R', W' and W/C were obtained.

Lamps Used.—About 20 special lamps were constructed having cylindrical bulbs 7–9 cm. in diameter. The leads were of very large size (.05 cm. platinum for the smaller filaments, and .08 cm. tungsten for the larger sizes) so that the total lead resistance, obtained by subsequently welding together the two leads, averaged only .017 ohm. The filaments, in the form of single hair-pin loops, were electrically welded to the leads in such a way that the filament close to the lead was straight, and the junction of the lead and the filament was sharply defined. The lengths of the filaments were carefully measured after they were welded to the leads, but before they were sealed into the bulbs.

Tungsten Wires.—Twelve different samples of tungsten wire were used. Of these, six were regular samples of commercial wire, used in the manufacture of lamps. Four out of these six contained thorium (thoriated tungsten).

The remaining six samples were special experimental lots of wire drawn down from three different rods. Two of the rods were made by the Pacz process, and were of an unusually high degree of purity. The other rod was made of pure tungsten to which thorium oxide had been added.

The diameter of the wires ranged from .0028 to .0252 cm.

Measurement of Diameters.—Experience with many ways of measuring diameter has finally resulted in the adoption of calculation from the density and weight per unit length. The density of drawn tungsten wire (as taken from the spool) ranges from 18.4 to 19.0, but recent measurements on the filaments of lamps which have run 24 hrs. or more shows that the density nearly always lies between 18.9 and 19.1, with an average of 19.0. These measurements were made by weighing the filaments in air and in bromoform. Since it is the diameter of the wire after running in the lamp that is wanted, the value 19.0 has been chosen for the density in the calculation of the diameter.

The diameter in cm. is thus given by the formula (w = mg. per cm.).

$$d = .008186\sqrt{w}. \tag{11}$$

The diameters obtained this way for wires which have been run in lamps agree within the experimental error with measurements made with a micrometer.

Pyrometer.—The pyrometer was the same as that described in connection with the determination of the melting-point of tungsten (*l. c.*).

Before undertaking the study of the characteristics of tungsten filaments the pyrometer was carefully recalibrated.

For this purpose Hyde's value $\lambda = .664\ \mu$ for the effective wave-length of the red screen (double thickness of Schott & Gnossen's red glass No. 4512) was taken instead of the value[1] $\lambda = .667$ previously used. Slight errors of 0–12° at high temperatures were found in the calibration curve giving the relation between pyrometer current and true temperature. Although the new calibration curve differed very little from the old, yet by its aid the results obtained with different screens (red or green) or sectors agreed among themselves better than before.[2]

In calibrating this pyrometer for true temperatures it was assumed: First, that the melting-point of gold is 1335.4° K. Second, the emissivity of tungsten (for $\lambda = .664\ \mu$) is 0.46. Third, the bulb absorption was 9 per cent. and fourth, the constant C_2 of the Wien equation is 1.4392.

Photometer Measurements.—A Lummer-Brodhun constant illumination photometer was used. In most of the work an illumination of 57 candle meters was used, but with filament temperatures below 1900° K. it was necessary to use lower illuminations. In photometering the filaments

[1] Phys. Rev., *6*, 70 (1915).

[2] The temperatures obtained with the new calibration are on the whole slightly higher than with the old. The values for the melting-point of tungsten previously obtained ranged from 3530–3566°. The present recalibration would indicate that the higher values are probably correct, so that the most probable value for the melting-point would seem to be 3570° K.

at very high temperatures, the large difference in color between the filament and the standard lamp was avoided by inserting a blue glass screen between the standard lamp and the photometer head. The transmission coefficient for this screen was determined in this laboratory by the use of a flicker photometer. It was also calibrated for us through the kindness of Dr. Ives, by the use of his physical photometer.[1] The results by the two methods were practically identical.

The object of the photometer measurements was primarily to determine the intrinsic brilliancy of the filaments in candles per sq. cm. To avoid uncertain corrections due to the cooling effects of the leads, a slit, usually 2.5 cm. wide, was placed horizontally in front of and close to the lamp to be photometered. The intrinsic brilliancy was then found by dividing the observed candle power by the effective area of the filament. This effective area was calculated as follows: The effective length of the filament was taken as twice the width of the slit multiplied by the ratio of the distance of the filament from the photometer head to that of the slit from the photometer head. The effective area was then obtained as the product of the effective length by the diameter. Of course care was always taken that the two portions of the filament seen through the slit were as nearly straight and parallel as possible. Correction was made, when necessary, for any lack of parallelism.

Electrical Measurements.—The volts and amperes taken by the lamps were measured by " laboratory standard " instruments which were calibrated both before and after the measurements. Corrections were made for the current taken by the voltmeter and for the resistance of the leads of the lamps (usually about .017 ohm).

Correction for the Cooling by Leads.—As indicated above, the candle-power measurements were made in a way which eliminated any effect due to leads. The voltage measurements, however, were influenced by this cause, since the cooling of the ends of the filaments lowered the resistance. To determine the necessary correction, the temperature of the filament at two or three points near the leads was determined. From these data, by methods which will be described in a subsequent paper,[2] the correction was found which should be *added* to the observed voltmeter reading in order to eliminate the effect of the leads. In all cases this correction was approximately equal to

$$.00026\ (T - 300)\ \text{volts},$$

where T is the temperature (° K.) of the central portion of the filament.

[1] PHYS. REV., *6*, 319 (1915).

[2] See also Worthing, PHYS. REV., 4, 538 (1914).

This correction is independent of the diameter of the filament in case the size of the lead bears a fixed ratio to that of the filament.

Preliminary Treatment of Filaments.—While the lamps were being exhausted the filaments were heated for one minute to a temperature of about 1500°. This treatment causes the evolution of an amount of gas (mostly carbon monoxide) which measured at atmospheric pressure is usually about 3 to 6 times the volume of the filament. There is also a simultaneous change in the filament whereby its cold resistance is lowered 15 to 20 per cent., and the temperature coefficient of its resistance increased by a like amount. Longer or more severe heat treatment causes the evolution of only insignificant quantities of gas.[1]

After sealing off from the pump, the characteristics of the lamps were studied and the temperatures were determined by the pyrometer. The observations were made at gradually increasing temperatures up to about 2600° and then a series of measurements at descending temperatures was made. When the observations with the descending temperatures are compared with the initial measurements at 1100 to 1500° it is found that at a given temperature the *wattage* decreased 30 to 50 per cent., whereas the resistance suffered little if any change. The effect of the first heating to 2600° is thus to decrease the total emissivity.

The filaments were then heated to 2400° K. for twenty-four hours before studying the characteristics. This heating produced a slight further decrease in emissivity and a slight (2 to 4 per cent.) decrease in resistance. More prolonged heat treatment than twenty-four hours produces (with drawn wire filaments) only very gradual changes. These changes may be of three kinds: First a roughening of the surface with slight increase in emissivity; second, a decrease in cross-section by evaporation and consequent increase in resistance of the filament, and third, a gradual elimination of traces of impurities by distillation, accompanied by a corresponding decrease in specific resistance.

With filaments of very large diameter the last of these processes takes place rather slowly, but with filaments up to 0.1 or 0.2 mm. diameter the change is nearly complete in twenty-four hours. The time of twenty-four hours was chosen for the heat treatment in these experiments because, if the time is extended beyond this, the changes due to the evaporation of tungsten and the resulting darkening of the bulb, are liable to more than offset the changes in resistance due to the elimination of minute traces of impurities.

[1] If the bulb has not been well baked out, or if stop-cocks (grease!) are used in the system the continued heating of the filament will, by the decomposition of water-vapor or hydrocarbons, cause the slow evolution of apparently unlimited quantities of gas. (See J. Amer. Chem. Soc., *35*, 105 (1913).

EXPERIMENTAL RESULTS.

Volt-Ampere Characteristics.—With each lamp tested the readings of volts and amperes for about 30 to 50 pyrometer settings from 1050 up to 3540° K. were obtained. Up to temperatures of about 3000° K. the changes in the filament and the blackening of the bulb occurred so slowly that readings could be taken without haste. For higher temperatures than this the readings were taken at greater intervals (usually about 100° apart) and were made as quickly as possible, and after each reading the volts and amperes were redetermined at one or two lower temperatures (about 2400 and 2600° K.). From these data it was possible to calculate the change in diameter of the filament (due to evaporation) and the amount of absorption of light by the bulb. This calculation was made by the aid of the function $V\sqrt[3]{A}/l$. We have seen that this function for any given filament temperature is independent of the diameter of the filament. The relation between $V\sqrt[3]{A}$ and temperature was plotted from the data obtained before the lamp was run up to 3000°. This curve could be used to determine the true temperatures even after the filament had lost material by evaporation. By then comparing the observed pyrometer reading with the calculated true temperature, the bulb absorption was determined. Knowing the bulb absorption it was then possible to determine the true temperature when the filament was subsequently raised again to a temperature above 3000° K.

In this way it was found practicable to study the characteristics of the filaments in vacuum right up to the melting-point of the filaments. Although in some cases corrections as large as 250° (at 3500°) needed to be applied for the effect of bulb absorption, yet it is felt that the accuracy with which such corrections could be made was so high that the uncertainty in the final result was not over 30 to 40° (at 3540° K.). As a matter of fact the agreement between the results obtained with different lamps was considerably better than this.

From the observed values of volts and amperes (corrected for lead resistance, etc.) the values of the functions V' and A' were found from the known lengths and diameters. The logarithms of these functions were then plotted on a large scale against log T. The points from all the lamps tested were found to lie very closely along a smooth curve. The average deviation of the points from the smooth curve was found to be about 0.2 per cent. in temperature (both for V' and A'). For filament temperatures above 1200° K. the maximum departure of any of the observations was 1.2 per cent. in temperature. The small deviations which did occur were not all irregular, for certain lamps tended to give low values while others gave consistently high values (within the above

TABLE I.

Characteristics of Tungsten Filaments.

T.	V'	A'	W'	R'	C'	W/C	$V'\sqrt[3]{A'}$	E	Log V'	Log A'	Log C'
°K.	$\frac{V\sqrt{d}}{l}$	$\frac{A}{d^{3/2}}$	$\frac{W}{ld}$	$\frac{Rd^2}{l}$	$\frac{C}{ld}$		$\frac{V\sqrt[3]{A}}{l}$				
				$\times 10^{-6}$					−10		−10
273				6.37							
300	.000050	6.9	.00034	7.24			00009	.0237	5.6959	0.8362	
400	.000341	32.7	.0112	10.43			.00109	.0330	6.5331	1.5148	
500	.000764	55.5	.0424	13.76			.00291	.0421	6.8828	1.7442	
600	.001396	81.0	.1131	17.23			.00604	.0514	7.1448	1.9085	
700	.002330	111.8	.2606	20.83			.01123	.0626	7.3673	2.0486	
800	.003648	148.6	.5420	24.55			.01932	.0755	7.5620	2.1720	
900	.005439	191.8	1.043	28.36			.03136	.0904	7.7355	2.2828	
1000	.007795	241.8	1.885	32.24	.00014	13,460.	.04856	.1069	7.8918	2.3834	6.1455
1100	.01080	298.5	3.225	36.20	.00120	2,680.	.07221	.1247	8.0336	2.4749	7.0803
1200	.01454	361.5	5.258	40.23	.00738	712.5	.1036	.1435	8.1627	2.5581	7.8680
1300	.01908	430.2	8.207	44.34	.03461	237.1	.1440	.1625	8.2805	2.6337	8.5392
1400	.02445	504.0	12.32	48.52	.1325	93.0	.1945	.1813	8.3883	2.7024	9.1222
1500	.03071	582.0	17.87	52.77	.4243	42.1	.2564	.1995	8.4873	2.7649	9.6277
1600	.03792	663.7	25.17	57.13	1.179	21.35	.3309	.2170	8.5789	2.8220	0.0715
1700	.04613	748.9	34.55	61.61	2.928	11.80	.4189	.2338	8.6640	2.8744	0.4666
1800	.05539	836.8	46.34	66.19	6.552	7.074	.5219	.2494	8.7434	2.9226	0.8163
1900	.06575	927.5	60.98	70.89	13.46	4.530	.6412	.2643	8.8179	2.9673	1.1291
2000	.07725	1,021.	78.87	75.67	25.90	3.045	.7778	.2785	8.8879	3.0090	1.4133
2100	.08995	1,117.	100.5	80.52	46.8	2.147	.9333	.2920	8.9540	3.0481	1.6702
2200	.1039	1,216.	126.3	85.41	80.6	1.568	1.109	.3046	9.0165	3.0850	1.9063
2300	.1192	1,318.	157.1	90.41	133.3	1.179	1.307	.3174	9.0762	3.1200	2.1249
2400	.1357	1,423.	193.2	95.39	209.8	0.9207	1.527	.3290	9.1327	3.1532	2.3218

TABLE I.—*Continued.*

T	V'	A'	W'	R'	C'	W/C	$V'\sqrt[3]{A'}$	E	Log V'	Log A'	Log C'
°K.	$\frac{V\sqrt{d}}{l}$	$\frac{A}{d^{3/2}}$	$\frac{W}{ld}$	$\frac{Rd^2}{l}$	$\frac{C}{ld}$		$\frac{V\sqrt[3]{A}}{l}$				
				$\times 10^{-6}$					−10		
2500	.1538	1,531.	235.5	100.48	319.6	0.7367	1.772	.3407	9.1870	3.1849	2.5046
2600	.1733	1,642.	284.5	105.56	471.0	0.6041	2.045	.3517	9.2388	3.2153	2.6730
2700	.1943	1,756.	341.1	110.69	674.9	0.5054	2.344	.3625	9.2885	3.2444	2.8293
2800	.2169	1,873	406.3	115.83	944.0	0.4303	2.674	.3733	9.3363	3.2725	2.9750
2900	.2410	1,993.	480.5	120.9	1,290.	0.3725	3.034	.3839	9.3821	3.2996	3.1106
3000	.2669	2,117.	565.2	126.1	1,729.	0.3270	3.428	.3941	9.4264	3.3258	3.2377
3100	.2944	2,244.	660.7	131.2	2,272.	0.2908	3.854	.4041	9.4689	3.3511	3.3564
3200	.3236	2,376.	768.8	136.2	2,941.	0.2615	4.318	.4142	9.5100	3.3758	3.4684
3300	.3543	2,511	889.6	141.1	3,763.	0.2364	4.816	.4239	9.5494	3.3998	3.5755
3400	.3863	2,649.	1,025.	146.0	4,725.	0.2169	5.352	.4334	9.5875	3.4231	3.6744
3500	.4213	2,792.	1,176.	150.9	5,869.	0.2004	5.932	.4428	9.6246	3.4459	3.7685
3540	.4355	2,850.	1,241.	152.8	6,373.	0.1948	6.175	.4467	9.6390	3.4549	3.8043

mentioned limits). No relationships, however, could be traced between these deviations from the mean and the diameter or other characteristics of the wire.

The values of log V' and log A' obtained from the smooth curve described above from 1300° up to 3540° have been tabulated in Table I., together with the corresponding values of V' and A'. For temperatures below 1300° the uncertainty in the cooling effect of the leads and the difficulties in the accurate temperature measurements have made the results of the direct experiments less trustworthy, so that the characteristics below 1300° have been determined by another method which will be described later.

The values of W', R' and $V'\sqrt[3]{A'}$ given in Table I. have been obtained directly from V' and A'.

Candle-Power Data.—To determine the relation between candle power per sq. cm. and temperature, three lamps were studied. Two of these were nitrogen-filled lamps with large (0.6 mm. diam.) single loop, straight (not helically wound) filaments, and the third was a vacuum lamp with a single loop filament, 0.25 mm. diameter. By the use of nitrogen-filled lamps it was possible to determine the relation between temperature and candle power up to temperatures of 3100° K. without any blackening of the bulbs or any permanent change in the characteristics of the filament.

TABLE II

Experimental Values of C′

Temp.	C' Candles per Sq. Cm.	Ratio C'/L_W.
1400	.109	216.2
1500	.381	235.8
1600	1.101	245.2
1700	2.824	253.4
1800	6.493	260.4
1900	13.43	262.0
2000	25.67	260.4
2100	46.85	262.8
2200	81.33	265.2
2300	133.3	262.6
2400	208.	260.8
2500	318.0	261.4
2600	470.5	262.4
2700	677.	263.4
2800	949.	264.2
2900	1295.	263.8
3000	1733.	263.4
3100	2273.	262.8
	Average.....	262.7

At low filament temperatures irregularities occur in the convection losses from the filaments of gas-filled lamps. It was to avoid these, and to find whether the nitrogen exerted any effect on the emissivity that the vacuum lamp was studied.

The values of log C' obtained from the three lamps were plotted against $1/T$. A smooth curve (not far from a straight line) was passed as nearly as possible through the points. The average departure of the points from the smooth curve was about 0.15 per cent. in temperature, and the maximum departure of any of the points was 0.8 per cent. in temperature.

The values of C' taken from the smooth curve for every hundred degrees from 1400 to 3100° are given in the second column of Table II.

Discussion of Experimental Data.

It will be of interest to compare the results of the above experiments with the data of other investigators. This will also have the advantage that it will enable us to extend our data on the characteristics of tungsten filaments down to temperatures below 1300° K., where our experiments gave unreliable results because of the cooling effects of leads and the difficulty of measuring temperatures by the optical pyrometer.

The ratio of hot to cold resistance for tungsten filaments has been determined by Somerville,[1] Pirani,[2] and Corbino.[3]

From these data it is possible to calculate R' provided we know the specific resistance of tungsten at room temperature.

Specific Resistance of Tungsten.—Fink,[4] has given the specific resistance of tungsten at 25° C. as 5.0×10^{-6} ohm-cm. units, and its temperature coefficient between 0° and 170° as .0051. Thus at 0° C. according to Fink the specific resistance would be 4.43×10^{-6} ohm-cm. More recent measurements made in this laboratory of the specific resistance of well-aged tungsten filaments have given at 0°, 5.0×10^{-6} ohm-cm.

The temperature coefficient from 0 to 300° is found to agree well with Fink's value, *i. e.*, .0051.

We have seen that the function R' is equal to $4\rho/\pi$ so that we may place at 0° C. $R' = 6.37 \times 10^{-6}$.

At higher temperatures we may now calculate R' from Somerville's, Pirani's and Corbino's data by simply multiplying their values of the ratio of hot to cold (0° C.) resistance by the above value for R' at 0° C. In this way the results given in the second, third and fourth columns of Table III. have been obtained.

[1] Phys. Rev., *30*, 433 (1910).
[2] Verh. deutsch. Phys. Ges., *12*, 301 (1910), and Phys. Zeit., *13*, 753 (1912).
[3] Phys. Zeit., *13*, 375 (1912).
[4] Trans. Amer. Electrochem. Soc., *17*, 229 (1910).

In our own experiments on the characteristics of tungsten filaments we have measured R' from 1300° K. up to 3540° K. and from 273° K. to 600° K. The agreement with Somerville's results at low temperatures is excellent, while at high temperatures the results agree with Pirani's. At lower temperatures Pirani's values appear to be too low.

To fill in the gap in our measurements between 600° and 1300° the values have been chosen by interpolation between the higher and lower values, laying due weight on the results of Somerville and Pirani.

Corbino's results above 1700° are too low, probably due, as Pirani has pointed out[1] to failure to correct for the cooling effect of the leads.

Table III.

Comparison of Values of R^1.

T	Somerville.	Pirani.	Corbino.	Langmuir.
273	6.37	6.37	——	6.37
300	7.24	7.24	——	7.24
400	10.45	10.06	——	10.43
500	13.76	13.19	——	13.76
600	17.4	16.5	——	17.23
700	20.8	20.0	——	20.83
800	24.5	23.6	——	24.55
900	28.2	27.4	——	28.36
1000	31.7	31.2	——	32.24
1100	35.6	35.2	——	36.20
1200	40.1	39.2	——	40.23
				——
1300	45.4	43.3	——	44.34
1400	——	47.3	——	48.52
1500	——	51.5	——	52.77
1600	——	56.1	——	57.13
1700	——	60.8	60.2	61.61
1800	——	65.6	64.5	66.19
1900	——	70.6	68.5	70.89
2000	——	75.5	72.8	75.67
2100	——	80.5	76.9	80.52
2200	——	85.5	81.2	85.41
2300	——	90.5	85.4	90.34

The values of R' obtained as above (from 300 to 1300°) have been inserted in Table I.

The specific resistance of tungsten at any temperature may be obtained by multiplying the corresponding value of R' by $\pi/4$. A table of specific resistances calculated as above has been published.[2]

[1] Phys. Zeitsch., *13*, 753 (1912).

[2] Phys. Rev., *7*, 154 (1916).

Total Emissivity.—Planck's law (Equation 7) gives J_λ the energy of wave-length λ radiated from a black body. The total energy W' radiated from a tungsten filament one cm. in diam. and one cm. long is then

(12) $$W' = \pi \int_0^\infty E_\lambda J_\lambda d\lambda.$$

On the other hand, by the Stephen-Boltzman law we may place

(13) $$W' = \pi E \sigma T^4,$$

where E is the *total emissivity* and the constant σ is equal to 5.633×10^{-12} watts per sq. cm.

The values of E, above 1200°, given in Table I. have been calculated from the corresponding values of W' by means of this equation.

For the lower temperatures (up to 600° K.) the value of E has been calculated by a formula given by Foote,[1] namely,

(14) $$E = 0.5736\sqrt{\rho T},$$

where ρ is the resistivity of ohm-cm.

The results obtained from this equation for the higher temperatures do not agree at all well with the values found from our experiments, as may be seen from the following comparison.

TABLE IV

T.	By Foote's Formula.	From Table I.
1200	.1118	.1435
1600	.1537	.2170
2000	.1978	.2785
3000	.3125	.3941

The failure of Foote's formula at the higher temperatures is to be expected if we take into account the work of Rubens and others on the temperature coefficient of the emissivity of metals. Rubens[2] by a study of the emissivity of platinum, platinum-rhodium, nickel and constantan at different temperatures, concludes that for wave-lengths below 2 μ the temperature coefficient of the emissivity is negligible or at least very small, whereas for wave-lengths greater than 6 μ the emissivity may be calculated accurately by the Hagen-Rubens formula

(15) $$E_\lambda = .365\sqrt{\frac{\rho}{\lambda}}.$$

For these longer wave-lengths the temperature coefficient of the emissivity is thus one half that of the electrical resistivity.

[1] Bull. Bur. Stand., *11*, 607 (1915).

[2] Verh. deutsch. Phys. Ges., *12*, 172 (1910).

Between the limits of 2 and 6 μ Rubens finds that the transition from one type of temperature coefficient to the other occurs gradually.

Investigations on many different metals during the last few years have shown that the Hagen-Rubens formula is universally applicable for the longer heat rays, whereas in the visible spectrum the emissivity seems to be determined by totally different causes and is, in general, very nearly independent of the temperature.

The Foote formula for the total emissivity is derived from the Hagen-Rubens equation and should therefore be applicable only where most of the energy radiated is limited to wave-lengths for which the emissivity is given by the Hagen-Rubens equation. Since the wave-length of maximum energy emission is about 5 μ for a black body at 600° K. and is about 2 μ at 1500° K. we should expect the Foote equation to apply quite accurately up to about 500 or 600° K. but above this temperature the Foote equation should give low values, and at temperatures as high as 1500° K. it should be nothing more than a rough approximation. This seems to be borne out by the experimental data.

In order to obtain values for the emissivity at temperatures between 600° and 1200° recourse has been taken to interpolation between Foote's formula at lower temperatures and the experimental data at the higher temperatures. The results obtained in this way have been entered in Table I.

The uncertainty involved in this interpolation probably does not exceed five per cent. in any single value given for the emissivity.

From the data on the total emissivity the values of W' for temperatures below 1200° were calculted by means of equation (13). At the lower temperatures, however, "back radiation" was taken into account so that W' was actually calculated from the equation

(16) $$W' = \pi\sigma(ET^4 - E_0T_0^4).$$

Here T_0 is room temperature which is taken to be 20° C. ($T_0 = 293$).

E_0 is the total absorption coefficient of a filament at a temperature T for energy radiated by a black body at T_0, that is, at 293°. Since the wave-length of the energy radiated at room temperature is longer than 6 μ Hagen and Rubens's formula should apply and we may therefore consider that E_0 increases in proportion to $\sqrt{\rho}$. For $T = 293$ we may place $E_0 = E$ and may thus calculate E_0 from Foote's formula.

(17) $$E_0 = 0.0230\sqrt{\rho/\rho_0},$$

where ρ_0 is the resistivity at 293°.

The values of W' from room temperature up to 1200° have been calculated from equations (16) and (17). The functions A', V', etc., for these temperatures have been calculated from W' and R'.

Total Intrinsic Brilliancy.—It will be of interest to compare the experimentally determined candles per sq. cm. (Table II.) with the results of the calculation of this quantity from the Planck equation and visibility function according to equation 8.

For this purpose we take the constants of the Planck equation to be

$$C_1 = 3.721 \times 10^{-12} \text{ watts per sq. cm.},$$

$$C_2 = 1.4392 \text{ cm.}$$

These values, according to Planck's theory, correspond to the following values of the fundamental constants h, N, and σ[1]

$$h = 6.580 \times 10^{-27} \text{ ergs. sec.},$$

$$N = 6.062 \times 10^{23},$$

$$\sigma = 5.633 \times 10^{-12} \text{ watts per sq. cm.}$$

Nutting's[2] visibility data has been chosen since Ives[3] considers this more accurate than his own.

From these data the light (in watts per sq. cm.) radiated from a black body has been calculated by the equation

$$L_B = \int_0^\infty V_\lambda J_\lambda d\lambda. \tag{18}$$

The integration was performed by the aid of Simpson's one third rule using 35 ordinates at each temperature. The calculations were carried out to four significant figures and the results should be accurate to 0.2 per cent. The values of L_B obtained in this way are given in Table V.

Similar calculations of the light radiated from tungsten were then made by the same method. For this purpose the emissivity of tungsten

TABLE V

Light Radiated from a Black Body and from Tungsten in Watts/Cm.² Calculated from the Planck Equation and Nutting's Visibility Data.

T °K.	L_B Black Body.	L_W Tungsten.
1000	.00000109	.000000532
1300	.000269	.000132
1500	.00329	.001615
1750	.03429	.01687
2000	.2000	.0986
2500	2.458	1.216
3000	13.26	6.580
3500	44.98	22.34

[1] Dushman, Gen. Electric Rev., December, 1915.

[2] Trans. I. E. S., *9*, 633, (1914).

[2] PHYS. REV., *5*, 287 (1915).

E_λ was taken to be 0.46 for $\lambda = .66\,\mu$ and 0.50 for $\lambda = 0.55$ while for other wave-lengths in the visible spectrum the emissivity was obtained by linear interpolation or extrapolation from these values.[1]

Thus the light radiated from tungsten was obtained by the equation

$$L_W = \int_0^\infty E_\lambda V_\lambda J_\lambda d\lambda. \tag{19}$$

The values of L_W have been recorded in Table V.

The quantity L_W should be proportional to C' the candles per sq. cm. found by experiment. In Table II. the ratio of C' to L_W has been tabulated.[2] It is seen that at temperatures above 1900° the ratio is very nearly constant. The absence of any systematic error is an excellent confirmation of the correctness of the pyrometric measurements. At temperatures below 1900° the ratio of C' to L_W decreases considerably. This, however, is to be expected, for it was necessary to work with very low field illuminations (even as low as 0.2 meter candles) at the lower temperatures, and under these conditions it is well known that the Purkinje effect tends to give low values in the measurement of red light. We are therefore justified in discarding the measurements made below 1900°. The average of the remaining values of the ratio gives

$$\frac{C'}{L_W} = 262.7. \tag{20}$$

The mechanical equivalent of light may be obtained from this ratio by equation 8:

$$M = \frac{L_W}{\pi C'} = .00121 \text{ watt per lumen.} \tag{21}$$

This value is considerably lower than the latest value of this constant given by Ives and Kingsbury,[3] namely .00159. Nutting[4] gives the value 66.2 candles per watt which corresponds to .00120 lumen per watt, and this he considers is uncertain within about five per cent.

Ives[5] has previously calculated the mechanical equivalent by the method given above, using Nernst's[6] data for the intrinsic brilliancy of a black body. This result, recalculated according to more recent visi-

[1] The value 0.46 is taken from the author's paper on the melting-point of tungsten (l. c.). The slope of the curve E_λ (λ) was taken from Coblentz (Bull. Bur. Stand., *7*, 198 (1911)).

[2] For this purpose the values of L_W for temperatures other than those given in Table V. were found by interpolation. This could be done with high accuracy since log L_W is very nearly a linear function of $1/T$.

[3] Phys. Rev., *6*, 319 (1915).

[4] Trans. I. E. S., *9*, 633 (1914).

[5] Electrical World, 1911, p. 1565.

[6] Phys. Zeit., *7*, 380 (1906).

bility data[1] leads to the value .00125 watt per lumen. Ives suggests that the discrepancy between this value and .00159 may be due to a failure of the Planck equation for short wave-lengths.

It should be noted that the value .00121 obtained from the experiments described in this paper does not depend to any great extent upon assumptions as to the emissivity of tungsten. This is clear when we consider that an error in emissivity would affect the temperatures obtained by the pyrometer in such a way as to offset the change in the brilliancy calculated by equation 19.

The ratio $C' : L_W$ in Table II. is so nearly constant that we may use it to calculate C' from L_W with a higher accuracy than that of the original values of C' given in the table.

The values of C' given in Table I. have therefore been obtained from the corresponding values of L_W by multiplying them by 262.7, the mean value of C'/L_W.

In a similar way we may obtain the intrinsic brilliancy of a black body in candles per sq. cm. from the corresponding value of L_B by multiplying these by 262.7. Results obtained this way are given in the second column of Table VI.

There have been several other determinations of the total intrinsic brilliancy of the black body, and it will be of interest to compare some of these with the above results.

In a recent publication Pirani[2] calculates the intrinsic brilliancy by a similar method to that which we have used. He chooses for the constants of the Planck equation

$$C_1 = 3.5 \times 10^{-12} \text{ watts per sq. cm.,}$$

$$C_2 = 1.440 \text{ cm.,}$$

and he uses Ives's visibility data. In this way he finds the slope of the curve (log C against $1/T$), and then in order to fix the position of the curve takes the value 1 Hefner candle per sq. cm. as the brilliancy of a black body at 2090° K. This value he obtains by extrapolation over a range of 400° from some data given by Lummer and Pringsheim in 1901.

In the third column of Table VI. are given Pirani's results converted to international candles per sq. cm.

At low temperatures Pirani's values for the brilliancy are 10 to 20 per cent. *higher*, and at high temperatures as much as 15 per cent. *lower* than those we have calculated. Both sets of values are calculated from the Planck Equation and visibility data. The discrepancy cannot be accounted

[1] Phys. Rev., *5*, 273 (1915).

[2] Verh. d. D. Phys. Ges., *13*, 219 (1915).

TABLE VI

Brilliancy of Black Body (C_B').

International candles per sq. cm.

T	Langmuir.	Pirani.	Nernst.	ΔT, $T_L - T_N$
1300	.0706	.086	.0482	−26.
1400	.270	.306	.199	−24.
1500	.864	.936	.648	−26.
1600	2.40	2.52	2.01	−18.
1700	5.95	6.03	5.19	−16.
1800	13.3	13.23	12.1	−13.
1900	27.3	27.9	25.7	− 9.
2000	52.5	53.1	50.8	− 5.
2100	94.9	94.5	94.1	− 2.
2200	163.	153.	164.7	+ 2.
2300	270.	252.	275.0	+ 4.
2400	424.	414.	439.	+ 8.
2500	646.	630.	674.	+10.
2600	951.	900.	1,000.	+13.
2700	1,362.	1,260.	1,450.	+17.
2800	1,904.	1,800.	2,040.	+21.
2900	2,600.	2,430.	2,810.	+25.
3000	3,485	3,330.	3,780.	+29.
3100	4,579.	———	5,000.	+33.
3200	5,920.	5,040.	6,480.	+36.
3300	7,580.	———	8,280.	+37.
3400	9,515.	8,280.	10,430.	+41.
3500	11,820.	———	12,960.	+43.

for by the slightly different values chosen for the constant C_2 nor does the fact that Pirani chose the Ives instead of the Nutting visibility curve account for the difference.[1]

The difference between the Pirani and Langmuir values for C_B' would seem to be caused by insufficient accuracy in the graphical methods used by Pirani in determining the area of his curves.

Nernst[2] has given the following relation between the temperature and the brilliancy of a black body:

$$T = \frac{11230}{5.367 - \log_{10} K} \tag{22}$$

where K is the brilliancy in Hefnerkerzen per sq. mm. If we express the brilliancy in international candles per sq. cm. this becomes

[1] In order to determine how much the values of C_B' would be changed by using Ives's visibility data in place of Nutting's, the values of L_B (Table V.) were recalculated from Ives's data. The new values were found to be uniformly 5.8 per cent. higher than those given in Table V. Thus the use of Ives's data cannot account for the fact that the Pirani and Langmuir curves (Table VI.) *cross each other.*

[2] Phys. Zeit., 7, 380 (1906).

$$T = \frac{11230}{7.321 - \log_{10} C_B'}. \quad (23)$$

The values given in the fourth column of Table VI. are calculated by means of this equation.

Until recently the temperature scale used in this laboratory has been based on this Nernst equation. The corrections that should be applied to this temperature scale to reduce it to the new scale are given in the fifth column. These figures may be used to correct temperature data which have been given in various publications from this laboratory. In most cases the errors involved are within the experimental error.

Color of Light Radiated by Tungsten.

A series of experiments was undertaken to determine whether the light radiated from tungsten differs materially in color from that radiated by a black body. A helix was formed by winding a 20 mil tungsten wire on a 40 mil mandrel using a 20 mil spacer. This filament was sealed into a bulb which was then filled with nitrogen. A very much enlarged image of the heated filament was thrown on a white screen and by means of a Weber portable photometer the intensity and color of various parts of the image were determined.

The intensity of the light coming from the interior of the helix was from 1.5 to 1.8 times as great as that coming from the outer portions. The color of the light from the interior was distinctly redder than that from the outside. By first color-matching the interior of the helix against the photometer lamp and then increasing the current through the helix until the outer portions gave a color-match with the photometer, it was possible to determine the difference in color quantitatively.

If we take the emissivity of tungsten for white light to be 0.50 and assume that the inner portion radiates like a black body, then the ratio of the brilliancy of the inner and outer portions should be 2.0. Actually the observed ratio as found by the photometer was less than this (1.5 to 1.8) showing that strictly black-body conditions did not obtain. The observed difference in color between the inner and outer portions was therefore corrected for this lack of blackness as measured by the ratio of brilliances.

Two methods may be used in making this correction: First, it may be assumed that the light from the interior is the light emitted from a tungsten surface which has been reflected n times before escaping. On this assumption

$$\frac{\Delta_2}{\Delta_1} = \frac{-\log E}{\log R}, \quad (24)$$

where Δ_1 is the observed difference in color between the inner and outer parts (n reflections), Δ_2 is the difference in color which would exist if the inner parts radiated like a black body ($n = \infty$), R is the observed ratio of brilliancy and E is the emissivity of tungsten.

Second, it may be assumed that the light from the inner parts is a mixture of light from some parts which radiate as a black body, with light from other parts which radiate like an exposed tungsten surface. On this basis we find:

$$\frac{\Delta_2}{\Delta_1} = \frac{1 - E}{1 - 1/R}, \tag{25}$$

where the symbols have the same meaning as before. The corrections actually used were obtained by averaging those obtained by the two methods. The corrections in most cases amounted to 20 to 40 per cent. The results obtained in this way are given in Table VII.

Table VII.

T_W	T_B	$T_B - T_W$	$\frac{d \ln E_\lambda}{d\lambda}$
1800	1841	41	−5,900
2000	2050	50	−5,800
2200	2257	57	−5,500
2400	2464	64	−5,100
2600	2672	72	−4,900
2800	2877	77	−4,500
3000	3082	82	−4,200
3200	3285	85	−3,800

In this table T_B is the temperature of a black body which will give light of the same color as that emitted by a tungsten filament at the temperature T_W.

No great accuracy is claimed for these results; the differences in temperature, $T_B - T_W$, may be in error by 20 or 30 per cent. But, in contrast to the conclusions of Paterson and Dudding[1] these results show definitely that there is a distinct difference in color between the light from a tungsten filament and from a black body of the same temperature.

This difference in color is evidence that tungsten is not a gray body. Tungsten even in the visible region of the spectrum is a selective radiator giving a larger proportion of blue light than a black body. The energy distribution in the visible spectrum of a black body is given by Wien's equation. If E_λ be the emissivity of tungsten for the wave length λ, the energy distribution of the light emitted by a tungsten filament will

[1] Loc. cit.

therefore be

$$J_W = C_1 E_\lambda \lambda^{-5} e^{-\frac{C_2}{\lambda T_W}}, \tag{26}$$

where T_W is the true temperature of the tungsten.

Experimentally it has been found that the color of the light emitted by tungsten is the same as that emitted by a black body at a slightly different temperature T_B. The energy distribution from the black body is

$$J_B = C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T_B}}. \tag{27}$$

Since the tungsten filament and the black body are of the same color, although not of the same brilliancy, the ratio of J_W to J_B must be constant for all wave lengths in the visible spectrum. It is thus possible to obtain from (26) and (27) the relation

$$\frac{d(lnE_\lambda)}{d\lambda} = \frac{C_2}{\lambda^2}\left(\frac{1}{T_B} - \frac{1}{T_W}\right). \tag{28}$$

We have seen that $T_B - T_W$ is small compared to T_W so that we may place approximately

$$\frac{dlnE_\lambda}{d\lambda} = \frac{C_2(T_W - T_B)}{\lambda^2 T^2}. \tag{29}$$

From this equation, with the data of Table VII., we may calculate the emissivity of tungsten for one wave-length if we know it for another wave-length. Let us place $C_2 = 1.439$, $\lambda = 550 \times 10^{-7}$ cm. and take $T_W - T_B$ from Table VII.

The values of $(d/d\lambda)lnE_\lambda$ thus found by (29) are given in the fourth column of Table VII. Since this quantity varies with the temperature it must follow that the emissivity of tungsten varies with the temperature. How large must this variation be in order to account for the figures obtained? If we assume that $E_\lambda = 0.46$ for $\lambda = 0.664\,\mu$ at 2400° K. then we may calculate the value of E_λ at 2400° K. for other wave-lengths by integrating (29). The resulting values are given in the third column of Table VIII.

It is probable from Rubens's work that the variation of the emissivity with temperature occurs mostly in the region of longer wave-lengths; so that we may safely consider that the temperature coefficient of E_λ for $\lambda = 0.4$ micron is negligible. We thus obtain $E_\lambda = 0.525$ at $\lambda = 0.4$ for all temperatures. Then by (29) we may calculate E_λ at these temperatures for other wave-lengths. The results at 1800° and 3000° are given in Table VIII.

The change in emissivity with wave-length is in fairly good accord

TABLE VIII.

Emissivity $E\lambda$ of tungsten as a function of wave-length and temperature as calculated from the color of the emitted light.

λ (Microns).	1800° K.	2400° K.	3000° K.	Coblentz 300° K.
0.7	0.440	.450	.463	.460
0.664	0.449	(.460)	.470	
0.60	0.465	.475	.483	.487
0.55	0.480	.486	.494	
0.5	0.495	.498	.503	.507
0.4	(0.525)	.525	(.525)	.530

with the measurements of Coblentz,[1] which are given in the fifth column of Table VIII.

The variation of E_λ with temperature indicated in the table is so slight that it does not materially affect our temperature scale which has been based on the assumption of constant emissivity. The accuracy of the results given in Table VIII. is probably not high, so that it is not desirable at present to correct the temperature scale for these rather small variations. The changes are in such a direction as to make the true temperature scale lie somewhere between the Nernst and the Langmuir scales given in Table VI. The corrections in the last column of this table should therefore probably be reduced to about one half the values given.

Thermal Expansion of Tungsten.

By cathetometer measurements of the length of a tungsten filament at temperatures from 1200° to 2500°, the following relation has been found, l being the length at 300° K.

$$\frac{\Delta l}{l} = .00245\left(\frac{T - 300}{1000}\right) + .000567\left(\frac{T - 300}{1000}\right)^2. \quad (30)$$

This equation is probably accurate within two or three per cent.

Summary.

The contents of this paper are briefly as follows:

1. A discussion of the various functions that may be used in estimating the temperature of tungsten filaments.

2. Experimental data (see Table I.) on the volt-ampere-candle-power characteristics of tungsten filaments as functions of temperature and the dimensions of the filament. Derived functions such as watts, ohms and watts per candle are also given. These data cover the range of temperature from room temperature up to the melting point of tungsten. All the data are corrected for the cooling effects of the leads.

[1] Bull. Bur. Stand., 7, 197 (1911).

3. Data on the specific resistance of tungsten from 300° K. to 3540° K. The specific resistance may be obtained by multiplying the values of R' in Table I. by $\pi/4$.

4. The total emissivity of tungsten (black body taken as unit) has been calculated from these data for temperatures up to 3540° K. and the results have been tabulated in the column marked E of Table I. At temperatures above 1200° K., Foote's formula for the total emissivity of metals gives results which are much too low. Below about 500° or 600° K. Foote's formula is probably quite accurate.

5. The intrinsic brilliancy of a black body as a function of the temperature has been calculated in terms of the mechanical equivalent of light from Nutting's visibility data, by means of the Planck equation. By comparing these results with the experimentally determined brilliancies of tungsten and correcting for the known emissivity of tungsten the mechanical equivalent of light is found to be .00121 watt per lumen. This is close to the value (.00125) calculated by Ives from Nernst's data on the brilliancy of a black body, but differs considerably from Ives and Kingsbury's direct determinations of the mechanical equivalent (.00159) The discrepancy is hard to account for.

6. The intrinsic brilliancy of a black body is calculated from the experimental data for tungsten and is compared with similar data obtained by Nernst and Pirani. These data are given in Table VI.

7. The color of the light from tungsten filaments is distinctly bluer than that from a black body at the same temperature and corresponds to that from a black body at a temperature 40 to 80° higher.

8. The linear thermal expansion of tungsten at temperatures from 1200 to 2500° K. is given approximately by the equation 30.

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