A Spherical Shell Nuclear Model

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A DISCUSSION of the energies of the beta-, gamma-, and alpha-rays from the naturally radioactive elements led the writer[1] in 1933 to suggest that the nuclei of these elements may have sets of equally spaced energy levels with spacings of 0.387 Mev.

Recently Wiedenbeck[2] has reported equally spaced levels in Au, Ag, In, Cd, and Rh with spacings between 0.36 and 0.50 Mev. Equally spaced levels with separations about 0.4 Mev. are also found with N, Be, Al, and Co. It seems probable therefore that most atomic nuclei have a set of equally spaced levels with separations near to 0.4 Mev.

An account of a simple classical mechanical model of a nucleus, which has nearly equally spaced levels with spacings equal to about 0.4 Mev for all nuclei from beryllium to uranium, follows.

The model is a thin spherical shell which is supposed to be flexible, inextensible, and uniformly charged with positive electricity. The possible frequencies of vibration of the shell assuming its area to remain constant are equal to $(Ze/4\pi r)(Amr)^{-\frac{1}{2}}[n(n+1)-2]^{\frac{1}{2}}$ where Z = atomic number, A = mass number, r = radius of the shell, and $n=2, 3, 4, 5 \ldots$. The vibration with $n=2$ would not be excited by x-rays. m = mass of one nuclear particle.

This gives nearly equally spaced frequencies with spacing $(Ze/4\pi r)(Amr)^{-\frac{1}{2}}$ for large values of n.

Letting $r=r_1A^{\frac{1}{3}}$ and putting in the numerical values of e and m we get

$$r_1=2.46\times 10^{-13}(Z/A\Delta E)^{\frac{2}{3}},$$

where ΔE is the energy level separation in Mev corresponding to the frequency spacing for large values of n.

The values of $(Z/A\Delta E)$ with ΔE about 0.4 Mev are nearly equal to unity so that $r=2.46\times 10^{-13}A^{\frac{1}{3}}$ approximately.

Table I gives values of ΔE and r for several elements. The third column gives values of $1+2.3A^{\frac{1}{3}}$ which agree nearly with the values of r in most cases.

The value of r for uranium given by Gamow's theory is 9×10^{-13} and the value given by the target area for fast neutrons is 10×10^{-13}. Gamow's theory gives the radius of the potential hole inside the nucleus. On the shell model the radius should be equal to Gamow's radius plus the radius of the alpha-particle which may well be about 4×10^{-13}. This makes the shell radius 13×10^{-13} which agrees as well as could be expected with 15×10^{-13}. The value 10×10^{-13} for uranium from the target area for fast neutrons was obtained by putting the target area equal to $2\pi r^2$. Classically the target area would be πr^2 so the classical radius would be 14×10^{-13} which is nearly equal to 15×10^{-13}.

The threshold levels for x-ray excitation for Au, In, Cd, Ag, and Rh were measured by Wiedenbeck[2] and found to be nearly equal with an average value of 1.2 Mev. According to the shell model the threshold should be $(10)^{\frac{1}{2}}\Delta E$ which is about 1.3 Mev.

The formation of a shell instead of a sphere may be caused by saturation of the nuclear forces. Thus if we suppose each particle cannot be strongly attracted by more than four nearby particles, it is easy to see that the electrostatic forces should pull out a sphere into a shell.

The area of the shell $S=4\pi r^2$ must be determined by the number N of neutrons and the number Z of protons in it. We may suppose, for example, that $S=aZ-bN$ where a and b are positive quantities which vary slowly with $A=N+Z$.

TABLE I.

Element	ΔE	$r\times 10^{13}$	$(1+2.3A)^{1/3}\times 10^{13}$
Uranium	0.387	15.25	15.25
Silver	0.39	12.6	11.95
Aluminum	0.43	7.9	7.9
Beryllium	0.388	5.8	5.8

[1] H. A. Wilson, Phys. Rev. **44**, 858 (1933); Proc. Roy. Soc. **144**, 280 (1934).
[2] M. L. Wiedenbeck, Phys. Rev. **68**, 237 (1945).

β-Ray Spectrum of K^{40}

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FOR the theory of β-decay the highly forbidden transmutations are of the greatest importance. From this point of view the investigation of β-decay $K^{40}\rightarrow Ca^{40}$ must be very interesting, as the spin K^{40} is 4[1] and the spin Ca^{40} is 0. A large half-period of decay (1.4×10^{9} years[2]) with relatively great hardness of β-spectrum also indicates that the third-or the fourth-order exclusion takes place here.[3] The β-spectrum investigation is difficult because of the low activity of natural potassium: one cm^2 of potassium emits about 0.6 electron/sec. For this reason the shape of the β-spectrum K^{40} has not yet been investigated; only rough determinations of upper limit have been made. The results were obtained in a wide region of 0.7 to 1.3 Mev.[4]

We have investigated the β-spectrum K^{40} with a magnetic spectrometer of special design (Fig. 1). The apparatus placed in a box corresponded to six general β-spectrometers of the Danysz-type. The central counter was common for all the spectrometers, six other counters being situated each near their own potassium source. The number of coincidences between the central counter and the six other counters was measured with different magnetic fields. The counters were made of Al foil 20μ thick. $K_2C_2O_4$ powder $69 mg/cm^2$ thick, plated to a celluloid strip, served as the source of electrons. It is possible that our β-spectrum was distorted in the region of low energies, because of electrons scattering in the source, in the first counter or in the gas. To correct this inaccuracy, the spectrum of RaE was obtained by us under the same conditions. For this pur-