of over two. (2) The magnetic anomaly, at the zenith, and with edges at 20° east and west, does not occur in atom-annihilation bands, but has an S-factor close to one. It seems possible there is here evidence for the dual nature of mesotron primaries, with only one kind able to generate soft electron showers.

APPENDIX

Notation and Formulas

α = azimuth; ζ = zenith angle; Ω = most probable mean square residual from "true" curve; $\Omega=\Sigma V_i^2/(n-m)$, where V_i = residual at point i, n = number of points fitted, and m = number of parameters adjusted from data. $X^2=\Sigma(V_i/\sigma_i)^2$, where σ_i = standard error of measurement whose residual is V_i. I.e., $\sigma_i=(1/0.675)\times$ probable error. When the measurement is a true frequency, this reduces to the more common definition $X^2=\Sigma V_i^2/f_c$, where f_c = theoretical frequency at point i. $P(>X^2)$ = probability of obtaining a value of X^2 greater than that observed. For the least-squares calculations by which lines were fitted to the logarithmic plot, the following formulas and definitions hold: j = relative intensity from data, $x=\log\cos\zeta_0-\log\cos\zeta$ ($\zeta_0=25°$ in references 1 and 2). The equation fitted has the form $\log j=a+bx$, where a = intercept of calculated line at ζ_0; b = slope of logarithmic line = exponent of generalized hyperbola whose equation is $j=A(\cos\zeta)^{-b}$. When two lines are used to represent the fine structure, a_H is for the higher line; a_L is for the lower; σ_{aH} = (standard error in a_H) = (probable error in a_H)/0.6745; σ_{aL} = standard error in a_L; and σ_b = standard error in b. In Table III, the standard deviations of these quantities are given for the azimuthal variation $s_b=\left\{\frac{1}{n}\Sigma(b-\bar{b})^2\right\}^{\frac{1}{2}}$. When a single constant term is used, $a=\frac{\Sigma x^2\Sigma y-\Sigma x\Sigma xy}{N\Sigma x^2-(\Sigma x)^2}$; $b=\frac{N\Sigma xy-\Sigma x\Sigma y}{N\Sigma x^2-(\Sigma x)^2}$, where $y=\log j$.

When two constant terms are used,

$$b=\frac{\frac{1}{n_1}\Sigma_1x\Sigma_1y+\frac{1}{n_2}\Sigma_2x\Sigma_2y-\Sigma_{1,2}xy}{\frac{1}{n_1}(\Sigma_1x)^2+\frac{1}{n_2}(\Sigma_2x)^2-\Sigma_{1,2}x^2},$$

$$a_1=\frac{1}{n_1}\Sigma_1y-(b/n_1)\Sigma_1x,$$

$$a_2=\frac{1}{n_2}\Sigma_2y-(b/n_2)\Sigma_2x.$$

In these equations, the subscripts 1 and 2 distinguish the points assigned to the high and low lines, respectively. $S=E_{\text{pri}}/E_{\text{sec}}$ = ratio of primary energy to most probable energy of the secondaries it generates.

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Radiation Losses in the Induction Electron Accelerator

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This paper discusses the possibility that radiation losses because of the high radial accelerations experienced by the electrons in an induction electron accelerator may introduce limitations in the design of accelerators for energies above 100 million electron volts. The effects of radiation losses on the electron orbits are calculated, and it is shown that not only should the orbit shift pulse necessary to bring electrons to a target inside the equilibrium orbit fall below the value expected in the absence of radiation, but also electrons should eventually arrive at the target with no orbit shift pulse whatever, at a phase of the field wave predictable from the theory. Both effects have been observed in the General Electric 100-Mev unit in a manner consistent with the predictions of the theory. The radiation itself has not yet been detected.

1. INTRODUCTION*

In the induction electron accelerator, the electrons are subjected continually to radial accelerations of the order of 10^{17} meters per second per second. It has been pointed out by

* *Symbols:*—Unrationalized m.k.s. units will be used throughout: The following symbols will be employed:
- A = peak value of applied magnetic flux density at the equilibrium orbit (webers per sq. m)
- A' = peak value of magnetic flux in orbit shrinking pulse at the equilibrium orbit (webers per sq. m)
- B_0 = applied magnetic flux density at the equilibrium orbit (webers per sq. m)
- B_r and B_z are components of magnetic flux density (webers per sq. m)
- c = velocity of light = 3.00×10^8 m per sec.
- e = charge on the electron = 1.602×10^{-19} Coulomb
- E_r and E_z are components of electric field (volts per m)
- f_n and f_t are normal and tangential components of the acceleration vector **f** (m per sec. per sec.)
- $F(\omega t)=(\omega t/\sin\omega t)-\cos\omega t-(2/3)\sin^2\omega t\cos\omega t$
- h = Planck's constant = 6.624×10^{-34} joule sec.
- H_r and H_z are components of magnetic field
- I = beam current (amperes)
- m_0 = rest mass of the electron = 9.107×10^{-31} kg

Iwanenko and Pomeranchuk[1] that radiation from these electrons will be appreciable and may set an upper limit to the energy attainable unless due attention is paid to new choices of operating parameters. The present paper discusses this conclusion and outlines the effects of radiation on the electron orbits.

The discussion will be based on the General Electric 100-Mev unit which will be described in detail in a forthcoming paper in the *Journal of Applied Physics*.[2] The radius of the "equilibrium orbit" in the G.E. accelerator is 0.833 meter. Magnetic fields of the order of 0.4 weber per square meter are used to attain the highest electron energies. Focusing is achieved by shaping the poles of the magnet so that the field varies inversely with the $\frac{3}{4}$ power of the radius in the neighborhood of the equilibrium orbit. The target to be bombarded by the high energy electrons is located 0.07 m in from the equilibrium orbit. At the appropriate moment, the orbit diameter is shrunk by a short magnetic field pulse applied by an auxiliary system of coils so that the beam hits the target.

If the electrons lose energy continually by radiation, they will tend to spiral inward. The strength of the orbit shrinking pulse necessary to bring the electrons to the target will then be less than that expected from a calculation neglecting radiation effects. If sufficient energy has been lost by radiation, the beam may reach the target without any orbit shrinking pulse. Both effects have been observed in our accelerator. Distortions and time delays in the field and flux waves which might cause such effects have been investigated and found to be too low by an order of magnitude. Since no other explanation has been offered, the possible results of radiation have been calculated and seem to offer an adequate description of the observed phenomena. The nature of the radiation is discussed in Section 3 below. The radiation itself has not yet been detected experimentally.

N = number of electrons per cu. m in the beam
n = index of variation of applied magnetic field with radius $= -(3/4)$ in our case
R_0 = radius of equilibrium orbit $= 0.833$ m in our case
U_r = rate of radiation from single electron (watts)
U_0 = radiation energy density in the beam (joules per cu. m)
v = electron velocity (m per sec.)
V = injector volts
w = maximum width of beam in plane of orbit (m)
W = electron energy (joules)
ϵ_0 = dielectric constant of free space $= 1.11\times 10^{-10}$ farad per m
μ_0 = permeability of free space $= 10^{-7}$ henry per m
ρ = charge density in beam (Coulombs per cu. m)
φ = magnetic flux density (webers)
$\omega = 2\pi\times 60 = 377$
$\omega' = \pi/$(duration of orbit shrinking pulse)
(Note: 1 weber per square meter = 10,000 gauss.)

[1] Iwanenko and Pomeranchuk, Phys. Rev. **65**, 343 (1944).
[2] W. F. Westendorp and E. E. Charlton, "A 100-million volt induction electron accelerator," J. App. Phys. **16**, 581 (1945).

2. RADIATION FROM A SINGLE ELECTRON

It is shown by Page and Adams[3] that the rate of radiation per solid angle $d\omega$ along the vector **c** from an electron having velocity **v** and acceleration **f** is given by:

$$\frac{dU_r}{d\omega}=\frac{10^{-7}e^2}{4\pi c^5\left(1-\dfrac{\mathbf{c}\cdot\mathbf{v}}{c^2}\right)^5}\times\{(\mathbf{f}\times(\mathbf{c}-\mathbf{v}))\times\mathbf{c}\}^2 \text{ watts.} \quad (1)$$

If v is approximately equal to c and if **f** is normal to **v**, the radiation has a strong maximum in the direction of **v**. The half intensity contour will be a rough cone whose vertical angle is less than 0.3° for energies of the order of 100 Mev.

The total rate of radiation from the electron is given by integrating (1) to obtain

$$U_r=\frac{2\times 10^{-7}e^2}{3c}\left\{\frac{f_t^2}{(1-v^2/c^2)^3}+\frac{f_n^2}{(1-v^2/c^2)^2}\right\} \text{ watts.} \quad (2)$$

This is Page and Adams' formula 75–6[4] expressed in m.k.s. units. As v approaches c, the ratio of the first term in Eq. (2) to the second term becomes so small that the first term is entirely negligible. The second term can be rewritten by use of the relations:

$$\frac{m_0 f_n}{(1-v^2/c^2)^{\frac{1}{2}}}=-evB_z, \quad (3)$$

$$f_n=v^2/r, \quad (4)$$

[3] L. Page and N. Adams, *Electrodynamics* (D. Van Nostrand and Company, New York, 1940), Chap. 7.
[4] See reference 3, p. 328.

whence

$$\frac{m_0 v}{(1-v^2/c^2)^{\frac{1}{2}}}=erB_z. \quad (5)$$

We substitute from (4) and (5) for f_n and $(1-v^2/c^2)$ in (2) and obtain:

$$U_r=\frac{2\times 10^{-7}e^6B_z^4r^2}{3cm_0^4}\text{ watts.} \quad (6)$$

Equation (6) is essentially the relation used by Iwanenko and Pomeranchuk. If we insert the values of the fundamental constants in Eq. (6), we obtain:

$$U_r=5.44\times 10^{-9}B_z^4r^2\text{ watts.} \quad (7)$$

The question may be raised as to the adequacy of special relativity in dealing with this problem. The equations of motion of a charged particle will, however, be found to be unchanged by any considerations introduced in the general theory of relativity.[5]

3. NATURE OF THE RADIATION

When the total radiation from a group of N closely spaced electrons is evaluated by the techniques which led to Eq. (1), the net energy radiated proves to be proportional to N^2 since the fields are summed up and the energy is proportional to the square of the net field. On the other hand, the radiation fields due to the elements of a continuous distribution of charge travelling in a beam are self-canceling, so that the energy radiated by a continuous beam adds up to zero. The phenomena observed in our accelerator and described below, however, are not affected by changing the current in the beam and are adequately described by the single electron radiation theory. It would appear that the fields radiated by the various electrons are not coherent.

The existence of radiation is probably attributable to statistical fluctuations in the beam density. The average density in a random fluctuation should be proportional to the square root of N, the electron density, so that the energy radiated would be proportional to N. If this is the case, the single electron calculations will be adequate to describe the behavior of the whole beam.

The energy radiated should be distributed among the harmonics of the rotation frequency of the electron in its orbit, which, in our case, is about 57 megacycles. It would appear at first glance that the distribution of energy between these harmonics should be deducible from a Fourier analysis of the radiation pattern described by Eq. (1). Since the radiation cone has a width of about $2\pi/1200$, we might expect the greater part of the energy to be found distributed more or less uniformly between the first thousand or so harmonics. This argument, however, has been shown to be fallacious by Schwinger for reasons which he discusses at length in a forthcoming paper in *The Physical Review.*[6] Schwinger has demonstrated that the energy is distributed among more than 10^7 harmonics, and that the energy distribution has its maximum in the near infra-red or in the visible spectrum.

While we were under the impression that the energy distribution included only a thousand or so harmonics, we made a search over the microwave range for the expected radiation. From the geometry and operating parameters of the accelerator and from the calculations outlined in Section 4 below, it seems that a total radiation power of the order of one watt should be available for detection. The range from 50 to 1000 megacycles was searched with receivers capable of detecting less than 10 microwatts, but no radiation associated with the beam was detected. This is easily understandable in the light of Schwinger's calculations which indicate that the power in a microwave harmonic is only about one part in 10^9 of the total energy radiated.

4. CHARGE DENSITY IN THE BEAM AND EFFECT OF OTHER ELECTRONS ON SINGLE ELECTRON RADIATION

We must consider three fields in evaluating the shape and charge density of the beam; the applied magnetic field whose components are B_r and B_z, the electric field due to Coulomb forces whose components we shall call E_r' and E_z', and the magnetic field due to the current in

[5] Cf. Tolman, *Relativity, Thermodynamics and Cosmology* (Clarendon Press, Oxford, England, 1934), Section 103, p. 259.

[6] J. S. Schwinger, "On Radiation by Electrons in a Betatron," to be submitted to The Physical Review.

the beam whose components we shall call H_r' and H_z'.

In the neighborhood of the beam the applied field

$$B_z = B_0(r/R_0)^{-\frac{3}{4}}, \tag{8}$$

to a high degree of approximation. From Maxwell's equations

$$B_r = -\tfrac{3}{4}B_0 z/R_0, \tag{9}$$

also to a high degree of approximation.

From Maxwell's equations it is easy to show that

$$H_z' = -v\epsilon_0 E_r', \tag{10}$$

$$H_r' = v\epsilon_0 E_z', \tag{11}$$

for small values of $(r-R_0)$ and z.

When the beam is in equilibrium, we shall have $\ddot{r}=\ddot{z}=0$. Taking this fact into account we write Newton's force equations and make the appropriate substitutions from Eqs. (8) to (11). The centrifugal force term

$$\frac{m_0 v^2}{r(1-v^2/c^2)^{\frac{1}{2}}}$$

in the radial force equation we write in terms of the centrifugal force at the center of the beam (i.e., on the equilibrium orbit) thus:

$$\frac{m_0 v^2}{R_0(1-v^2/c^2)^{\frac{1}{2}}}(r/R_0)^{-1}.$$

Since the fields due to the beam will disappear at its center, we can substitute from Eq. (3) so that the centrifugal force term reduces to $-evB_0(r/R_0)^{-1}$. With these substitutions, in the neighborhood of $r=R_0$, $z=0$, the radial and axial force equations reduce to:

$$E_r'(1-\epsilon_0\mu_0 v^2) = E_r'(1-v^2/c^2) = \frac{vB_0}{4R_0}(r-R_0), \tag{12}$$

$$E_z'(1-\epsilon_0\mu_0 v^2) = E_z'(1-v^2/c^2) = \frac{3vB_0 z}{4R_0}. \tag{13}$$

From Poisson's equation and Eq. (5), the charge density:

$$\rho = \frac{\epsilon_0 B_0 v}{4\pi R_0(1-v^2/c^2)} = \frac{\epsilon_0 e^2 R_0 B_0^3}{4\pi m_0^2 v}$$
$$= 775 B_0^3 c/v \text{ Coulombs per cubic meter.} \tag{14}$$

The number of electrons per cubic meter

$$N = \rho/e = 4.84\times 10^{21} B_0^3 c/v. \tag{15}$$

These densities are considerably higher than densities achieved in conventional electronic devices but are still materially lower than the densities of 10^9 Coulombs per cu. m found in metals.

If we know the current in the beam, our knowledge of the charge density enables us to calculate the dimensions of the beam. The boundary of the beam will be an equipotential. But from Eqs. (12) and (13), the equipotentials have the form

$$(r-R_0)^2 + 3z^2 = a \text{ constant}. \tag{16}$$

Evidently the beam is elliptical in cross section. If w represents the width of the beam in the plane of the orbit; then its width in the z direction will be $w/\sqrt{3}$. The cross-sectional area of the beam will be $\pi w^2/4\sqrt{3} = 0.45w^2$. The beam current will be given by

$$I = 0.45 w^2 \rho v \text{ amperes}, \tag{17}$$

whence, if we substitute from (14) for ρ,

$$w = 3.06\times 10^{-6} I^{\frac{1}{2}} B_0^{-\frac{3}{2}} \text{ meters}. \tag{18}$$

For a current of one ampere, at the peak field of 0.4 weber per square meter, the beam has the surprisingly small radial extension of 10^{-5} meter.

Since Eq. (14) can be useful in estimating the amount of charge which can be started in the beam by an injector voltage V, it has been rewritten in terms of V instead of B_0 and v. The substitutions follow from the ordinary relativistic form of the force equations. Equation (14) becomes:

$$\rho = 4.96\times 10^{-23} V(V+5.11\times 10^5)$$
$$\times (V+10.22\times 10^5) \text{ Coul./cu. m.} \tag{19}$$

For example, for 50 kv injection voltage,

$$\rho = 1.5\times 10^{-6} \text{ Coul./cu. m.}$$

The volume of our vacuum envelope is of the order of 0.1 cu. m so if the injector fills the whole tube with charge, the total charge present will be about 1.5×10^{-7} Coulomb.

If this quantity of charge is injected 60 times per second, the average input current will be about 10 microamperes. If this charge were all accelerated to 100 Mev, the output of the machine would be about 1000 watts. When the charge has reached approximately the velocity of light, the circulating current in the accelerator will be of the order of 10 amperes. Unfortunately, no technique is available for measuring this current, and its actual value is probably materially lower than 10 amperes because of the initial transients which result in collection on the tube walls and because of interception of electrons by the electron source.

We now attack the question as to the effect of the radiation field caused by all of the electrons on the radiation from a single electron. We shall show that this effect is so small as to be negligible. First we evaluate the energy density U_0 in the beam.

In Section 2 above, it was shown that the radiation is mostly confined to a narrow beam in the direction of the electron velocity vector. On the average the beam of radiation will travel a distance of the order of $(R_0w)^{\frac{1}{2}}$ before it emerges from the electron beam. Hence, at any one time, the energy in the beam caused by the radiation from one electron will be about $U_r(R_0w)^{\frac{1}{2}}/c$. The energy density in the beam because of the radiation from all the electrons will be obtained by multiplying this quantity by the total number of electrons in the beam and dividing by the volume of the beam. This procedure yields the relation

$$U_0 = NU_r(R_0w)^{\frac{1}{2}}/c. \qquad (20)$$

We make the appropriate substitutions from Eqs. (15) and (18) and obtain

$$U_0 = 1.29\times10^{10}I^{1/4}B_0^{9/4}U_r \text{ joules/cu.m.} \qquad (21)$$

This energy distribution may be considered to be associated with an electric field. We shall assume the worst possible case in which this field acts along the radius. In this case, the field

$$E_r = (8\pi U_0/\epsilon_0)^{\frac{1}{2}} \qquad (22)$$

$$= (2.91\times10^{21}I^{1/4}B_0^{9/4}U_r)^{\frac{1}{2}} \text{ volts per meter.} \qquad (23)$$

To obtain the radial acceleration for the radiation formula, we must now change Eq. (3) to

$$\frac{m_0 f_n}{(1-v^2/c^2)^{\frac{1}{2}}} = eE_r - evB_z. \qquad (24)$$

Squaring:

$$f_n^2 = (e/m_0)^2(1-v^2/c^2)(E_r - vB_z)^2$$
$$= (e/m_0)^2(1-v^2/c^2)(E_r^2+v^2B_z^2), \qquad (25)$$

since E_r is an incoherent a.c. field, and so its product with B_z will average out to zero.

We now substitute in the radiation formula which, from Section 2, has the form

$$U_r = \frac{2\times10^{-7}e^2f_n^2}{3c(1-v^2/c^2)^2} \text{ watts.} \qquad (26)$$

When we substitute from Eq. (25), this becomes

$$U_r = \frac{2\times10^{-7}e^2(e/m_0)^2(E_r^2+v^2B_z^2)}{3c(1-v^2/c^2)} \text{ watts.} \qquad (27)$$

From this relation and Eqs. (5) and (23), if $r=R_0$ and $v=c$,

$$U_r = 3.80\times10^{-9}B_0^2(3.24\times10^4I^{1/4}B_0^{9/4}U_r+B_0^2).$$

Therefore

$$U_r(1-1.23\times10^{-4}I^{1/4}B_0^{9/4}) = 3.80\times10^{-9}B_0^4. \qquad (28)$$

For currents as high as one ampere and magnetic fields up to 0.4 weber per sq. m, the correction term in Eq. (28) makes a difference in the radiation of less than one part in 30,000. It seems legitimate to neglect the effects of other electrons on the single electron radiation.

5. DECREASE IN ORBIT SIZE CAUSED BY RADIATION

The rate of change of electron energy W plus the rate of radiation of energy U_r must equal the power fed into the electron by the changing magnetic field, so that:

$$\dot{W} + U_r = \frac{ve}{2\pi r}\frac{\partial\varphi}{\partial t}. \qquad (29)$$

Also, from Eq. (5)

$$W = \frac{m_0c^2}{(1-v^2/c^2)^{\frac{1}{2}}} = erB_zc^2/v$$
$$= erB_zc \text{ approximately.} \qquad (30)$$

In the neighborhood of the equilibrium orbit, B_z is given by

$$B_z = B_0(r/R_0)^n, \qquad (31)$$

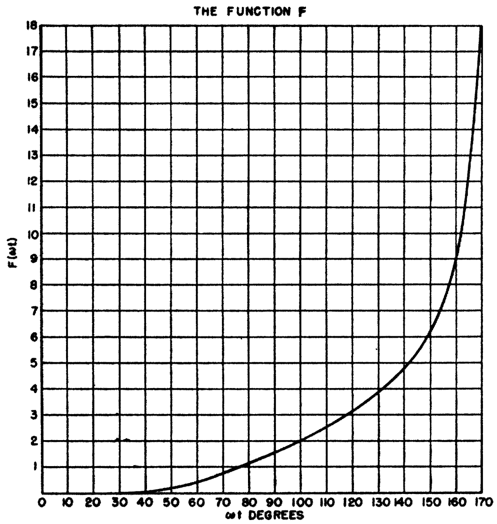

FIG. 1. The function $F(\omega t)$.

where

$$B_0 = A\sin\omega t, \quad n = -\tfrac{3}{4} \text{ in our case.}$$

The accelerator magnet is so designed that the flux is given by

$$\varphi = 2\pi R_0^2 B_0 + \int_{R_0}^{r} 2\pi r B_z dr, \tag{32}$$

$$= 2\pi R_0^2 B_0 \left\{ \frac{n+1}{n+2} + \frac{1}{n+2}(r/R_0)^{n+2} \right\} \text{ webers.} \tag{33}$$

From (29)

$$\dot{W} + U_r = veR_0\dot{B}_0(r/R_0)^{-1} \times \left\{ \frac{n+1}{n+2} + \frac{1}{n+2}(r/R_0)^{n+2} \right\}. \tag{34}$$

From (30) and (31)

$$W = ce\dot{B}_0R_0(r/R_0)^{n+1} + (n+1)ceB_0\dot{r}(r/R_0)^n. \tag{35}$$

In the range over which radiation will be of importance, we can set $v=c$ in Eq. (34). We subtract (35) from (34) and rewrite to obtain:

$$U_r = \frac{n+1}{n+2}\frac{ecR_0^2}{r}\frac{d}{dt}\{B_0(1-(r/R_0)^{n+2})\} \text{ watts.} \tag{36}$$

But from Eq. (7)

$$U_r = 5.44\times 10^{-9} B_z^4 r^2$$
$$= 5.44\times 10^{-9} B_0^4 R_0^{-4n} r^{4n+2}. \tag{37}$$

From (36) and (37)

$$\frac{d}{dt}\{B_0(1-(r/R_0)^{n+2})\} = 113\frac{n+2}{n+1}B_0^4 r^{4n+3} R_0^{-(4n+2)}. \tag{38}$$

Since $B_0 = A\sin\omega t$ webers per sq. m and $n = -\frac{3}{4}$, Eq. (38) becomes:

$$\frac{d}{dt}\{A\sin\omega t(1-(r/R_0)^{5/4})\} = 565A^4\sin^4\omega t R_0. \tag{39}$$

We now integrate Eq. (39) from $t=0$ to $t=t_0$ and rearrange terms to obtain:

$$(r/R_0)^{5/4} = 1 - \frac{212A^3R_0}{\omega\sin\omega t_0}(\omega t_0 - \sin\omega t_0\cos\omega t_0 - \tfrac{2}{3}\sin^3\omega t_0\cos\omega t_0)$$
$$= 1 - \frac{212A^3R_0F(\omega t_0)}{\omega}, \tag{40}$$

where

$$F(\omega t) = \omega t/\sin\omega t - \cos\omega t - \tfrac{2}{3}\sin^2\omega t\cos\omega t.$$

Substitution in (40) of $R_0 = 0.833$ m, $\omega = 377$, gives:

$$(r/R_0)^{5/4} = 1 - 0.473A^3F(\omega t_0). \tag{41}$$

If δr $(=r-R_0)$ is small compared with R_0, Eq. (38) can be integrated before the substitution is made for n, to give

$$\delta r = -\frac{42R_0^2A^3F(\omega t_0)}{(n+1)\omega} \text{ meters.} \tag{42}$$

With the appropriate substitutions for R_0, n, and ω, Eq. (42) becomes

$$\delta r = -0.315A^3F(\omega t_0). \tag{43}$$

The function F is plotted in Fig. 1. This graph, used in conjunction with Eq. (41) or (43) gives a complete picture of the decrease in r caused by radiation losses as a function of time for our accelerator. For example, at 90° phase, for $A=0.4$ weber per sq. m, $\delta r = 0.032$ meter.

In Fig. 2 we have plotted δr as a function of phase for $A=0.4$ weber per sq. m.

6. CALCULATION OF NECESSARY ORBIT SHIFT PULSE

We have already mentioned in Section 1 the fact that radiation losses will make the strength of the pulse necessary to bring the electrons to the target less than that expected from a calculation neglecting radiation effects.

We shall make the simplifying assumption that the pulse is so short in duration that we can neglect radiation effects during the pulse and can assume that the value of B_0 does not change during the pulse. This should be a valid assumption, since the first half of the pulse, which is the active part, lasts less than 100 microseconds.

The pulse will have two effects on the beam, first, the electron energy will be increased slightly because of the slight change in the flux enclosed by the beam, and second, the orbit radius will be changed. The second effect, of course, represents the primary function of the pulse; the first effect is incidental, but will be included in the calculations.

The orbit shift system consists of two coils, one of radius 0.71 m and one of radius 0.96 m. The same current flows through the two coils, but its direction in the inner coil is opposite to its direction in the outer coil. Because of the shape of the iron core, this arrangement results in no field inside the inner coil, but does result in a field which varies with the inverse three-quarters power of the radius between the coils. We represent this field by $A' \sin \omega' t' (r/R_0)^{-3/4}$, where t' is measured from the beginning of the pulse. At the beginning of the pulse the orbit will have a radius r which is less than R_0 because of radiation losses and whose value is given by Eq. (41). The pulse now shrinks the orbit to the circle on which the target lies and whose radius is 0.763 m. The flux due to the orbit shift is

$$\int_{0.71}^{r} 2\pi r A' \sin \omega' t' (r/R_0)^{-3/4} dr$$
$$= (8/5)\pi A' \sin \omega' t' R_0^2((r/R_0)^{5/4} - 0.820). \quad (44)$$

Therefore, the rate of change of energy in the beam due to the pulse (cf. Section 5)

$$\dot{W} = (4/5) evA'\omega' \cos \omega' t' (R_0^2/r) \times ((r/R_0)^{5/4} - 0.820). \quad (45)$$

But

$$W = ecR_0(B_0 + A' \sin \omega' t')(r/R_0)^{1/4}. \quad (46)$$

Since we assume B_0 to remain constant during the pulse, differentiation of (46) yields

$$\dot{W} = ecR_0 A'\omega' \cos \omega' t' (r/R_0)^{1/4} + (1/4) ec(B_0 + A' \sin \omega' t')(r/R_0)^{-3/4} \dot{r}. \quad (47)$$

We eliminate $\dot{W}$ between Eqs. (45) and (47) letting $v = c$ as we did in Section 5. The resulting equation is easily integrated between the limits $r = r$ to $r = 0.763$ m and $t' = 0$ to $t' = \pi/2$. The result of this integration is

$$B_0\{(r/R_0)^{5/4} + 3.280\} = 4.176(B_0 + A'). \quad (48)$$

Substitution from (41) for (r/R_0) and rearrangement of terms gives

$$A' = B_0(0.0249 - 0.1132A^3F(\omega t))$$
$$= A \sin \omega t (0.0249 - 0.1132A^3F(\omega t))^{\text{webers/sq.m}}. \quad (49)$$

Figure 3 shows the orbit shift pulse peak value in webers per sq. m for peak magnetic fields A of 0.2, 0.3, and 0.4 webers per sq. m.

The observations on the orbit shift pulse have been made on the current flowing through the orbit shift coils. Direct comparison of experiment and theory will involve a conversion factor from amperes to webers per sq. m which has not yet been measured. The general form of the experimental curves agrees with that of the theoretical curves plotted from Eq. (49); the maxima of the theoretical curves occur at about the right phase

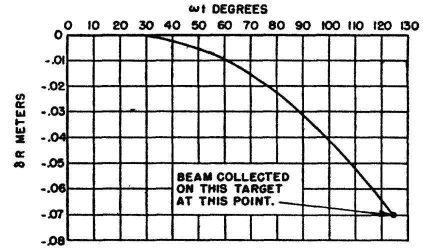

FIG. 2. Change in orbit radius as a function of 60-cycle phase for a peak magnetic field of 0.4 weber per sq. m.

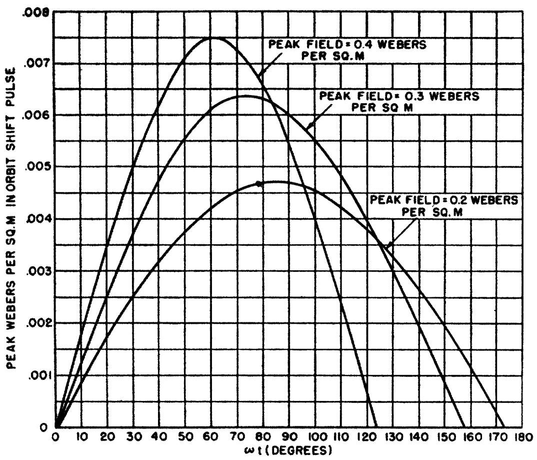

FIG. 3. Magnitude of orbit shift pulse required to move beam to target, as a function of 60-cycle phase.

and the shapes of the theoretical curves agree at least qualitatively with the shapes of the experimental curves.

7. CALCULATION OF PHASE OF COLLECTION IN ABSENCE OF ORBIT SHIFT PULSE

It is evident from either Eq. (41) or Eq. (49) that in time the beam will arrive at the target even if no orbit shift pulse is applied. The phase of collection may be deduced by setting $r=0.763$ in Eq. (41) or by setting $A=0$ in Eq. (49). In either case we obtain

$$A^3=0.220/F(\omega t),$$

or

$$A=0.604F^{-\frac{1}{3}}. \qquad (50)$$

There are no undetermined parameters in this relation, and we can compare it directly with experiment. In Fig. 4 we have plotted ωt against A from Eq. (50). A set of experimental points are included on the same graph. For low values of A the agreement is good. At higher values the experimental values diverge slightly from the theoretical curve until a maximum disagreement is reached of about 12 percent in A or about 15° in phase. If this divergence is real, it is in such a direction as to indicate a slightly greater loss in energy than that given by radiation alone. It is quite possible, however, that experimental errors in phase measurements, magnetic field measurement, determination of position of the equilibrium orbit, and assignment of the value $-\frac{3}{4}$ to the index of magnetic field variation with radius could add up to an error of this order.

Recent experiments on the range of orbit shift pulse magnitudes which result in electron collection at the target indicate that the beam does not have the very small extension indicated by Eq. (18) above. The diameter of the beam would appear from these experiments to be of the order of two centimeters. The reason for this discrepancy between experiment and theory is not yet understood. If the beam is as large as this, its first interception by the target will occur before the time indicated by Eq. (50) by just about the difference between the theoretical and experimental points of Fig. 4. If the divergence between the experimental results and the predictions of Eq. (50) is real, the large extension of the beam evidently provides an adequate explanation.

8. TRUE ENERGY OF THE COLLECTED ELECTRONS

Although we have shown that changes in orbit radius of the order of 10 percent occur in the range of operation of our accelerator, the associated energy losses will generally be less than

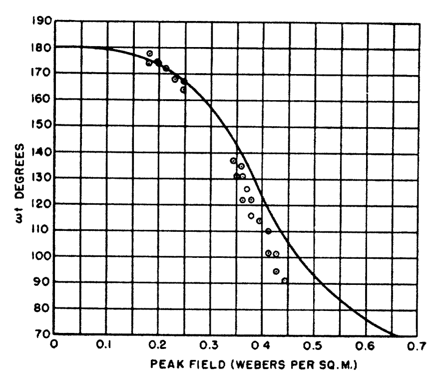

FIG. 4. Phase of collection with no orbit shift pulse, as a function of peak magnetic field.

2 percent. The true energy of the electrons striking the target will be given by (cf. Eq. (30)):

$$ecR_0(A\sin\omega t+A')(r/R_0)^{\frac{1}{4}}\text{ joules.} \qquad (51)$$

If we substitute $r=0.763$ m, $R_0=0.833$ m and express this energy in Mev, we obtain

$$\text{True energy}=250\times 0.978(A\sin\omega t+A')\text{ Mev.} \qquad (52)$$

The value of A' is given by Eq. (49). Substitution of this value in (52) gives

$$\text{True energy} = 250A \sin \omega t \times (1.003 - 0.111A^3 F(\omega t)) \text{ Mev.} \quad (53)$$

If we collect at 90° phase, $F(\omega t_0) = \pi/2$ and

$$\text{True energy} = 250A(1.003 - 0.175A^3) \text{ Mev.} \quad (54)$$

The increase of 0.3 percent in the first term in the bracket represents the amount by which the energy would be increased because of the orbit shift pulse flux linked by the beam if no energy were lost by radiation. The second term represents the radiation loss. As an example, we consider the "100-Mev" case ($A = 0.4$ weber per sq. m). In this case the true energy turns out to be 99.2 Mev.

9. PROCEDURE FOR ATTAINMENT OF HIGHER ENERGIES

If the radiation hypothesis is definitely established, it will be necessary to make some changes in operating parameters of the accelerator before energies are achieved which are appreciably higher than those already reached. The primary objective will be to speed the electrons up faster so that they do not have so much time in which to lose energy by radiation. We shall also gain by increasing the radius of the orbit so that the radial accelerations are not so high. Analytically these factors follow from the considerations of Section 5. From Eq. (40) we find that if the ratio of the target radius to R_0 is to be kept at its present value, the ratio R_0A^3/ω must be kept constant or must be less than its present value. Since the energy W is proportional to the product AR_0, it follows that if the energy is to be increased by increasing the magnetic flux density, ω must be increased as the cube of the energy. This procedure does not look promising, both because of the fast increase of ω and because the central part of the magnet is already being operated uncomfortably close to saturation. It would be far preferable to attain the higher energy by increasing R_0 in which case ω must increase linearly with the energy.

Further gains will be effected by moving the target farther in from the equilibrium orbit and by letting the magnetic field fall off more slowly than according to the inverse three-quarters power of the radius (cf. Eq. (42)). Neither of these changes will decrease the energy loss caused by radiation. They will simply delay the time at which the beam spirals into the target without the assistance of the orbit shift pulse.

One feature of the present operating technique which is undesirable from the point of view of radiation losses is the procedure of collecting at 90° phase. During the period from 65° to 90° the beam energy is increased by less than 10 percent, yet more than half of the radiation energy loss takes place during this time. It would seem to be desirable to raise the peak magnetic flux by 10 percent and collect at about 65° phase.

10. ACKNOWLEDGMENTS

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