plane varies as r^{-n} and if the frequency of the r-f field is ω_0, then $\omega_1=(1-n)^{\frac{1}{2}}\omega_0$ and $\omega_3=n^{\frac{1}{2}}\omega_0$. The remaining frequency, ω_2, depends upon the magnitude of the r-f potential and upon the energy of the electron. In general, it is much smaller than ω_0 and in a reasonable numerical example, ω_2 varies from $\omega_0/30$ at $\frac{1}{2}$Mev to $\omega_0/400$ at 150 Mev. In zeroth and first orders of approximation the orbits are stable and the amplitudes of the oscillations decrease slowly as the magnetic field increases.

The azimuthal variation in the magnetic field may be introduced by assuming that the z component of the field has the form

$$H_z=\sum_{p=0}\frac{\lambda^p A_p\cos 2p\theta}{r^n}.$$

λ is the parameter of smallness, and shows that a development of the equations of motion is contemplated. The quantities A_p are even functions of z and the median plane is defined by $z=0$. The remaining field components may be derived from H_z in the usual way.

Since we are interested in discovering any elements of instability due to the introduction of the azimuthal field variation, it will be sufficient to examine the case where the magnetic field is constant in time. The first-order solution resembles that for the circular synchrotron except that the r coordinate is harmonic in the three frequencies, $2\omega_0$, ω_1, and ω_2, while z oscillates with the single frequency ω_3. In still higher order of approximation no new fundamental frequencies occur but there appear overtones and combination tones of the oridinal four frequencies. A commensurability, or near commensurability, between the frequencies, may, in some cases, lead to a vanishing resonance denominator and a consequent instability in the motion. Our analysis has been carried through the fourth order of approximation.

The results may be summarized as follows. The sharpness of resonance is defined by

$$\epsilon=\frac{2j\omega_0+k\omega_1+l\omega_2+m\omega_3}{\omega_0},$$

where j, k, l, and m are positive or negative integers (including zero). High order resonances may develop large amplitudes but only after many turns. When $\epsilon\leqslant 1/N$, the amplitude of the motion will grow to the value C after N turns. If r_0 is the mean radius of the orbit, A is some average of the initial r and z amplitudes due to the ω_1, ω_2, and ω_3 oscillations, and B is the amplitude of the r motion due to the azimuthal variation of the field, we find that,

$$N\sim\frac{Cr_0^{|j|+|k|+|l|+|m|-2}}{B^{|j|}A^{|k|+|l|+|m|-1}}.$$

Reasonable numerical values would be $A=C=2$ cm, $B=20$ cm, and $r_0=110$ cm.

In Table I the harmful resonances through fourth order are listed. The fourth row of the table gives N, the number of turns necessary to produce a catastrophe (also the sharpness of the required resonance), while the last row gives the value of the magnetic fall-off which would produce that resonance. [It will be noted that resonances where $l\neq 0$ are not listed. These resonances do occur and have been studied but since ω_2 is small compared with ω_0 and since it changes as the electrons acquire energy, they are never dangerous. The resonances leading to values of $n\geqslant 1$ or $n\leqslant 0$ have been omitted since for these, the motion is unstable even in first order.]

TABLE I. Harmful resonances.

2nd order		3rd order			4th order			
j	0	0	−1	−1	0	0	−1	−1
k	−1	2	1	3	1	3	4	0
m	2	−2	2	0	−4	−2	0	4
N	50	3000	300	300	170,000	170,000	17,000	17,000
n	0.20	0.50	0.36	0.56	0.06	0.69	0.75	0.25

It is clear from Table I that for a magnetic fall-off of $n=0.56$ to 0.75 the orbits will be stable. The resonance at $n=0.69$ is so sharp as to be innocuous and for orders above the fourth, the resonances will be still sharper.

We therefore conclude that for suitable values of n the racetrack orbits will be stable. The reasoning has been based upon a perturbation treatment. The deviation of the racetrack from the circular synchrotron represents a rather large perturbation and hence the calculated values for N may be somewhat in error. The method is completely adequate, however, for it rigorously predicts all the values of magnetic fall-off for which instabilities may occur. Analogous calculations have been made for other types of periodic azimuthal variations and it is interesting to note that the stability is improved by going to a racetrack with four rather than two straight sections.

[1] For example, see D. M. Dennison and T. H. Berlin, Phys. Rev. (in print).

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Elastic Deficiency and Color of Natural Smoky Quartz

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IN a recent paper,[1] the writer has drawn attention to the fact that the elastic properties of quartz are altered by exposing the substance to radiation. The effective radiations include electron beams, deuterons, alpha-particles and x-radiation over the region from gamma-rays of radioactive origin up to at least 2.28A. The effect is conveniently investigated by means of peizoelectric quartz oscillator-plates. These offer a coupled electro-mechanical system that is sensitive to minute variations in the density and elasticity of the medium. During the irradiation of a quartz oscillator-plate, the frequency of oscillation decreases according to an exponential law until a saturation value is reached. This value appears to be independent of the type of radiation, but varies by a factor of at least 20 among different specimens of quartz. The quartz concomitantly becomes smoky in color, and the depth of color is proportional to the change in frequency. The several elastic moduli are affected unequally with the C_{14} modulus relatively highly responsive.[2] Baking the irradiated plates at temperatures over *ca.* 180°C or exposing them to short

wave ultraviolet radiation bleaches the color and restores the frequency of the plate to the original value.

It has now been found that natural smoky quartz possesses an elastic deficiency associated with the color analogous to that of artificially colored smoky quartz. When oscillator-plates fashioned from a natural smoky quartz crystal are decolorized by baking, the frequency of oscillation is found to increase, as in artificially irradiated plates, and the increase is proportional to the original depth of color. Natural colorless quartz is not so affected. The effect is small and not easily measured in pale smoky crystals. To cite a specific instance, a number of 8900 Kc BT-cut oscillator-plates were made from a single, black and almost opaque raw quartz crystal. These plates increased in frequency by 1000 cycles when decolorized by baking. Control plates of the same frequency cut from various colorless, water-clear, raw crystals, and identically manufactured by the etch method, changed randomly in frequency, when baked, in amounts up to +50 cycles. This change is believed caused by unloading of films of water. If the smoky plates were irradiated before baking, the color somewhat darkened and the frequency decreased by 800 cycles; and when these plates were baked, they increased in frequency by 1800 cycles. Similarly, the baked and decolorized plates when re-irradiated decreased by 1800 cycles, and this amount of change could be thereafter repeatedly effected upwards and downwards by successive baking and irradiation. The original smoky color is thus equivalent to 1000 cycles of frequency, so to speak; or alternatively the frequency-thickness constant, K, can be said to increase by an amount (0.011 percent) equivalent to 1000 cycles in a 8900-kc plate when the color is bleached out by baking. The absolute value of K in this case and its relation to that of ordinary colorless quartz is not known.

These observations are of interest in connection with the generally held theory of origin of natural smoky quartz, which ascribes the color to natural radiations of radioactive origin.[3] Several points of difference exist, however, between the effects of the artificial and supposed natural irradiation. The natural smoky color does not begin to bleach until about 225°C, as compared to *ca.* 180° for the artificial color. The rate of bleaching at these temperatures is extremely slow, requiring months for completion, but increases rapidly with increasing temperature. Natural smoky quartz also shows a relatively weak thermo-luminescence during heating, but this effect may luminescence during heating, but this effect may not be related to the color[4] or elastic deficiency. It is also observed that natural smoky quartz usually is unevenly pigmented and contains adjoining and often alternating bands of smoky and colorless or nearly colorless material. When such specimens are irradiated, the colorless or relatively light colored bands always are observed to be the more deeply affected, and this is true whether the sample has first been decolorized by baking or not. The natural smoky zones themselves always are further deepened in color by artificial irradiation and hence are not saturated in terms of these radiations. The few specimens measured were from one-half to two-thirds saturated. The depth of color in the darkest colored natural smoky quartz crystals observed, which were jet black and opaque except on thin edges, is only about one-tenth the maximum and one-half the average depth of color, as measured by the associated frequency change in oscillator-plates, observed in artificially irradiated quartz. The effects of the artificial and presumed natural irradiation thus seem to differ in several respects, although the smoky color produced in both instances has an identical absorption spectrum[5] and is associated with an elastic deficiency.

[1] C. Frondel, Am. Min. **30**, 432 (1945).
[2] V. E. Bottom, and V. Nowicki, Long Branch Signal Lab., Eng. Memo. No. 1 (Sept. 1945).
[3] E. F. Holden, Am. Min. **10**, 203 (1925).
[4] S. P. Choong, Proc. Phys. Soc. London **57**, 49 (1945).
[5] N. Mohler, Am. Min. **21**, 258 (1936).

Erratum: Bose-Einstein Condensation of Trapped Electron Pairs. Phase Separation and Superconductivity of Metal-Ammonia Solutions

[Phys. Rev. **69**, 243 (1946)]

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ON page 244, lines 12 to 14 should read, "but at a concentration small enough to lower the degeneracy temperature to the prevailing temperature."