The Racetrack: A Proposed Modification of the Synchrotron*

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EXPERIMENTS are being carried on at the University of Michigan on an electron accelerator which, we believe, contains enough novel features to warrant a brief description at this time. Basically, the machine employs the principle of the adiabatic acceleration of electrons by a radiofrequency electric field, in a slowly changing magnetic field, a principle which was discovered by Veksler[1] and by McMillan.[2] The vacuum tube is in the form of two half-circles connected by straight sections. That there are no essential difficulties in regard to the stability of the orbits in this shape of field is shown by D. M. Dennison and T. H. Berlin in an accompanying letter. One of the straight sections of the tube connecting the half-circles contains the injection apparatus and target, and the other contains the accelerating cavity. In order to maintain a nearly constant radius for the equilibrium orbit, the radiofrequency is modulated so that throughout the acceleration cycle it is proportional to the velocity of the electron. The electron beam is initially bent into the orbit by means of a pair of electrostatic deflection plates.

The sketch in Fig. 1 and the data given in the table below are the result of measurements on several scale-model magnets, and constitute the approximate specifications for the full scale machine which is now under construction.

Orbit radius = 100 cm
Length of straight section = 60 cm
Geometrical cross section of magnet gap, $7\frac{1}{2}\times 17\frac{1}{2}$ cm
Injection energy, 500 kev ($\beta=0.86$)
Maximum frequency modulation, 14 percent
Maximum field, 8000 oersteds at center of each magnet
Final energy, 200 Mev
Weight of iron, 7 tons
Condenser kva, 9000 at 60 cycles
Power loss at 60 cycles continuous; copper 18 kw, condensers 27 kw, iron 10 kw.

Our preliminary investigations have not given us much information on the important question of current yield, but some of the differences between the racetrack and the betatron (or the synchrotron with betatron injection) which affect the yield can be pointed out. In order to keep the amount of frequency modulation within reasonable limits, say 14 percent, the injection energy of the electrons must be high, about 500 kev. This has several consequences. 1. The injected beam is quite "stiff," which may increase the difficulty of filling the tube at the beginning of the acceleration cycle. 2. Space charge effects in the initial electron cloud will be smaller, but it is difficult to say whether this will be an advantage or a disadvantage. 3. Scattering by the residual gas in the tube will be smaller than it is in the betatron.

The advantages of the racetrack are mainly of an engineering and economic nature. 1. The type of magnet used is efficient in regard to both weight of iron and condenser kva. 2. The injection box and the accelerating cavity may be made of solid metal, in spaces that are free from the magnetic field. 3. Since the r-f accelerating device does not occupy space within the magnet gap, practically all of the volume of magnetic field which has the proper "fall-off" is available for electron orbits. 4. The energy output can be increased by enlarging the half-circles, with an increase in cost which is not far from linear.

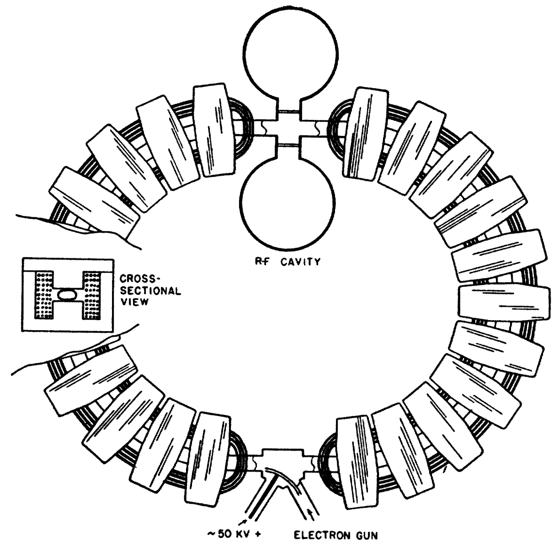

FIG. 1. Sketch of proposed accelerator.

While the advantages mentioned may not be decisive in the intermediate energy range, we believe that they will take on real importance when voltages above a billion are contemplated, and it is with this prospect in view that we intend to test the feasibility of the method.

[1] V. Veksler, J. Phys. U.S.S.R. 9, 153 (1945).
[2] E. M. McMillan, Phys. Rev. 68, 143 (1945).
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The Stability of Orbits in the Racetrack*

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IN the preceding letter by H. R. Crane, a modification of the synchrotron has been described in which the magnetic field is contained in two semicircular regions joined by two regions in which the magnetic field is absent or very small. The equilibrium orbits resemble an oval racetrack. It would be very awkward, analytically, to treat the stability of the orbits of the problem just described. The essential question, however, is whether an azimuthal variation in the magnetic field which is periodic in $\theta/2$ produces any elements of instability and this problem may be readily solved by the method of successive approximations.

The orbits of the circular synchrotron have been studied[1] and it is known that the radial coordinate of the electron may oscillate harmonically with the two frequencies ω_1 and ω_2 while the z coordinate oscillates with the single frequency ω_3. If the magnetic field in the median plane varies as r^{-n} and if the frequency of the r-f field is ω_0, then $\omega_1=(1-n)^{\frac{1}{2}}\omega_0$ and $\omega_3=n^{\frac{1}{2}}\omega_0$. The remaining frequency, ω_2, depends upon the magnitude of the r-f potential and upon the energy of the electron. In general, it is much smaller than ω_0 and in a reasonable numerical example, ω_2 varies from $\omega_0/30$ at $\frac{1}{2}$Mev to $\omega_0/400$ at 150 Mev. In zeroth and first orders of approximation the orbits are stable and the amplitudes of the oscillations decrease slowly as the magnetic field increases.

The azimuthal variation in the magnetic field may be introduced by assuming that the z component of the field has the form

$$H_z=\sum_{p=0}\frac{\lambda^p A_p \cos 2p\theta}{r^n}.$$

λ is the parameter of smallness, and shows that a development of the equations of motion is contemplated. The quantities A_p are even functions of z and the median plane is defined by $z=0$. The remaining field components may be derived from H_z in the usual way.

Since we are interested in discovering any elements of instability due to the introduction of the azimuthal field variation, it will be sufficient to examine the case where the magnetic field is constant in time. The first-order solution resembles that for the circular synchrotron except that the r coordinate is harmonic in the three frequencies, $2\omega_0$, ω_1, and ω_2, while z oscillates with the single frequency ω_3. In still higher order of approximation no new fundamental frequencies occur but there appear overtones and combination tones of the oridinal four frequencies. A commensurability, or near commensurability, between the frequencies, may, in some cases, lead to a vanishing resonance denominator and a consequent instability in the motion. Our analysis has been carried through the fourth order of approximation.

The results may be summarized as follows. The sharpness of resonance is defined by

$$\epsilon=\frac{2j\omega_0+k\omega_1+l\omega_2+m\omega_3}{\omega_0},$$

where j, k, l, and m are positive or negative integers (including zero). High order resonances may develop large amplitudes but only after many turns. When $\epsilon\leqslant 1/N$, the amplitude of the motion will grow to the value C after N turns. If r_0 is the mean radius of the orbit, A is some average of the initial r and z amplitudes due to the ω_1, ω_2, and ω_3 oscillations, and B is the amplitude of the r motion due to the azimuthal variation of the field, we find that,

$$N\sim\frac{Cr_0^{|j|+|k|+|l|+|m|-2}}{B^{|j|}A^{|k|+|l|+|m|-1}}.$$

Reasonable numerical values would be $A=C=2$ cm, $B=20$ cm, and $r_0=110$ cm.

In Table I the harmful resonances through fourth order are listed. The fourth row of the table gives N, the number of turns necessary to produce a catastrophe (also the sharpness of the required resonance), while the last row gives the value of the magnetic fall-off which would produce that resonance. [It will be noted that resonances where $l\neq 0$ are not listed. These resonances do occur and have been studied but since ω_2 is small compared with ω_0 and since it changes as the electrons acquire energy, they are never dangerous. The resonances leading to values of $n\geqslant 1$ or $n\leqslant 0$ have been omitted since for these, the motion is unstable even in first order.]

TABLE I. Harmful resonances.

	2nd order	3rd order			4th order			
j	0	0	−1	−1	0	0	−1	−1
k	−1	2	1	3	1	3	4	0
m	2	−2	2	0	−4	−2	0	4
N	50	3000	300	300	170,000	170,000	17,000	17,000
n	0.20	0.50	0.36	0.56	0.06	0.69	0.75	0.25

It is clear from Table I that for a magnetic fall-off of $n=0.56$ to 0.75 the orbits will be stable. The resonance at $n=0.69$ is so sharp as to be innocuous and for orders above the fourth, the resonances will be still sharper.

We therefore conclude that for suitable values of n the racetrack orbits will be stable. The reasoning has been based upon a perturbation treatment. The deviation of the racetrack from the circular synchrotron represents a rather large perturbation and hence the calculated values for N may be somewhat in error. The method is completely adequate, however, for it rigorously predicts all the values of magnetic fall-off for which instabilities may occur. Analogous calculations have been made for other types of periodic azimuthal variations and it is interesting to note that the stability is improved by going to a racetrack with four rather than two straight sections.

[1] For example, see D. M. Dennison and T. H. Berlin, Phys. Rev. (in print).

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Elastic Deficiency and Color of Natural Smoky Quartz

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IN a recent paper,[1] the writer has drawn attention to the fact that the elastic properties of quartz are altered by exposing the substance to radiation. The effective radiations include electron beams, deuterons, alpha-particles and x-radiation over the region from gamma-rays of radioactive origin up to at least 2.28A. The effect is conveniently investigated by means of peizoelectric quartz oscillator-plates. These offer a coupled electro-mechanical system that is sensitive to minute variations in the density and elasticity of the medium. During the irradiation of a quartz oscillator-plate, the frequency of oscillation decreases according to an exponential law until a saturation value is reached. This value appears to be independent of the type of radiation, but varies by a factor of at least 20 among different specimens of quartz. The quartz concomitantly becomes smoky in color, and the depth of color is proportional to the change in frequency. The several elastic moduli are affected unequally with the C_{14} modulus relatively highly responsive.[2] Baking the irradiated plates at temperatures over *ca.* 180°C or exposing them to short