average value for τ_{at} from the velocity measurements is 1.9×10^{-8} atmos.-sec. and from the absorption data is 1.7×10^{-8} atmos.-sec. Van Itterbeck gives 2.0×10^{-8} atmos.-sec. at 15°C and 1 atmos. from absorption measurements made at 600 kc.

The failure of these data to fit the f curve is now interpreted as caused by two assumptions in the Kneser theory of vibrational dispersion which are not applicable to the case of rotational dispersion. Since the relaxation times for adjustment between the unequally spaced rotational levels obviously are unequal, the curve should be a step curve, to which the f curve is a first approximation. The deviations from the f/p law are believed to be caused by the failure of the assumption that the number of favorable collisions is proportional to the pressure and are presumably caused by the effect of three-body collisions. A more rigorous interpretation of this dispersion, in the light of the above discussion, will be the topic of a subsequent paper.

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Theory of Long Period Magnetic Relaxation

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A discussion is given of the long period changes in magnetization in a mild steel specimen subjected to alternating stresses while in a weak magnetic field. The magnetization changes appear to occur with two time constants, of the order of three months and five years. A formal treatment of the phenomena is given along the lines of the time-dependent barrier potential used by Snoek in discussing other magnetic relaxation processes. The increase in the potential barrier is pictured as caused by the mechanical relaxation of local strains.

INTRODUCTION

DURING the war there was built up by physicists attached to the Naval Ordnance Laboratory, Washington, D. C., and to the Director of Scientific Research, British Admiralty, a body of experimental data relating to long period variations of magnetization of mild steel subject to alternating mechanical stresses while in a weak magnetic field. There has been little fundamental research, however, into the mechanisms involved. The existence of such time effects have been known for a very long time—see for example Ewing's book[1] or any manual on the compensation of ships' compasses. In view of the possible importance of the effects to our general understanding of ferromagnetism, it is desirable to summarize the main conclusions derived from the data and to indicate a phenomenological representation similar to that employed by Snoek[2] in discussing other magnetic relaxation processes.

The basic facts derived from observation at ordinary temperatures are given below. The applied fields involved are of the order of 0.5 gauss.

(*a*) After a period of the order of five years in a constant applied field the magnetization of a mild steel specimen subjected to alternating stresses *approximates* a state represented by infinite effective permeability, as was pointed out first by Dr. D. W. Ver Planck.

(*b*) When the applied field is changed suddenly, a considerable fraction of the magnetization changes practically instantaneously—this fraction is determined by the ordinary permeability and by the demagnetization coefficient.

[1] J. A. Ewing, *Magnetic Induction in Iron* (The Electrician Publishing Company, London, 1892).

[2] J. L. Snoek, Physica 5, 663 (1938).

(*c*) Following a sudden change in the applied field, there is a slow change in magnetization with a time constant of the order of three months. This change does not proceed all the way to the condition of infinite effective permeability discussed in (*a*) above.

Dr. H. Iskenderian has shown that a single time constant of about three months, as proposed originally by the author, does not fit all of the data; subsequently Dr. H. R. Hulme and Mr. E. C. Holmberg of the British Admiralty showed that the remainder of the magnetization changed with a time constant of the order of five years.

The existence of a period of the order of five years is also evident in the work of R. L. Downdell[3] on the aging of magnet steels.

REPRESENTATION OF EXPERIMENTAL RESULTS

A phenomenological picture generally consistent with the above facts can be made along the following lines; it should be remembered that this picture is bound to be oversimplified.

Suppose that an initially demagnetized steel specimen is placed at time $t=0$ in a uniform constant applied field H_0. The specimen is subjected to alternating stresses with a frequency of the order of 60 cycles per minute while in the field; the magnitude of the stresses is not known but is probably of the order of 1/20 of the yield stress.

After a period of time which measurements suggest should be long in comparison with five years, the magnetization of the specimen will have reached a steady or equilibrium state in the field H_0. There are some measurements which suggest that for mild steel in fields of the order of 0.5 gauss the equilibrium state is characterized by nearly infinite effective permeability, so that

$$M_0^e = H_0/N, \tag{1}$$

where M_0^e is written for the total equilibrium magnetization (magnetic moment per unit volume) and N in the demagnetization coefficient in the direction of the field. It is supposed that the shape of the specimen is such that a demagnetization coefficient can be defined, at least approximately. Equation (1) is obtained from the well-known relation

$$M = H_0/[N+4\pi/(\mu-1)] \tag{2}$$

by letting $\mu \to \infty$.

It appears that the equilibrium magnetization is composed of three somewhat distinct parts which we shall denote by M_1^e, M_2^e, M_3^e, so that

$$M_0^e = M_1^e + M_2^e + M_3^e. \tag{3}$$

Here M_1^e is the ordinary induced magnetization which changes practically instantaneously when the applied field is changed, according to Eq. (2). We shall write

$$M_1^e/H_0 = c_1. \tag{4}$$

The second term M_2 represents the magnetization which increases to its equilibrium value M_2^e in a time of the order of three months, according to existing measurements. It would be tempting to try to describe the time variation of M_2 by an equation of the form

$$dM_2/dt = H_i/\tau_2, \tag{5}$$

where H_i is the internal field in the specimen after allowing for the demagnetizing field, but with this equation M_2 increases until H_i is zero, meaning that total equilibrium is reached with a time constant of three months. This contradicts the known existence of a five-year time constant. Instead of Eq. (5) we shall assume

$$dM_2/dt = (M_2^e - M_2)/\tau_2; \tag{6}$$

that is, the rate of change of M_2 is taken to be proportional to the difference between the equilibrium value M_2^e and the instantaneous value M_2; we specify further that

$$M_2/H_0 = c_2, \tag{7}$$

a constant.

The third term M_3 represents the magnetization which increases to its equilibrium value M_3^e in a time of the order of five years. We shall assume that the time variation of M_3 is governed by equations similar to those written above for M_2, so that

$$dM_3/dt = (M_3^e - M_3)/\tau_3 \tag{8}$$

and

$$M_3^e/H_0 = c_3. \tag{9}$$

Combining Eqs. (3), (4), (7), and (9) we get

$$M_0^e = M_1^e + M_2^e + M_3^e = (c_1+c_2+c_3)H_0, \tag{10}$$

[3] R. L. Downdell, Am. Soc. Metals Trans. **22**, 19 (1934); Trans. Am. Soc. Steel Treat. **5**, 27 (1924).

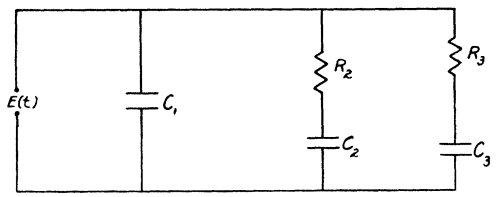

FIG. 1. Equivalent electrical circuit.

so that, by Eq. (1),

$$c_1+c_2+c_3=1/N. \tag{11}$$

To study the time variation of the total magnetization as the applied field H_0 is varied, we write

$$\begin{aligned} dM_0/dt&=(dM_1/dt)+(dM_2/dt)+(dM_3/dt)\\ &=(c_1dH_0/dt)+([c_2H_0-M_2]/\tau_2)\\ &\quad+([c_3H_0-M_3]/\tau_3). \end{aligned} \tag{12}$$

This equation is equivalent in behavior to the electric circuit shown in Fig. 1 if we make the following substitutions:

H_0 = voltage $E(t)$
M_i = charge Q_i
c_i = capacitance C_i
τ_i = time constant R_iC_i.

In terms of electrical quantities Eq. (12) becomes

$$\begin{aligned} dQ_0/dt&=C_1dE/dt\\ &\quad+(C_2E-Q_2)/\tau_2+(C_3E-Q_3)/\tau_3, \end{aligned} \tag{13}$$

an equation which has been studied in connection with dielectric relaxation. The solution in terms of magnetic quantities is

$$M_0(t)=\left[c_1H_0(t)+\int_0^\infty \varphi(\theta)H_0(t-\theta)d\theta\right], \tag{14}$$

where

$$\begin{aligned} \varphi(\theta)&=(c_2/\tau_2)\exp[-\theta/\tau_2]\\ &\quad+(c_3/\tau_3)\exp[-\theta/\tau_3] \end{aligned} \tag{15}$$

is the Boltzmann "memory function."

Before we discuss special solutions of Eq. (14) we must make some decision as to the values of the constants c_1, c_2, c_3. Now c_1 is determined from Eqs. (2) and (4), giving

$$c_1=[N+4\pi/(\mu-1)]^{-1}. \tag{16}$$

From Eq. (11),

$$c_2+c_3=(4\pi/N)/[N(\mu-1)+4\pi]. \tag{17}$$

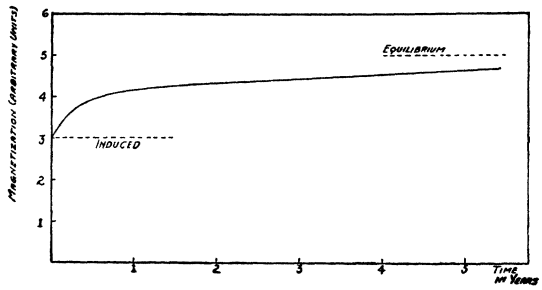

FIG. 2. Growth of magnetization following switching-on at $t=0$.

There is little information as to the relative importance of c_2 and c_3. In the case of a steel tube-like structure characterized by an equivalent volume permeability of about 4, and for which $N=2\pi$, giving $c_1=0.3/\pi$, there is evidence suggesting $c_2\sim c_3\sim 0.1/\pi$. The relative extent to which c_2 and c_3 share in satisfying Eq. (17) in other situations is not known.

Two solutions of Eq. (14) are of particular interest. The first is for the case of switching on the field at $t=0$; that is $H_0=0$ for $t<0$, while $H_0=h$ for $t>0$. Here the solution is $M_0(t)=0$ for $t<0$ and

$$\begin{aligned} M_0(t)&=h[c_1+c_2(1-\exp(-t/\tau_2))\\ &\quad+c_3(1-\exp(-t/\tau_3))], \end{aligned} \tag{18}$$

for $t>0$; for large t this approaches $M_0{}^e$. Figure 2 shows a plot of M_0 *vs.* t, for $\tau_2=3$ months, $\tau_3=60$ months, and is chosen so that $hc_1=3$, $hc_2=hc_3=1$, which corresponds to the situation discussed above for the ratio of the c's.

Another solution of interest is found by letting $H_0=he^{j\omega t}$, which gives, apart from transients,

$$\begin{aligned} M_0(t)/H_0(t)&=c_1+c_2/(1+j\omega\tau_2)\\ &\quad+c_3/(1+j\omega\tau_3). \end{aligned} \tag{19}$$

The magnetization lags the applied field. A plot of M_0 *vs.* H_0 gives a loop closely resembling a hysteresis loop.

THEORY

The existence of magnetic relaxation effects in iron with a time constant at room temperature of the order of minutes is well known. A review is given by Ewing[1] and by Becker and Doering.[4] A unified theory has been given by Snoek[2] which

[4] R. Becker and W. Doering, *Ferromagnetismus* (Edwards Brothers, Ann Arbor, Michigan, 1943).

accounts successfully for several different associated phenomena on the basis of domain theory. For a general discussion of domain theory Becker and Doering, and Bitter[5] may be consulted.

In order to eliminate non-essential features from the picture, Snoek supposes the magnetization at low field strength to be caused only by 180° reversals—that is, by the movement of boundaries separating domains magnetized in opposite directions. He assumes that the material is subject to "elastic after-effect" so that it can adapt itself locally to the condition of strain prevailing in the barrier layer itself. The loss of potential energy in the boundary layer in the course of adaptation of the material to local strain is thought of as a sort of supplementary potential hole coming gradually into existence after the barrier layer has moved from one spot to another. The characteristic time constant for the establishment of the potential hole is identified with the magnetic relaxation time constant.

The relaxation processes considered by Snoek are quite temperature dependent and are characterized by an activation energy of the order of one electron-volt per atom. His theory does not give any account of the effects of alternating stresses on the time constant. From the experimental standpoint nothing quantitative is known about the relative importance of thermal agitation and of alternating external stresses on the long period relaxation of barrier potentials. For convenience we shall proceed on the assumption that *elapsed time*, rather than *number of stress cycles*, is the significant independent variable. We shall apply Snoek's theory in a formal way to the long period relaxation processes, since it is attractive to try to associate them with long period mechanical relaxation of local strains. Furthermore, Snoek's theory can be generalized in a natural way to take account of the co-existence of several time constants. This is a very important point. In the discussion which follows the very short period effects (of the order of minutes) are omitted for the sake of compactness.

Changes in magnetization in a specimen may be discussed by considering the displacements of the boundaries separating domains magnetized in opposite directions. The displacement of such a boundary is determined by the potential function describing the strain energy of the boundary as a function of the position of the boundary, and also by the pressure exerted on the boundary by the applied magnetic field.

The strain potential will be written

$$V_0(x,t) = V_1(x) - \text{time-dependent terms.} \quad (20)$$

Here $V_1(x)$ is the potential energy due to the *permanent* part of the strain, while the time-dependent terms describe supplementary potentials with relaxation time constants of the order of three months and five years. The latter terms will be written $V_2(x,t)$ and $V_3(x,t)$, respectively; and their equilibrium values after a very long time at $x=x_1$ will be described by $V_2^e(x-x_1)$ and $V_3^e(x-x_1)$, so that

$$V_0^e(x) = V_1(x) - V_2^e(x-x_1) - V_3^e(x-x_1). \quad (21)$$

The V_2^e and V_3^e are bell-shaped curves (Fig. 3) of half-width comparable with the wall thickness.

It is supposed that the course of the supplementary potential V_2 is given by an equation somewhat similar to Eq. (6):

$$\partial V_2(x,t)/\partial t = [V_2^e(x-x_1) - V_2(x,t)]/\tau_2; \quad (22)$$

that is, the time rate of change of V_2 at a given point x (when the barrier wall is at x_1) is directly proportional to the difference between the equilibrium potential $V_2^e(x-x_1)$ and the instantaneous potential $V_2(x,t)$. The solution for the growth of V_2 at a new position x_1 for the barrier is given by

$$V_2(x,t) = V_2^e(x-x_1)[1-\exp(-t/\tau_2)]. \quad (23)$$

Similar equations hold for V_3.

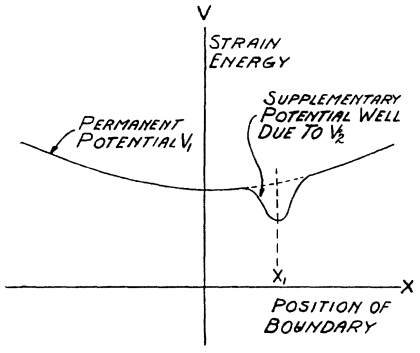

FIG. 3. Potential energy curve for domain barrier.

[5] F. Bitter, *Introduction to Ferromagnetism* (McGraw-Hill Book Company, Inc., New York, 1937).

TABLE I.

Approximate relaxation time	Probable cause
~5 years	Relaxation of local strains
~3 months	Relaxation of local strains
$\sim 10^{-2}-10^{2}$ second	Relaxation of local strains
$\sim 10^{-7}$ second	Eddy current damping of irreversible processes
$\sim 10^{-9}$ second	Eddy current skin depth less than domain dimensions.

Consider what happens to a boundary which has been at $x=0$ for a very long time, when a field H is applied suddenly. The pressure on the domain wall due to the field is $2M_sH$, where M_s is the saturation magnetization. The wall will move until the pressure is balanced by a force caused by the boundary strains. Since the force due to the boundary strains relaxes in the course of time, the wall will slowly creep farther out, thus increasing the macroscopic magnetization. All this time the wall is creating at its new positions new supplementary potentials, but these have no important influence on the qualitative aspects of the motion, and ultimately the wall will reach an equilibrium position determined by the fixed potential function $V_1(x)$.

Neglecting the effect of the new potentials created as the wall moves out, we treat the motion of the wall in the original set of potentials given by:

$$V_0(x,t)=V_1(x)-V_2^e(x)\exp(-t/\tau_2)-V_3^e(x)\exp(-t/\tau_3). \quad (24)$$

The instantaneous position of the wall is given by

$$\partial V_0(x_1)/\partial x \equiv V_1'(x_1)-V_2'(x_1,t)-V_3'(x_1,t)=2M_sHA, \quad (25)$$

where A is the area of the boundary wall. Since $x=0$ is the equilibrium position without a field we can write for small x, $V'(x)=xV''(0)$.

When the field is switched on the wall moves to x_1, where

$$x_1(t)V_0''(0,t)=2M_sHA. \quad (26)$$

Here x_1 is directly proportional to the macroscopic magnetization of the material, so that we can study the time variation of magnetization by studying the variation of x_1.

Immediately after the applied field is switched on the movement of x proceeds until limited by the combined effect of the three potential functions: the signs of V_1'', $-V_2^{e\prime\prime}$, and $-V_3^{e\prime\prime}$ are all positive. After a time comparable with τ_2 the effect of V_2^e practically vanishes and the magnetization has increased to approximately $M_1^e+M_2^e$. Finally after a time comparable with τ_3 the effect of V_3^e also vanishes and the equilibrium state M_0^e is reached, where $x_1(\infty)=2M_sHA/V_1''(0)$. Since the effective permeability is very high in the equilibrium state, the value of V_1'' must be small in comparison with $V_2^{e\prime\prime}$ or $V_3^{e\prime\prime}$.

If accurate values of the time constants of strain relaxation are required for any purpose, the calculation should be made including the effects due to the new potentials created by the barrier motion. Such calculations may be made by an extension of the method indicated by Snoek.

MAGNETIC RELAXATION SPECTRUM

The characteristic relaxation times for magnetization in iron may be summarized as in Table I. It should be emphasized that both the relaxation times and the assigned causes are only tentative.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor Francis Bitter and Dr. D. W. Ver Planck for stimulating discussions of this subject. I am obliged to Dr. E. E. Schumacher of the Bell Telephone Laboratories for references to the metallurgical literature. This work was begun while the author was at the Naval Ordnance Laboratory, Washington, D. C.