Production of Mesotrons in the Stratosphere

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The intensity of the vertically moving ionizing penetrating cosmic-ray particles in the stratosphere is derived as a function of altitude, magnetic latitude, and energy, the purpose being to test the validity of the initial assumptions: that all the cosmic-ray mesotrons have the same mean life at rest of 2.15μsec. and are produced, nine at a time, by primary protons in the fields of air nuclei, that the cross section of an air nucleus for mesotron production by a proton is independent of the proton's energy and is equal to 2.5×10^{-25} cm^2, and that the kinetic energy of a primary proton is divided equally among the total energies of the mesotrons it produces. Comparison of theory and experiment shows that the multiplicity of mesotron production by protons is approximately nine for proton energies above 7×10^9 ev, for a differential energy spectrum of protons, of the form $N_0\mathcal{E}^{-2.9}$, that for proton energies below 7×10^9 ev the multiplicity of mesotron production is lower than nine and the power law energy spectrum is modified, that mesotrons with mean lifetimes much less than 2.15×10^{-6} sec. probably must be postulated to account for the soft component in the stratosphere, and that, as mesotrons are produced, nuclear particles are knocked forward, taking a small fraction of the available energy.

I. INTRODUCTION

THE purpose of this paper is to determine to what extent the observed energy, latitude, and altitude dependence of the penetrating component of cosmic radiation is compatible with the assumption that mesotrons are produced in multiples by primary protons in the fields of atomic nuclei.

That some of the cosmic-ray primaries might be protons was first suggested by Compton and Bethe[1]; later, Johnson[2] concluded from measurements of the east-west asymmetry of the hard component at low altitudes that protons were the primaries of practically all the mesotrons present at sea level. Then Schein, Jesse, and Wollan[3] discovered that there were only very few electrons of energies between 10^9 and 10^{12} ev at very high altitudes, almost conclusively demonstrating that practically all the ionizing primaries of energies greater than about 10^9 ev are protons. According to Schein,[4] only mesotrons of very low energies (about 10^8 ev) are produced by non-ionizing radiation.

The primary differential energy spectrum $p_0(\mathcal{E})=N_0\mathcal{E}^{-2.9}$ used by Euler and Heisenberg[5] is also used here. Its validity, at least for high energies, is indicated by the fact that the energy distribution of many high energy cosmic-ray phenomena seems to follow approximately such a law. Its validity for low energies will be discussed in a later section.

It is assumed that the cross section of an air nucleus for mesotron production by a proton is independent of proton energy for proton energies above about 2.5×10^9 ev; the value of the cross section used, $\sigma=2.5\times10^{-25}$ cm^2, was obtained from the approximate formula,

$$\sigma=\pi r_0^2 Z^{\frac{4}{3}},$$

where Z is the atomic number of air (approximately 8), and r_0, the range of nuclear forces, is 1.4×10^{-13} cm.[6] It is shown in a later section that this assumption is very nearly valid for proton energies above 3×10^9 ev.

Schein, Jesse, and Wollan; Schein, Iona, and Tabin; and Schein and Stroud[7] have measured the number of coincidences involving a counter or counters above a block of lead or paraffin and two or more of a set of counters arranged in a horizontal plane below the block; these measurements were made at high altitudes, and a coincidence of

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[1] A. H. Compton and H. A. Bethe, Nature 134, 734 (1934).
[2] T. H. Johnson, Rev. Mod. Phys. 11, 208 (1939).
[3] M. Schein, W. P. Jesse, and E. O. Wollan, Phys. Rev. 59, 615 (1941).
[4] M. Schein, private communication.
[5] H. Euler and W. Heisenberg, Ergeb. d. Exakt. Naturwiss. 17, 1 (1938).
[6] J. Tabin, Phys. Rev. 66, 86 (1944).
[7] W. G. Stroud and M. Schein, Phys. Rev. 67, 62 (1945).

the sort described was attributed to multiple production of mesotrons by a proton. The relative frequencies of two-, three-, and fourfold coincidences in the lower set of counters led Schein to the conclusion that the average multiplicity (i.e., the number of mesotrons produced by a single proton) was in the neighborhood of nine. In the present derivation, multiplicity is set at nine and is assumed to be independent of proton energy for energies above about 2.5×10^9 ev (the geomagnetic cut-off energy at Chicago). Other evidence regarding the multiplicity is discussed in Part III.

Further postulates on which the ensuing analysis is based are that, in the process of mesotron production, the kinetic energy of the primary proton is divided equally among the total energies (rest plus kinetic) of the mesotrons and that the latter all proceed in the direction in which the proton was traveling when it produced them (as indicated by the observed east-west asymmetry of the hard component in moderate altitudes).

The only part of the hard component considered herein is that part which moves vertically. If the direction of motion of each primary proton is perpetuated throughout the path of each mesotron produced by that proton, the derivation of the intensity of the hard component moving in any given direction proceeds in much the same way as the derivation carried out here for vertically moving particles. The soft (non-penetrating) component of cosmic radiation is not treated here at all; hence no definite conclusion is reached as to whether it is necessary to postulate the existence of rapidly decaying (lifetime of the order of 10^{-9} sec.), so-called "transverse" mesotrons of the type considered by Moeller and Rosenfeld[8] in the theory of nuclear forces. The only mesotrons dealt with here are those whose mean life at rest is $2.15\pm0.07\mu$ sec.[9]; however, considerations mentioned in Section III, 4, make it appear likely that transverse mesotrons are needed to account for the soft component in the stratosphere. The soft component at lower altitudes is, according to Stanton,[10] satisfactorily accounted for by the observed long-lived mesotrons.

[8] M. Moeller and L. Rosenfeld, Kgl. Danske Vid. Sels. Math.-Fys. Medd. **17**, No. 8 (1940).

[9] N. Nereson and B. Rossi, Phys. Rev. **64**, 199 (1943).

[10] H. E. Stanton, Phys. Rev. **66**, 48 (1944).

Derivations of the dependence of the intensity of the mesotron component upon altitude, energy, and latitude have previously been made by Euler and Heisenberg[5] and Rathgeber,[11] and, more recently, by Swann,[12] and Hamilton, Heitler, and Peng.[13] Euler and Heisenberg and Rathgeber assumed that the cosmic-ray mesotrons are all produced by the soft component; this assumption is now pretty well ruled out by the absence of electrons at very high altitudes.[3] Swann made extensive calculations on the assumption that the primaries are heavy charged particles each of which splits up into a number of mesotrons of equal energy. He did not perform the integration over the production region which is here carried out in Section II, 4, and hence could not predict the mesotron distribution at very high altitudes. Hamilton, Heitler, and Peng used primary protons each of which produces many mesotrons in a series of single production processes, multiplicity and effective cross section of an air nucleus being functions of energy. As will be seen in Section III, 4, this picture is not compatible with the results recently obtained by Schein and his collaborators.[7]

II. THEORY

1. Proton Flux as a Function of Altitude

According to the assumptions, the differential proton spectrum at the top of the atmosphere is

$$p_0(\mathcal{E})=N_0\mathcal{E}^{-2.9},$$

where $\mathcal{E}$ is kinetic energy in ergs. Let z_0 be altitude in centimeters; if the stratosphere is assumed to be isothermal, air density at altitude z_0 in the stratosphere is given by

$$\rho(z_0)=\rho_0\exp(-az_0),$$

where

$$\rho_0\equiv1.970\times10^{-3}\text{ g cm}^{-3},$$

and

$$a\equiv1.550\times10^{-6}\text{ cm}^{-1}.^{14}$$

Thus, if $p(\mathcal{E}, z_0)d\mathcal{E}dz_0$ is the number per cm² sec. unit solid angle of vertically moving protons in dz_0 at z_0 with energies in $d\mathcal{E}$ at $\mathcal{E}$,

$$p(\mathcal{E}, z_0)=p_0(\mathcal{E})\exp[-\alpha\exp(-az_0)],$$

[11] H. D. Rathgeber, Phys. Rev. **61**, 207 (1942).

[12] W. F. G. Swann, J. Frank. Inst. **236**, 111 (1943).

[13] J. Hamilton, W. Heitler, and H. W. Peng, Phys. Rev. **64**, 78 (1943).

[14] These values are obtained from tables in Humphreys *Physics of the Air*, and are valid for $T=219$°K.

and

$$\frac{\partial p}{\partial z_0}=p_0(\mathcal{E})a\alpha\exp(-az_0)\exp[-\alpha\exp(-az_0)],$$

where

$$\alpha\equiv\frac{6.03\times10^{23}\times2\sigma\rho_0}{29a}=13.2,$$

$\sigma=2.5\times10^{-25}$ cm^2 being the proton absorption cross section of an air nucleus.

These results will be seen to depend on the assumption that the protons in question lose no energy by ionizing the air; for protons of energy greater than 10^9 ev, in the stratosphere, ionization loss is practically negligible.

2. Spectrum of Mesotrons at Point of Production

Let $q_0(E_0, z_0)dE_0dz_0$ be the number of vertically moving mesotrons per cm^2 sec. unit solid angle produced in dz_0 at z_0 with total energies (rest plus kinetic) in dE_0 at E_0. Then, if $\phi(E_0, \mathcal{E})dE_0$ is the number of mesotrons with initial total energies in dE_0 at E_0 produced by a single proton with kinetic energy $\mathcal{E}$,

$$q_0(E_0, z_0)=a\alpha\exp(-az_0)\exp[-\alpha\exp(-az_0)]\int_{\mathcal{E}_0}^{\infty}p_0(\mathcal{E})\phi(E_0,\mathcal{E})d\mathcal{E};\quad(1)$$

here $\mathcal{E}_0$ is the lowest proton energy present, or the lowest energy at which mesotrons can be produced. It is assumed that the mesotrons travel in the same direction as the proton which produced them.

3. Spectrum of Mesotrons Produced at a Given Height, z_0, as a Function of Altitude, z

A mesotron produced at z_0 with energy E_0 loses energy by ionizing the air. Let E be its energy at height z:

$$dE/dz=\gamma p(z)=\gamma p_0e^{-az},$$

or

$$E=E_0-\frac{\gamma p_0}{a}[\exp(-az)-\exp(-az_0)],\quad(2)$$

where γ is rate of energy-loss in ergs g^{-1} cm^2. γ is here assumed, for mathematical simplicity, to be independent of energy; its value is taken to be 4.0×10^{-6} erg g^{-1} cm^2.[15]

[15] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 245 (1941).

While they are losing energy, the mesotrons are disintegrating at a rate which depends on their energy. Let $q(E, z)dE$ be the number of mesotrons at height z with energies in dE at E; then, if t is the time in the earth's frame of reference and τ is the mesotrons' mean life in the earth's frame of reference, for mesotrons of a given downward velocity $v(=c\beta)$,

$$\partial q(E, z')/\partial t=-q/\tau,$$

where z' is the changing altitude of the moving mesotrons. If τ_0 is the mesotrons' mean life in their own frame of reference,

$$\partial q(E, z')/\partial z'=q/v\tau=q(1-\beta^2)^{\frac{1}{2}}/v\tau_0.$$

Since

$$E=\mu c^2/(1-\beta^2)^{\frac{1}{2}}\cong\mu cv/(1-\beta^2)^{\frac{1}{2}},\quad(3)$$

where μ is the rest mass of a mesotron,

$$\frac{dq}{q}=\frac{\mu c}{E\tau_0}dz.\quad(4)$$

If the expression for E is substituted from Eq. (2) and the resulting expression is integrated, the following equation is obtained for the number of vertically moving mesotrons per cm^2 sec. erg unit solid angle at altitude z which were produced at altitude z_0:

$$q(E, z_0, z)=q_0(E+G[y-y_0], z_0)\times\left(\frac{y_0}{y}\frac{E}{E+G(y-y_0)}\right)^{\mu c/[a\tau_0(E+Gy)]},\quad(5)$$

where

$$y\equiv e^{-az},\quad y_0\equiv e^{-az_0},$$

and

$$G\equiv\gamma p_0/a.$$

4. Mesotron Spectrum as a Function of Altitude

Let $q(E, z)dE$ be the number per cm^2 sec. per unit solid angle of the vertically moving mesotrons at height z with energies in dE at E.

$$q(E, z)=\int_z^{\infty}q(E, z_0, z)dz_0$$

$$=\int_0^y\frac{1}{ay_0}\left(\frac{y_0}{y}\frac{E}{E+G(y-y_0)}\right)^{\mu c/[a\tau_0(E+Gy)]}q_0(E+G[y-y_0], z_0)dy_0.\quad(6)$$

Substitution for $q_0(E+G[y-y_0], z_0)$ from Eq (1)

TABLE I. Calculated differential mesotron spectrum q at 90° magnetic latitude, expressed in terms of N_0, the number of protons at the top of the atmosphere per cm² sec. erg unit solid angle at one erg energy. q is given for 8 energies and 8 altitudes.

E (Total energy, ergs)	Kinetic energy, electron-volts	y z(altitude in cm) Pressure in cm Hg	0 ∞ 0	0.02 2.52 $\times 10^6$ 1.88	0.05 1.93 $\times 10^6$ 4.70	0.10 1.49 $\times 10^6$ 9.40	0.15 1.22 $\times 10^6$ 14.1	0.16 1.18 $\times 10^6$ 15.0	0.17 1.14 $\times 10^6$ 16.0	0.18 1.11 $\times 10^6$ 17.0
5.6×10^{-4}	2.52×10^{8}	$\frac{10^{-7}q}{N_0}=$	0	4.89	8.37	9.15	7.43	7.02	6.63	6.24
2×10^{-3}	1.16×10^{9}	$\frac{10^{-6}q}{N_0}=$	0	1.66	2.90	3.33	2.93	2.92	2.71	2.59
2×10^{-2}	1.25×10^{10}	$\frac{10^{-3}q}{N_0}=$	0	3.10	6.50	8.32	8.98	9.02	9.05	9.04
2×10^{-1}	1.26×10^{11}	$\frac{q}{N_0}=$	0	4.16	8.28	11.8	13.3	13.4	13.6	13.7
2	1.26×10^{12}	$\frac{10^{3}q}{N_0}=$	0	5.28	10.5	15.1	17.0	17.2	17.4	17.6
2×10^{1}	1.26×10^{13}	$\frac{10^{6}q}{N_0}=$	0	6.65	13.3	19.0	21.5	21.8	22.0	22.2
2×10^{2}	1.26×10^{14}	$\frac{10^{9}q}{N_0}=$	0	8.37	16.7	23.9	27.0	27.4	27.7	28.0
2×10^{3}	1.26×10^{15}	$\frac{10^{11}q}{N_0}=$	0	1.05	2.10	3.01	3.40	3.45	3.49	3.52

yields

$$q(E,z)=\alpha\int_0^y e^{-\alpha y_0}\left(\frac{y_0}{y}\,\frac{E}{E+G(y-y_0)}\right)^{\mu c/[\alpha\tau_0(E+Gy)]}\left\{\int_{\mathcal{E}_0}^{\infty}p_0(\mathcal{E})\phi(E+G[y-y_0],\,\mathcal{E})d\mathcal{E}\right\}dy_0.$$

In the case under consideration, each proton produces 9 mesotrons, each having total energy equal to $\frac{1}{9}$ of the kinetic energy of the proton, or

$$\int_{\mathcal{E}_0}^{\infty}p_0(\mathcal{E})\phi(E_0,\,\mathcal{E})d\mathcal{E}=9p_0(9\{E+G[y-y_0]\})\frac{d\mathcal{E}}{dE_0}=81N_0\{9[E+G(y-y_0)]\}^{-2.9}.$$

Hence,

$$q(E,z)=9^{-0.9}\alpha N_0\int_0^y e^{-\alpha y_0}\left(\frac{y_0}{y}\,\frac{E}{E+G(y-y_0)}\right)^{A(E,y)}[E+G(y-y_0)]^{-2.9}dy_0, \tag{7}$$

where

$$A(E,y)\equiv\frac{\mu c}{\alpha\tau_0(E+Gy)}.$$

The substitution $u=\alpha y_0$ yields

$$\begin{aligned}q(E,z)&=\left(\frac{E}{y}\right)^A\frac{N_0\alpha^{-A}}{(E+Gy)^{A+2.9}9^{0.9}}\int_0^{\alpha y}e^{-u}u^A\left(1-\frac{Gu}{\alpha E+\alpha Gy}\right)^{-A-2.9}du\\&=\left(\frac{E}{y}\right)^A\frac{N_0\alpha^{-A}}{9^{0.9}(E+Gy)^{A+2.9}}\int_0^{\alpha y}e^{-u}\sum_{n=0}^{\infty}\frac{u^{A+n}}{n!}\left(\frac{G}{\alpha(E+Gy)}\right)^n(A+2.9)\cdots(A+n+1.9)du\\&=\left(\frac{E}{y}\right)^A\frac{N_0\alpha^{-A}}{9^{0.9}G^{A+1}D'^{A+1}(E+Gy)^{1.9}}\sum_{n=0}^{\infty}\frac{(A+2.9)\cdots(A+n+1.9)}{n!D'^n}\Gamma(A+n+1,\,\alpha y),\end{aligned} \tag{8}$$

where

$$D' \equiv \frac{\alpha(E+Gy)}{G},$$

and $\Gamma(m, h)$, the incomplete gamma-function of m and h, is defined as

$$\Gamma(m, h) \equiv \int_0^h t^{m-1} e^{-t} dt.$$

The differential mesotron spectrum, $q(E, z)$, has been computed for eight values of energy (E) and seven altitudes (z) in the stratosphere, at four magnetic latitudes (λ). No account has been taken of variation of mesotron intensity with temperature or longitude. The computed values of q for the magnetic pole are tabulated in Table I; introduction of the latitude effect is discussed in the next section.

5. Latitude Effect

The equation

$$\mathcal{E}^* = 2.39 \times 10^{-2} \cos^4 \lambda$$

defines the cut-off proton energy $\mathcal{E}^*$ for any magnetic latitude. Mesotrons with energies greater than $E_0^* \equiv \mathcal{E}^*/9$ will be unaffected in intensity by action of the earth's magnetic field; at any altitude z, there will be no mesotrons of energy less than $E^* \equiv E_0^* - Ge^{-az} = E_0^* - Gy$. Therefore, once $q(E, z)$ has been determined for $\lambda = 90°$, its determination for other latitudes is already accomplished except for the energy range $E_0^* > E > E^*$. In this energy range, for any height z and energy E, there will be a height z_0^* such that a mesotron of energy E_0^* originating at z_0^* will arrive at z with energy E. $y_0^* \equiv \exp(-az_0^*)$ is obtained thus:

$$E_0^* - E = G(y - y_0^*),$$

or

$$y_0^* = y - (E_0^* - E)/G.$$

Since all the mesotrons of energy E at height z must have been produced at height z_0^* or higher, the differential spectrum for this energy range can be obtained from Eq. (8), provided the upper limit of integration is made y_0^* instead of y:

$$q(E, z) = \int_0^{y_0^*} \frac{1}{ay_0} \left(\frac{y_0}{y} \frac{E}{E+G(y-y_0)} \right)^{\mu c/[a\tau_0(E+Gy)]} q_0(E+G[y-y_0], z_0) dy_0$$
$$= \left(\frac{E}{y}\right)^A \frac{N_0 \alpha^{-A}}{9^{0.9} G^{A+1} D'^{A+1} (E+Gy)^{1.9}} \sum_{n=0}^{\infty} \frac{(A+2.9)\cdots(A+n+1.9)}{n! D'^n} \Gamma(A+n+1, \alpha y_0^*). \quad (9)$$

6. Integral Mesotron Spectrum

Let $Q(E, z)$, the integral mesotron spectrum, be defined as

$$Q(E, z) \equiv \int_E^\infty q(E'', z) dE''$$

If $E_1, E_2 \cdots E_i \cdots$ are a sequence of energies such that $E_i > E_{i-1}$ and $\lim_{i \to \infty} E_i = \infty$, and if

$$Q_i(z) \equiv \int_{E_i}^{E_{i+1}} q(E'', z) dE'',$$

then

$$Q(E_j, z) = \sum_{i=j}^{\infty} Q_i(z).$$

For any altitude and any latitude, if $E > E_0^*$, $q(E, z)$ can, over some energy ranges, be quite closely approximated in the form

$$q(E, z) = \bar{q}(z) E^k,$$

where $k \cong -2.9$. If this expression is valid for $E_i \leqslant E \leqslant E_{i+1}$, then

$$Q_i(z) = \bar{q}(z) \int_{E_i}^{E_{i+1}} E''^k dE'',$$

or

$$Q_i(z) = \frac{1}{k+1} [q(E_{i+1}, z) E_{i+1} - q(E_i, z) E_i]. \quad (10)$$

$$Q(E_j, z) = \sum_{i=j}^{\infty} Q_i(z)$$

was computed for the first four of the eight energies for which $q(E, z)$ was found. $Q_i(z)$ was obtained by Eq. (10) for $E \geqslant E_0^*$; for $E < E_0^*$, graphical means were employed.

7. Integral Spectrum for All Penetrating Particles

The integral mesotron spectrum, $Q(E, z)$, accounts for only part of the penetrating ionizing particles observed under absorbers. In addition

FIG. 1. Calculated intensity of the ionizing cosmic-ray particles which can penetrate 18 cm of Pb at 52° magnetic latitude, plotted against y (defined in Section II, 3) and pressure in the stratosphere. P' = proton curve; Q = mesotron curve; $Q+P'$ = total hard component. Intensities are expressed in terms of N_0, the number of protons at the top of the atmosphere per erg cm² sec. unit solid angle at one-erg energy.

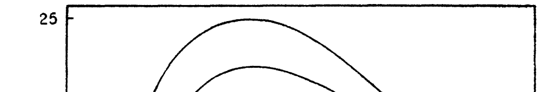

FIG. 2. Calculated intensity of the ionizing cosmic-ray particles which can penetrate 18 cm of Pb at the magnetic equator, plotted against y (defined in Section II, 3) and pressure in the stratosphere. P' = proton curve; Q = mesotron curve; $Q+P'$ = total hard component. Intensities are expressed in terms of N_0, the number of protons at the top of the atmosphere per erg cm² sec. unit solid angle at one-erg energy.

to mesotrons, protons are present; their differential spectrum is

$$p(\mathcal{E}, z) = N_0\mathcal{E}^{-2.9} \exp\left[-\alpha e^{-az}\right]$$
$$= N_0\mathcal{E}^{-2.9}e^{-\alpha y} \quad \text{for } \mathcal{E} \geqslant \mathcal{E}^*;$$
$$p(\mathcal{E}, z) = 0 \quad \text{for } \mathcal{E} < \mathcal{E}^*.$$

The integral proton spectrum is

$$P(\mathcal{E}, z) \equiv \int_{\mathcal{E}}^{\infty} p(\mathcal{E}'', z)d\mathcal{E}'',$$

or

$$P(\mathcal{E}, z) = \frac{N_0}{1.9}\mathcal{E}^{-1.9}e^{-\alpha y} \quad \text{for } \mathcal{E} \geqslant \mathcal{E}^*;$$
$$P(\mathcal{E}, z) = \frac{N_0}{1.9}\mathcal{E}^{*-1.9}e^{-\alpha y} \quad \text{for } \mathcal{E} < \mathcal{E}^*. \tag{11}$$

For comparison of theory with experiment, it is necessary that $Q(E, z)$ be added to $P'(E, z)$, the number of protons that can penetrate the same thickness of, say, lead as can a mesotron of energy E. If $\mathcal{E}'(E)$ is defined as the kinetic energy a proton must have in order to penetrate as much lead as can a mesotron of total energy E,

$$P'(E, z) = P[\mathcal{E}'(E), z],$$

and can be computed from Eq. (11), with the aid of Table II.[15] Graphs of P', Q, and $P'+Q$ *vs.* y appear in Figs. 1 and 2, and a graph of $\log(P'+Q)$ *vs.* $\log E$ appears in Fig. 3.

TABLE II. Kinetic energy, $\mathcal{E}'$, needed by a proton if it is to penetrate the same amount of lead as a mesotron of total energy E. Energies in ergs.

E(ergs)	5.6×10^{-4}	2×10^{-3}	2×10^{-2}	2×10^{-1}	2	20
$\mathcal{E}'$(ergs)	6.41×10^{-4}	2.09×10^{-3}	1.93×10^{-2}	1.99×10^{-1}	2	20

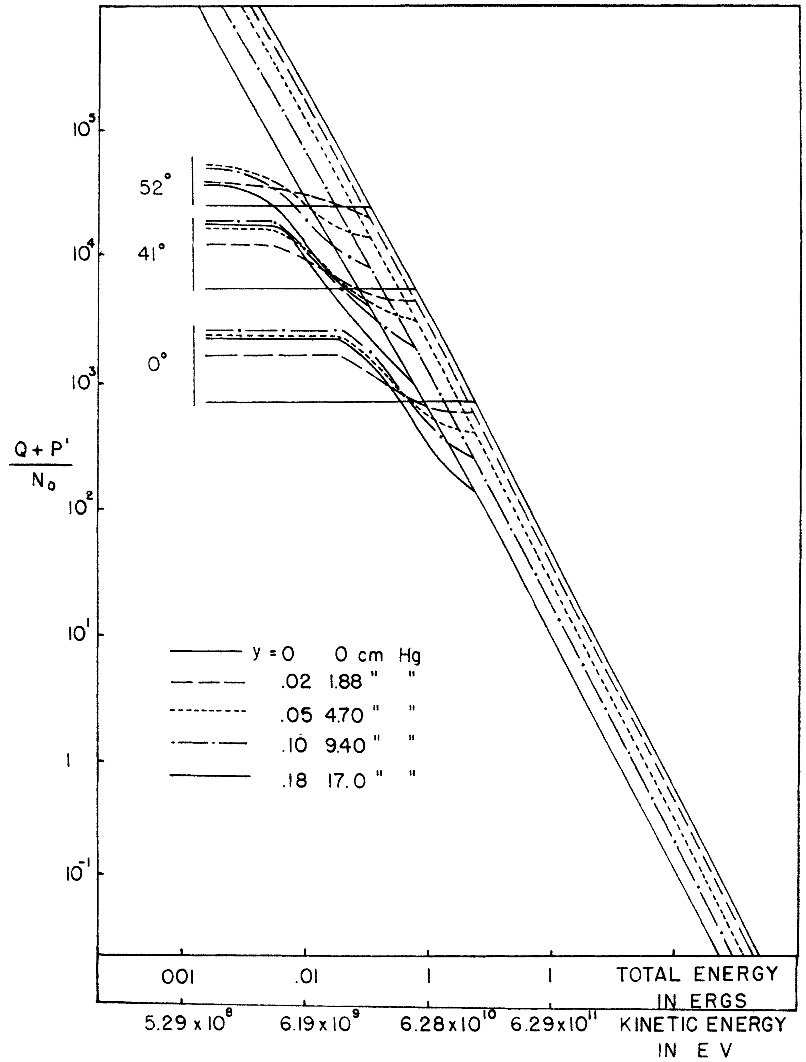

FIG. 3. Logarithmic graph of the calculated integral spectrum of mesotrons plus equally penetrating protons (expressed in terms of N_0, the number of protons at the top of the atmosphere per erg cm^2. Unit solid angle at one-erg energy) *vs.* E, total energy in ergs, at five different atmospheric pressures and four different magnetic latitudes.

8. Intensity of Cosmic Rays Underground

To any distance D underground there corresponds an energy E^* such that a mesotron of total energy E^* at the bottom of the stratosphere ($z=1.11\times10^6$ cm) has only its rest energy when it reaches depth D. If D is expressed in meters of water equivalent, then

$$E^*=4.0\times10^{-4}(D+10).$$

The total number of mesotrons present at depth D is thus

$$N(D)=Q(E^*,\ 1.11\times10^6)f,$$

where f is a factor introduced to take account of decay. Let $E^{*\prime}$ be the energy at sea level of a mesotron with energy E^* at $z=1.11\times10^6$. Table III lists, for three values of D, the corre-

TABLE III. Depth D below the ground, expressed in meters of water equivalent; energy E^* which a vertically moving mesotron must have at the bottom of the stratosphere if it is to penetrate to depth D; energy $E^{*\prime}$ which a vertically moving mesotron has at the surface of the ground if it had energy E^* at the bottom of the stratosphere; $T(D)$, time required for a high energy mesotron to reach depth D from the surface of the ground; and $\tau(E^{*\prime})$, mean life of a mesotron with energy $E^{*\prime}$.

D Meters water equivalent	E^* ergs	$E^{*\prime}$ ergs	$T(D)$ seconds	$\tau(E^{*\prime})$ seconds
89.1	3.85×10^{-2}	3.55×10^{-2}	8.91×10^{-8}	4.82×10^{-4}
163.2	6.81×10^{-2}	6.49×10^{-2}	1.63×10^{-7}	8.79×10^{-4}
245.4	1.01×10^{-1}	9.77×10^{-2}	2.45×10^{-7}	1.32×10^{-3}

sponding values of E^*, $E^{*\prime}$, $\tau(E^{*\prime})$ (mean life at energy $E^{*\prime}$), and $T(D)$ (time required for a mesotron to reach depth D from sea level in a medium of density$=3.3$). Since $\tau_0=2.15\times 10^{-6}$ sec., it is seen that decay in the ground is altogether negligible, for $T(D)\ll\tau_0$ for all D; this is true even if the density of the ground is as low as 2. It is also evident that $\tau(E^{*\prime})$ is large compared to $\frac{1}{3}\times 10^{-4}$ sec., the time required for a mesotron to reach sea level from the bottom of the stratosphere. Thus, the effect of decay may be altogether neglected. This is especially true because the foregoing comparisons of transit-times with mean lives were made for the shortest mean lives of the lowest energy mesotrons under consideration. Therefore, $f=1$, and

$$N(D)=Q(E^*,\ 1.11\times 10^6).$$

The fact that $Q(E^*, z)$ has effectively reached the asymptotic value at $z=1.11\times 10^6$ indicates that, at this height, the number of protons is already negligible. Certainly below ground the number of protons is extremely small compared to the number of mesotrons, and the high energy mesotrons produced below the stratosphere will be few compared to the number present at the bottom of the stratosphere. So the total intensity of vertically moving penetrating cosmic-ray particles at any depth, D, underground is simply $Q(E^*, 1.11\times 10^6)$.

In Table IV are given the values of $Q(E^*, 1.11\times 10^6)$ for 3 values of D.

III. DISCUSSION

1. Total Number of Penetrating Particles as a Function of Altitude

Schein, Jesse, and Wollan; Schein, Iona, and Tabin; and Schein and Stroud[7] measured the intensity of the penetrating component of cosmic radiation in the stratosphere. They measured, as

TABLE IV. Calculated numbers of mesotrons per cm² sec. unit solid angle at three depths D underground (meters water equivalent). Numbers are expressed in terms of N_0, the number of protons at the top of the atmosphere per erg cm² sec. unit solid angle at one-erg energy.

D (meters H_2O equivalent)	$Q(E^*, 1.11\times 10^6)$
89.1	$31.62N_0$
163.2	$10.97N_0$
245.4	$5.25N_0$

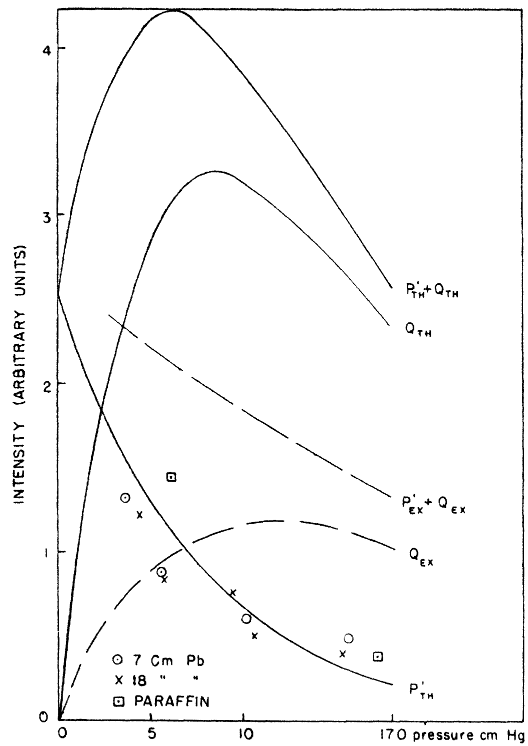

FIG. 4. Observed (broken) and calculated (solid) curves of mesotron (Q) and proton plus mesotron ($Q+P'$) intensity, and theoretical curve and experimental points of proton (P') intensity, plotted against pressure in the stratosphere. The relative scales are so adjusted that the theoretical and indicated experimental proton curves meet at the top of the atmosphere.

a function of altitude, the total number of vertically-moving ionizing cosmic-ray particles which penetrated 18 cm of lead without producing secondaries in the lead. This number proved to be approximately the same as the number which penetrated 8 cm of lead without producing secondaries. Measurements were also made of the number of ionizing penetrating particles each of which, in a block of paraffin or lead, produced several ionizing penetrating secondaries. It will be assumed that the particles which produced secondaries were primary protons. Approximately the same number of multiple production processes occurred in 6 cm of paraffin as in 10 cm of paraffin or in 7 or 18 cm of lead. In Fig. 4, experimental and theoretical curves of mesotron, proton, and total penetrating intensity at 52° magnetic latitude are shown.

It can be seen that the theoretical and indicated experimental proton curves, having been adjusted to meet at the top of the atmosphere, fit quite closely throughout the stratosphere. Thus the indication is that the assumed proton absorption cross section of an air nucleus (2.5×10^{-25} cm^2) is approximately correct for proton energies in the neighborhood of 6×10^9 electron volts (the average energy of the primaries at geomagnetic latitude 52°N) and that this cross section is in fact practically independent of energy above 6×10^9 ev since the experimental proton curve is in first approximation exponential.

The theory predicts approximately 3.5 times too many mesotrons near the maximum of the mesotron curve, and the theoretical rate of decay of these mesotrons is too rapid. This discrepancy could obviously be removed if a lower effective multiplicity than nine were assumed for mesotron production by a proton of approximately 6×10^9 ev energy. This possibility will be discussed in Section III, 4.

2. Absolute Number of Primaries

Bowen, Millikan, and Neher[16] have estimated, from ionization chamber measurements of the total cosmic-ray intensity, that there are 0.09 primaries per cm^2 sec., with energies between 6.7×10^9 ev and 15×10^9 ev at the top of the atmosphere. From this can be computed N_0, the number of primaries per erg cm^2 sec. unit solid angle with one-erg energy. Integration of the differential spectrum $N_0\mathcal{E}^{-2.9}$ from 6.7×10^9 ev to 15×10^9 ev gives $N_0=6.56\times10^{-6}$.

N_0 for the same proton spectrum can also be calculated from comparison of the theoretical integral mesotron spectrum with the underground intensity of mesotrons. The values obtained by Wilson,[17] using vertical counter telescopes,* for $N(D)$ at three depths, reduced to number of particles per cm^2 sec. unit solid angle, are given in Table V; with these values are tabulated the theoretical values as derived in Section II, 8. Since the theoretical values are proportional to N_0, equation of theoretical and experimental values permits calculation of N_0.

TABLE V. Depth D (meters water equivalent); $N(D)$, observed number of mesotrons per cm^2 sec. unit solid angle at depth D; $Q(E^*, 1.11\times10^6)$, calculated number of mesotrons at depth D per cm^2 sec. unit solid angle; values of N_0, the number of protons at the top of the atmosphere per erg cm^2 sec. unit solid angle at one-erg energy, obtained by equation of calculated and observed numbers of mesotrons at each depth.

D	$N(D)$	$Q(E^*, 1.11\times10^6)$	N_0
89.1	2.11×10^{-4}	$31.62N_0$	6.69×10^{-6}
163.2	7.72×10^{-5}	$10.97N_0$	7.05×10^{-6}
245.4	3.52×10^{-5}	$5.25N_0$	6.70×10^{-6}

The agreement between the values of N_0 obtained by the two methods is very satisfactory, although, as will be seen in Section III, 4, the value of N_0 given by Bowen, Millikan, and Neher may be somewhat too small. The indication is that from about 7×10^9 ev to at least 5×10^{11} ev the differential primary spectrum can be represented as $N_0^{-2.9}$ and the multiplicity is approximately nine, although the facts given here do not rule out the possibility of a different primary spectrum combined with a multiplicity which depends on energy.

3. Latitude Effect

Schein, Jesse, and Wollan[18] have measured the intensity of the hard component of cosmic rays

TABLE VI. Observed and calculated values of the ratio of the intensity of the hard component at 52° magnetic latitude to intensity at 41° magnetic latitude, at various atmospheric pressures.

Pressure (cm Hg)	$\frac{(Q+P')_{52°}}{(Q+P')_{41°}}$ (Experimental)	$\frac{(Q+P')_{52°}}{(Q+P')_{41°}}$ (Theoretical)
0	—	4.68
1.88	—	3.02
4.70	1.2	2.88
9.40	1.2	2.19
17.0	1.1	1.70

[16] I. S. Bowen, R. A. Millikan, and H. V. Neher, Phys. Rev. **53**, 217 (1938).

[17] V. C. Wilson, Phys. Rev. **53**, 337 (1938).

* Wilson's counter telescope was equivalent to a rectangular parallelepiped such that any ionizing particle which passed through both the top and bottom faces was counted. The data are given in terms of counts per minute, and must be expressed in terms of particles per cm^2 sec. unit solid angle. Let F be the factor by which counts per second must be divided to obtain particles per cm^2 sec. unit solid angle. F is approximately given by the formula

$$F=4\int_0^{\tan^{-1}L/h}\int_0^{\tan^{-1}W/h}\cos\theta\cos\phi(L-h\tan\theta)\times(W-h\tan\phi)d\theta d\phi,$$

where $L=71$ cm is the length of the parallelepiped, $W=9$ cm is the width, and $h=34.5$ cm is the height. In this case, $F=196$ cm^2. Strict accuracy would require the use of an F somewhat smaller than this value (leading to larger values of N_0), because an obliquely moving particle must start with a higher energy than a vertically moving particle to reach the same depth, and hence a larger fraction of the obliquely moving mesotrons are cut out by the ground. This correction may be as large as 25 percent.

[18] M. Schein, private communication.

as a function of altitude at magnetic latitudes 41° and 52°N. The values they obtained for the ratio of intensity at 52° to intensity at 41° at various pressures appear in Table VI; also tabulated are the theoretical values of this ratio.

The excessively large theoretical values of this ratio will be reduced (1) if the multiplicity of mesotron production is uniformly greater than nine, (2) if the multiplicity is an increasing function of proton energy, increasing rapidly in the neighborhood of 6.7×10^9 ev (the magnetic cut-off energy for 41°), or (3) if the integral proton spectrum falls off with increasing energy less rapidly than $\mathcal{E}^{-1.9}$ in the neighborhood of 5×10^9 ev.

The first possibility can be discarded, in the light of the measurements of proton and mesotron intensity discussed in Section III, 1. These same measurements indicate that multiplicity may increase with energy in the energy range around 5×10^9 ev; however, this assumption is not sufficient in itself to give the correct latitude effect, for even if multiplicity is zero for proton energies less than 7×10^9 ev and nine for energies greater than 7×10^9 ev, the theoretical latitude effect is still far too big, and, as was seen in Section III, 2, the multiplicity is probably not much greater than nine even at very high energies. Hence, it seems necessary to assume that the integral proton spectrum falls off less rapidly than $\mathcal{E}^{-1.9}$ between 3×10^9 ev and 7×10^9 ev.

4. Indications from Other Experiments

The foregoing facts seem to show that the differential proton spectrum is $N_0\mathcal{E}^{-2.9}$ for energies greater than about 7×10^9 ev, and falls off less rapidly with increasing energy for energies less than 7×10^9 ev. Furthermore, it appears that the multiplicity of the process of mesotron production by protons is about nine for proton energies greater than about 7×10^9 ev, and is an increasing function of energy for lower proton energies.

In addition, the fact that the knee of the curve of total intensity *vs.* latitude occurs at about 40° magnetic latitude at sea level, and at slightly higher latitudes at higher altitudes,[19,20] indicates that, for proton energies below about 3×10^9 ev, the integral proton spectrum is nearly constant and the multiplicity is not more than three or four.[21] It is known[22] that the sun's magnetic field prevents charged particles with energies less than approximately 3×10^9 ev from reaching the earth at all. It is probably significant, too, that, in the measurements referred to in Section III, 1, Schein and his collaborators found very few mesotrons which could penetrate 8 but not 18 cm of lead (i.e., few mesotrons of kinetic energy between about 1.2×10^8 ev and 2.5×10^8 ev).

If the primary spectrum is $N_0\mathcal{E}^{-2.9}$ above 7×10^9 ev, the underground measurements indicate that the multiplicity must be approximately nine in this range. If account is taken of the possibility that some of the energy present in the primaries goes into neutrinos which do not contribute to the total intensity curves of Bowen, Millikan, and Neher[16] (see Section III, 2), the number of primaries at the top of the atmosphere, as computed from the total intensity measurements, comes out larger by about 50 percent. In that case, agreement between their results and those obtained underground would require a multiplicity of about 13 for primary energies in the neighborhood of 5×10^{11} ev. If account is taken of the correction for oblique rays to the value of N_0 calculated from Wilson's results (see footnote on calculations of solid angle $\times$ area for counter telescopes, Section III, 2), the estimated multiplicity at high energies will be somewhat reduced, to a value of eleven or so.

Millikan, Neher, and Pickering[23] found that the total vertical intensity at the equator in the first radiation unit from the top of the atmosphere (2 cm Hg pressure) is greater, by a factor of about eight, than the intensity of the primaries as computed from the total ionization in the atmosphere. This fact indicates that, for primaries of about 3×10^{10} energy (the average energy of primaries at the equator), multiplicity of mesotron production is approximately eight.

The large (about 20 percent)[24] positive excess of the hard component supports the supposition

[19] A. H. Compton and R. N. Turner, Phys. Rev. **52**, 799 (1937).

[20] H. Carmichael and E. G. Dymond, Proc. Roy. Soc. **171A**, 321 (1939).

[21] L. W. Nordheim, Phys. Rev. **56**, 502 (1939).

[22] P. S. Epstein, Phys. Rev. **53**, 862 (1938).

[23] R. A. Millikan, H. V. Neher, and W. H. Pickering, Phys. Rev. **61**, 397 (1942).

[24] D. J. Hughes, Phys. Rev. **57**, 592 (1940).

that, at lower energies, multiplicity is not more than about five. While the soft component has not been considered at all in this treatment, it does not seem possible that, with so low a multiplicity for production of low energy mesotrons, there will be enough mesotrons decaying to account for the observed soft component, unless it is postulated that there are also rapidly decaying "transverse" mesotrons.[8] If these transverse mesotrons are produced in approximately the same numbers as the "pseudoscalar" mesotrons (those for which $\tau_0 = 2.15 \times 10^{-6}$ sec.) they must constitute a portion of the hard component which is at all points nearly proportional to the proton intensity. The experimental results discussed in Section III, 1, can perhaps be accounted for on the assumption that a large fraction of the energy of the primary protons goes into the production of rapidly decaying mesotrons.

The theory of Hamilton, Heitler, and Peng,[13] although it seems very satisfactory in many respects, appears to meet serious difficulties when used to determine the energy and altitude dependence of the intensities of mesotrons and protons separately. The measurements cited in Section III, 1, show that the proton absorption cross section of an air nucleus is nearly independent of energy and is much smaller than the cross section used by Hamilton, Heitler, and Peng for production of low energy mesotrons.

It seems worth while to note that energy and momentum are not simultaneously conserved in a process in which all or part of the *kinetic* energy of a primary goes to make up the *total* energies (rest plus kinetic) of any number of secondaries, unless some of the primary's kinetic energy is imparted to one or more other particles which existed before the inception of the process. Therefore, the sum of the total energies of the mesotrons produced by a given proton cannot be exactly equal to the original kinetic energy of the proton; furthermore, each mesotron production process is probably accompanied by the release of at least one neutron or proton from the nucleus in whose field the process occurs. This may be the natural explanation of the neutrons observed by Korff at high altitudes; since Bethe, Korff, and Placzek[25] concluded that most of these neutrons were probably produced with initial energies no greater than about 3×10^7 ev, it appears that the energy which goes into such particles does not significantly reduce the amount of energy left for mesotrons.

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[25] H. A. Bethe, S. A. Korff, and G. Placzek, Phys. Rev. **57**, 573 (1940).