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Absorption of Slow Mesotrons in Lead, Iron, Aluminum, and Water

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The absorption of slow mesotrons in lead, iron, aluminum, and water was measured by means of counters arranged in anticoincidence. In all cases, the statistical error in the number of mesotrons stopped by the absorbing layers was less than 3 percent. This accuracy was made possible by the high efficiency (about 99 percent) of the anticoincidence group of counters. The values obtained for the relative absorptions in the different substances were compared with those calculated from the theory of energy losses by collision. As is well known, the theory predicts a smaller absorption in heavier elements for a given superficial mass. The uncorrected experimental values did not agree with the theory. However, by taking into account the effect of scattering, which is of particular importance in heavy elements, the experimental data were brought into satisfactory agreement with the theory. For water, no reliable theoretical value could be calculated.

INTRODUCTION

IT is generally admitted that the hard component of the cosmic radiation at sea level is absorbed in different substances according to a mass-proportional law. It is also assumed that the absorption is essentially due to energy losses by collision. The theory of these losses indicates that the absorption should not be exactly proportional to the superficial mass of the absorber, but that there should be a slight dependence on the atomic number of the substance. However, the existing absorption measurements are not accurate enough to establish definitely whether these small discrepancies from the mass-proportional law exist or not.

In spite of the fact that many measurements of the absorption of the hard component in different materials have been published, there seems to exist no experiment in which the absorption in elements of widely different atomic number was measured with sufficient accuracy to test the above-mentioned point.

The lack of accurate data is due to the following reason. The absorption of mesotrons in matter is usually measured in two different ways. One may measure the intensity of a mesotron beam before and after the absorber is placed above the counter set or one may measure the intensity of the beam before and after the absorber is placed between the coincidence counters. Owing to the weak absorption of the hard component, very thick layers of matter are required to absorb more than a few percent of the radiation. For this reason when the absorber is placed above the counters, the divergence of the beam makes it necessary to employ inconveniently large volumes of the absorbing material. When the absorber is placed between the counters, a given thickness requires a smaller volume of absorber; but on the other hand the counters then have to be placed further apart, strongly decreasing the intensity of the beam. Moreover, the above mentioned methods are sensitive to variations in the intensity of the cosmic radiation. A quantitative analysis of the problem shows that for a light element the measurement of the absorption to within an accuracy of only ten percent would require an exceedingly long series of measurements.

There is another method that may be employed which involves the use of anticoincidence counters. In this experiment, one records directly the number of absorbed mesotrons instead of the difference in intensity with and without absorber. Hence the statistical error can only affect the small fraction of particles which is absorbed instead of the total number of particles in the beam.

A review of the available literature shows but one experiment performed by the anticoincidence method.[1] The accuracy attained in this experiment, however, was not sufficient for a quantitative test of the theory.

It was therefore considered worth while to

[1] M. A. Pomerantz and T. H. Johnson, Phys. Rev. **59**, 143 (1941).

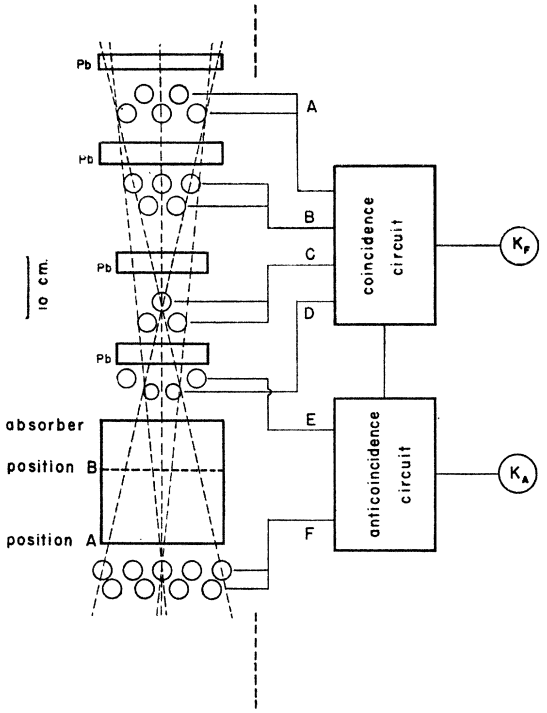

FIG. 1. Position of counters and absorbers in the first arrangements. Also showing connections to the recording circuits.

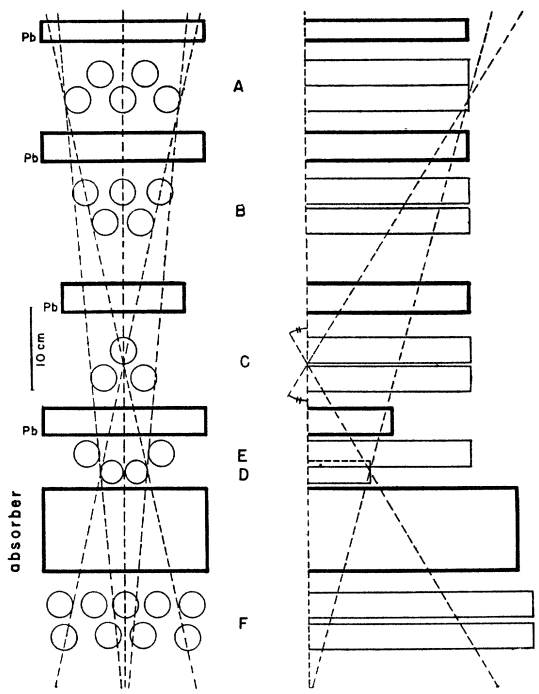

FIG. 2. Position of counters and absorbers in the second arrangement. Also showing the sections of the beam in two perpendicular planes.

undertake a new measurement of the absorption of mesotrons in different materials by the anticoincidence method. The precision attained was considerably higher than that achieved by any of the previous experimenters, owing chiefly to the high efficiency of the anticoincidence counter set and to the large number of particles recorded.

The geometrical arrangement of the counters and absorbers used in our experiment is shown in Fig. 1. The four groups of counters A, B, C, and D constitute a cosmic-ray "telescope" and define a beam of mesotrons. All counters within a given group are connected in parallel and each group is connected to one terminal of a fourfold coincidence set. This set actuates the mechanical recorder K_F when a coincidence $(ABCD)$ occurs and in most cases this event corresponds to the passage of a charged particle in the telescope. The soft radiation is absorbed by 14 cm of lead distributed into several layers placed between the groups forming the telescope. In this way a beam of mesotrons is obtained which we believe to consist of 94 percent of mesotrons and 6 percent of electrons. These remaining electrons cannot be eliminated since they are in equilibrium with the hard component.

Referring to the diagram one sees that the absorber is placed immediately under the telescope and above the anticoincidence group F. The maximum thickness of absorber that can be conveniently used is limited by the fact that the area of group F must be sufficient to fill the solid angle defined by the telescope.

The purpose of the group F of counters is to detect the mesotrons which have not been stopped by the absorber. This group is connected to the anticoincidence unit. The anticoincidence unit always receives the signal of the occurrence of a fourfold coincidence $(ABCD)$ and this signal will be transmitted to the mechanical recorder K_A except when accompanied by a simultaneous impulse from the group F. It is clear that K_A will record the number of particles stopped.

We will indicate by A the number of particles registered in K_A and by F the number of those registered by K_F. Thus, the above description makes it clear that the absorption increases

TABLE I. Measurements of absorption of slow mesotrons in lead, iron, aluminum, and water. The column headings have the following significance: t, thickness of material in cm; σ, mass per unit area in g/cm²; Abs. pos., position of absorber; N_b, number of particles in the beam; N_s, number of stopped particles; R_s, percent of particles stopped; R(abs.) corr., percent absorption corrected for background; R(abs.) for 10 g/cm², percent absorption in 10 g/cm² of substance.

	t cm	σ g/cm²	Abs. pos.	N_b	N_s	R_s	R(abs.) corr.	R(abs.) for 10 g/cm²
				First arrangement				
Measurements with lead								
	2.43	27.7	A	78,400	2220	2.83	0.94±0.08	0.34±0.03
	9.87	112.5	A	156,174	9651	6.18	4.29±0.08	0.381±0.008
Background			A	178,736	3372	1.89		
	9.87	112.5	B	106,736	7291	6.84	4.89±0.09	0.435±0.009
Background			B	106,168	2070	1.95		
				Graphically extrapolated value:				*0.35±0.015*
Measurements with iron								
	10.2	80.2	A	84,500	4467	5.29	3.40±0.08	*0.424±0.008*
Background			A	178,736	3372	1.89		
Measurements with water								
	20.0	20.0	AR*	174,027	4918	2.83	0.97±0.05	*0.48±0.025*
Background			AR	277,138	5151	1.86		
				Second arrangement				
Background			B	235,800	2075	0.878		
Lead	9.87	112.5	B	57,800	2965	5.13	4.25±0.1	0.38±0.01
Iron	10.20	80.2	B	58,200	2559	4.40	3.52±0.09	0.44±0.01
Al	10.16	27.4	B	122,600	2627	2.14	1.26±0.015	0.465±0.004

* AR means that the absorber in position A is in a reservoir.

linearly with the fraction A/F. To obtain a true value of the absorption we need only measure A/F with the absorber and subtract the corresponding value of A_0/F_0 measured without the absorber.

In view of the fact that the absorbers used stopped only a small percentage of the mesotrons, experiments were performed to reduce the background A_0/F_0 to a minimum.

Two slightly different arrangements were employed. In the first (Fig. 1) the maximum permitted thickness of absorber was 20 cm and in this case the number of background coincidences A_0/F_0 was about 1.9 percent. In the second arrangement (Fig. 2) the anticoincidence counters were placed only 10 cm below the telescope. This arrangement reduced the maximum thickness of absorber but allowed a more favorable disposition of the anticoincidence counters, with a consequent decrease of A_0/F_0 to 0.9 percent. Finally two anticoincidence counters E were placed one on each side of the counters of group D in order to minimize the number of spurious fourfold coincidences caused by showers.

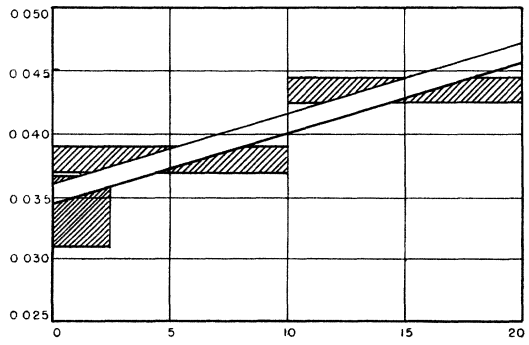

FIG. 3. Graphical extrapolation of the experimental results for the absorption in lead. Abscissae: distance in cm between the absorber and the counters F. Ordinates: percent absorption of mesotron beam in one g/cm².

The coincidence circuit used was of the conventional Rossi type and had a resolving time of about 16 microseconds. The anticoincidence circuit was of the same type as that employed in this laboratory in connection with a previous work.[2] The high voltage source included the usual pentode-stabilized circuit.[3] All the circuits were fed through a Raytheon stabilizer. The Geiger-Mueller counters used were of the self-quenching type and had a plateau of over 200 volts. Within a given group, the anodes of the

[2] F. Rasetti, Phys. Rev. **60**, 198 (1941).

[3] H. Victor Neher and W. H. Pickering, Rev. Sci. Inst. **10**, 53 (1939).

TABLE II. Most probable values of the mesotron absorption in lead, iron, aluminum, and water. The column headings have the following significance, R(abs.) for 10 g/cm², percent absorption in 10 g/cm² of substance from Table I; Arr., experimental arrangement; Av., average for arrangement 1 and 2; Av. (corr.), average corrected for divergence of the beam; σ(1 percent), mass per cm² (g/cm²) which absorbs one percent of the beam; σ(1 percent) corr., preceding values corrected for multiple scattering in absorber; Range (theor.), theoretical range for mesotrons of $p/\mu=1.257$ (adjusted for iron).

	R(abs.) for 10 g/cm²	Arr.	Av.	Av.(corr.)	σ(1%) g/cm²	σ(1%)corr. g/cm²	Range (theor.)
Lead	0.35±0.015	1	0.36±0.010	0.36±0.01	27.8	30.6±1	33.5
	0.38±0.01	2					
Iron	0.424±0.008	1	0.432±0.011	0.425±0.01	23.5	24.1±1	24.1
	0.44±0.01	2					
Al	0.465±0.004	2	0.465±0.004	0.453±0.004	22.1	22.4±1	21
Water	0.48±0.025	1	0.48±0.025	0.47±0.02	21.3	21.3±1	17

different counters were not connected together, but each one was coupled to the grid of the amplifying tube through a separate capacitor (10 micromicrofarads) and connected to the ground through a separate resistor (100 megohms). This type of coupling was necessary in the case of the anticoincidence counters in order to make sure that the discharge in one counter did not make the whole group insensitive.

The efficiency of the apparatus was periodically tested by disconnecting the anticoincidence counters from the high voltage source. In this case it is obvious that both mechanical recorders should count the same number of impulses.

The measurements with and without absorbers were performed alternatively for periods of one week in order to compensate for slow variations in the characteristics of the counters and the circuits.

EXPERIMENTAL RESULTS AND DISCUSSION

The measurements performed with the first arrangement (Fig. 1) in which the space available for the absorber was 20 cm were the following: (1) Absorption in 20 cm of water; (2) absorption in 10 cm of iron placed either in position A or in position B; (3) absorption in 10 cm of lead placed either in position A or in position B; (4) absorption in 2.5 cm of lead placed in position A.

The purpose of placing the absorber in two different positions was to obtain a measure of the importance of scattering compared to true absorption.

Several measurements were also performed with the second arrangement (Fig. 2) in which the maximum thickness of absorber that could be used was 10 cm. In this case the absorption in 10 cm of lead, iron, and aluminum was measured. The readings obtained from the recorders for both arrangements are summarized in Table I.

THE EFFECT OF SCATTERING

In order that the experimental data may be compared with the theoretical values they must be corrected for several perturbing factors. The most important of these is the effect of scattering which influences the apparent absorption in two different ways. A mesotron may be deflected from the beam by an elastic impact with the nucleus of the absorbing substance and hence will not be recorded by the anticoincidence counter. The multiple Coulomb scattering of mesotrons by nuclei has been calculated;[4] but it would be very difficult to evaluate, even approximately, the number of anticoincidences due to this effect because of the complicated geometrical conditions of the experiment. It was, therefore, considered more practicable to determine the effect of scattering empirically. This was done chiefly for lead, since the effect is much less important for the lighter elements. For this purpose, the apparatus in its first arrangement was used to measure the absorption in 2.5 cm of lead in position A, 10 cm of lead in position A,

[4] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 241 (1941).

and 10 cm of lead in position B. It is obvious that a much larger scattering angle is required in the A position than in the B position in order to deflect a mesotron out of the anticoincidence counters. Hence a rough consideration of the geometry of the experiment indicates that the effect of scattering should be much larger in the second case than in the first case.

Table I shows the results obtained for these three experiments. It appeared that the best way of utilizing them was to extrapolate graphically the value of the absorption per g/cm² of lead as a function of position as shown in Fig. 3. The abscissa indicates the distance of the lead absorber from the anticoincidence counter. The ordinate is the percentage absorption per g/cm² of lead. The horizontal length indicates the extension of the absorber, the vertical length the statistical error. A linear extrapolation was made and gave an absorption of 0.35 ± 0.015 percent for 10 g/cm² of lead for the ideal case of an absorber placed at zero distance from the group F. In the case of lead and iron, absorption measurements have been taken with the apparatus in both the first and the second arrangements. It is interesting to note that the results agree perfectly. This is particularly true in the case of lead where the extrapolated value of the absorption coincides with the value obtained in the second arrangement in which the geometry of the apparatus is such that the scattering is practically eliminated.

Because of this agreement, it has been deemed legitimate to consider both series of measurements as equally good and their average has been adopted as the most probable value of the absorption for the above mentioned substances (Table II).

Scattering will also affect the results through the lengthening of the path of the particle in the absorber. A correction for this effect was evaluated by using theoretical formulas given by Rossi and Greisen.[4] These authors give an expression for the square of the average scattering angle of a mesotron of initial momentum p_0, and possessing a momentum p after traversing a distance R (expressed in g/cm²) in an element of atomic number Z and of atomic weight A. They give:

$$\theta_m^2=16\pi\frac{\mu_e^2}{\mu^2}Nr_0^2\frac{Z^2}{A}\log\left(\frac{183}{Z^{\frac{1}{3}}}\right)\times\left[R\left(z,\frac{p_0}{\mu}\right)-R\left(z,\frac{p}{\mu}\right)\right]\frac{\log\left(\frac{1+y}{1+y_0}\frac{p_0}{p}\right)}{y_0+\frac{1}{y_0}-\left(y+\frac{1}{y}\right)}$$

The symbols used in this formula have the following meaning: N is Avogadro's number; r_0 is the classical radius of the electron; μ_e is the mass of the electron; μ is the mass of the mesotron; and $y=[(p^2/\mu^2)+1]^{\frac{1}{2}}$. The masses are expressed in ev/c² and the momenta are expressed in ev/c. We now wish to calculate what thickness T of absorber corresponds to a path of length R, actually followed by the particle in its zig-zag motion through matter.

To do this we write:

$$dT=dR\cos\theta_m,$$

or

$$dT=dR\left(1-\frac{\theta^2m}{2!}+\frac{\theta^4m}{4!}+\cdots\right),$$

and since θ_m is generally small only the first two terms need be considered. Integrating, we have:

$$T\left(z,\frac{p}{\mu}\right)=R\left(z,\frac{p}{\mu}\right)-\int_0^{p_0/\mu}\frac{\theta_m^2}{2}\frac{dR}{d(p/\mu)}d\left(\frac{p}{\mu}\right).$$

The preceding integration has been performed for several values of the initial momentum p_0 ranging from $p_0/\mu=0$ to $p_0/\mu=3$ and for the substances on which our measurements have been made. It is interesting to note that T and R can be approximately expressed by the following linear relations:

Lead	$T=0.907R$,
Iron	$T=0.975R$,
Aluminum	$T=0.986R$,
Water	$T=1.0R$.

These correcting factors have been applied to the experimental values of the ranges that are listed in the fifth column of Table II. The corrected ranges are listed in the sixth column of the same table.

CORRECTION FOR THE DIVERGENCE OF THE BEAM

The purpose of this experiment was to obtain relative values of the absorption of slow meso-

trons in matter. Consequently the usefulness of our measurements would not be impaired by a slight divergence of the beam because this effect increases all the values of the absorption by a constant factor. This factor has been roughly evaluated from the geometry of the apparatus and found to be equal to 1.02. However, we thought it preferable to give our absorption measurements in terms of the actual thickness traversed and for this reason we have corrected them for the divergence of the beam. The true values are given in the fourth column of Table II.

THE EFFECT OF THE ELECTRONS IN EQUILIBRIUM WITH THE MESOTRON RADIATION

These absorption measurements have been made with a beam of mesotrons containing approximately six percent of electrons and it is important to make sure that the presence of these electrons in the apparatus does not affect our interpretation of the results. In this connection we must first note that these equilibrium electrons are produced by head-on collisions of the mesotrons with the electrons of the lead blocks placed in the telescope. Since only a few centimeters of lead are required for the radiation to attain equilibrium, we may safely assume that these equilibrium electrons are all produced in the layer placed immediately above the group D of counters. On the other hand, it can be shown that a mesotron which would be stopped in 10 cm of lead (our thickness of absorber) could not produce in the lead block above the group D an electron having more energy than 25×10^6 ev. The range of this electron in lead is only a few millimeters, and in all probability it could not emerge from the lead block in which it was produced. Consequently we may safely admit that within the precision of the experiment all the equilibrium electrons which had sufficient energy to emerge from the lead block in which they were produced must have been produced by energetic mesotrons which are not likely to be deflected out of the beam or stopped in the absorber. These mesotrons may reach the anticoincidence group alone or accompanied by their equilibrium electrons, but in both cases only one pulse will be given by the F group and the apparatus will behave normally.

CONCLUSION

We now wish to express the results of our absorption measurements in lead, iron, aluminum, and water, by means of four quantities which will be proportional to the ranges of the mesotrons in the above substances. To do this we make use of the fact that the absorption, in a thin absorber, is proportional to the thickness of absorber. We then calculate what thickness of absorber would stop a given (small) fraction, say 1 percent of the total radiation (see column 5, Table II).

The thickness of absorber thus calculated is such that it would stop all particles having a momentum less than a critical momentum p_c. Consequently, this thickness is equal (except for the correction due to scattering) to the range, in that absorber, of mesotrons of momentum p_c.

The thicknesses of lead, iron, aluminum, and water thus obtained from experimental measurement must be corrected for the multiple Coulomb scattering in the absorber, as previously explained. The corrected values for the experimental ranges are given in column 6, Table II.

On the other hand, the theoretical range-momentum curves[4] for different elements have the same shape and to a good approximation, differ only by a constant factor. Hence, to test the relationship between atomic number and range, we do not need to know p_c. It is only necessary to compare the corrected experimental values of the range with the theoretical values, for some convenient value of the momentum. If theory and experiment agree the two sets of values should be proportional. The theoretical and experimental values were made to agree for iron (see column 7, Table II).

Let us first consider the absorption in water. In this case, no satisfactory theoretical value of the energy loss is available. In fact, the calculations are based on a statistical model of the atom and cannot be expected to apply to hydrogen. Moreover, even if the correct value for hydrogen were known, it is by no means certain that the hydrogen in the water molecule would possess the same stopping power as in the free state. Hence a significant comparison between theory and experiment is possible only for lead, iron, and aluminum.

From the values in Table II one sees that the agreement is fairly good, but that there is a discrepancy which is definitely outside of the statistical error. The difference is in the sense that the experimentally observed absorption is less dependent upon the atomic number than should be expected according to the theory. There are several perturbing factors involved, and although an attempt has been made to evaluate their effects, one cannot be certain that they have all been accounted for with sufficient accuracy. In particular, it is possible that the electrons in equilibrium with the mesotrons affect the measured absorption to some extent. Moreover, there is some uncertainty in the number of mesotrons that are scattered out of the beam. In conclusion, we believe that the theoretical values are in approximate agreement with the experimental results and it appears probable that the discrepancies are not due to any fundamental inadequacy of the theory.

I wish to acknowledge my indebtedness to Dr. F. Rasetti for suggesting the problem and for valuable advice and discussion.

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Performance of a Hot Wire Clusius and Dickel Column*,**

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The separation factor for argon isotopes has been measured as a function of the gas pressure in a hot wire type of Clusius and Dickel column. The variation obtained is in accord with the predictions of the Furry, Jones, and Onsager theory. It is found empirically that the explicit expressions given by Furry, Jones, and Onsager for the flat, parallel wall case can be modified to give correct results for the extreme cylindrical case if the constants of the gas are evaluated at the proper mean temperatures. The existence of turbulence in the gas causes a slight, if any, decrease in the separation factor determined by extrapolation from the experimental data obtained under conditions of lamellar flow.

INTRODUCTION

IN 1938 Clusius and Dickel,[1] and subsequently Brewer and Bramley,[2] cascaded the small effect of thermal diffusion in separating isotopes by putting the gaseous isotopic mixture into the annular space between two concentric cylindrical surfaces, mounted vertically. By maintaining the inner surface at a higher temperature than the outer, convection currents are set up which carry the isotope that concentrates toward the hot surface to the top of the column, thus greatly increasing the transverse separation caused by the radial temperature gradient. Batteries of Clusius and Dickel columns coupled in series have since been used successfully to obtain large concentrations of important rare isotopes.[3]

The theory of the Clusius and Dickel method was first developed by Furry, Jones, and Onsager[4] in 1939 (hereinafter referred to as FJO) and also by Waldmann[5] and Debye.[6] FJO calculated explicit expressions for the vertical change in concentration from their general theory for the case of a temperature gradient between two flat,

* Part of a dissertation presented for the degree of Doctor of Philosophy at Yale University.

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