Radii of the Nuclei of Natural α-Emitters

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THE standard theories[1] of α-radioactivity based on the one-body rectangular hole potential appear to obtain only one relation between λ, the decay constant, E, the energy of the α-particle, and r_0, the radius of the nucleus. However in the course of their arguments, all these writers make an assumption involving the potential of the hole, *viz.*:

$$\mu k r_0 = \pi, \tag{1}$$

where $\mu=(1-U/E)^{\frac{1}{2}}$, U=constant potential for $r<r_0$, $k=(2mE)^{\frac{1}{2}}/\hbar$, m=mass of α-particle.

Sexl[2] has shown that a more rigorous treatment yields the following two formulae.

$$\mu = -\tan\alpha_0 \tan(\mu k r_0), \tag{2}$$

$$\lambda = \frac{2v}{r_0}\cdot\frac{\mu^2\tan\alpha_0}{\mu^2+\tan^2\alpha_0}\cdot\exp\{-\kappa(2\alpha_0-\sin 2\alpha_0)\}, \tag{3}$$

where $\alpha_0=\mathrm{arc}\cos(Er_0/2e^2Z)^{\frac{1}{2}}$, $\kappa=4e^2Z/hv$, v=velocity of α-particle, and Z=charge number of product nucleus. Equation (2) replaces (1) and accurate calculations using (2) and (3) show that $\mu k r_0$ is always less than π by about 0.2 radian, raising the values found for r_0 by about 5 percent.

Equations (2) and (3) hold only for zero spin difference between original and product nuclei. Sexl also gives formulae for $l\neq 0$, but they appear to be based on inaccurate approximations. The same holds true for all other formulae for $l\neq 0$.

It will be shown elsewhere that Sexl's result can be obtained in a simpler and more rigorous way and also generalized correctly for $l\neq 0$. In the meantime the results of calculations for some r_0's are given here.

The most recent calculations of nuclear radii are those given by Bethe.[1] The following criticisms can be raised against them. Firstly, they are derived from the less accurate theories. Secondly, they are not based on the new values of the fundamental physical constants.[3] Thirdly, in the case of an α-ray spectrum of more than one line, Bethe employs the total decay constant. But the λ used should refer to the probability of decay with the one energy E only; the error is often appreciable. Such cases are indicated by an asterisk in Table I where I have used the partial decay constants.

Table I compares the new values of r_0 with Bethe's and gives also the deepness B of the potential hole, calculated from (2) and (3). $B=(2Ze^2/r_0)-U$. The table contains only those α-rays for which nuclear spin difference is almost certainly zero. Values of λ and E are those found generally in tables,[4] except where indicated by a note. Birge's[3] values of e, m, h, etc., were used.

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[1] G. A. Gamow, *Constitution of Atomic Nuclei and Radioactivity* (Cambridge, 1937); H. A. Bethe, Rev. Mod. Phys. **9**, 161 (1937); F. Rasetti, *Elements of Nuclear Physics* (Prentice-Hall, Inc., New York, 1936); von Weizsäcker, *Die Atomkerne* (Akad. Verlagsgesellschaft, Leipzig, 1937).
[2] Sexl, Zeits. f. Physik **81**, 163 (1933).
[3] R. T. Birge, Rev. Mod. Phys. **13**, 233 (1941).
[4] Feather, *Nuclear Physics* (The Cambridge University Press, New York, 1936); Curie, *Radioactivité* (Hermann, Paris, 1935), Vol. II; Hoag, *Electron and Nuclear Physics* (D. Van Nostrand Company, Inc., New York, 1938); Stranathan, *The "Particles" of Modern Physics* (The Blakiston Company, Philadelphia, 1942); Lewis and Bowden, Proc. Roy. Soc. **145**, 235 (1934); International Commission, J. de phys. et rad. **2**, 273 (1931).

TABLE I. Values of r_0 and B. (The letters refer to the remarks below.)

Nuclei	Radii $\times10^{13}$ cm New	Radii $\times10^{13}$ cm Bethe	Total decay energy-Mev New	Total decay energy-Mev Bethe	λ-per sec. New	λ-per sec. Bethe	B-Mev	$R=r_0A^{-1/3}$ New	$R=r_0A^{-1/3}$ Bethe
UI-UXI	9.3_7	9.8	4.31[a]	4.15	4.82×10^{-18}[c]	5.0×10^{-18}	23.9	1.52	1.59
	9.3_4		4.32[b]				23.9	1.52	
UII-Io	9.2_6	9.6	4.89[a]	4.76	8.14×10^{-14}[c]	7×10^{-14}	25.6	1.51	1.57
	9.2_1		4.91[b]				25.7	1.51	
Io-Ra	9.2_6	9.5	4.80[d]	4.67	2.6×10^{-13}[e]	2.9×10^{-13}	23.1	1.52	1.56
*Ra-Rn	9.2_9	9.0	4.879	4.88	1.35×10^{-11}	1.42×10^{-11}	22.3	1.53	1.49
Rn-RaA	9.2_8	9.0	5.589	5.59	2.097×10^{-6}	2.10×10^{-6}	21.0	1.54	1.50
RaA-RaB	9.1_4	9.0	6.112	6.11	3.78×10^{-3}	3.8×10^{-3}	20.3	1.53	1.51
RaF-RaG	8.2_7	8.2	5.410[f]	5.40	5.886×10^{-8}[f]	5.7×10^{-8}	23.8	1.40	1.39
Th-MThI	9.9_2	8.7	3.99[g]	4.34	1.2×10^{-8}	1.2×10^{-8}	22.0	1.62	1.42
	10.0_1				1.7×10^{-8}[h]		21.8	1.64	
*RdTh-ThX	9.3_3	8.8	5.517	5.52	9.7×10^{-9}	1.15×10^{-8}	22.1	1.54	1.45
ThX-Thn	9.2_9	9.0	5.786	5.79	2.20×10^{-6}	2.20×10^{-6}	21.4	1.54	1.49
Thn-ThA	9.2_8	9.1	6.400	6.40	1.27×10^{-2}	1.27×10^{-2}	20.2	1.55	1.52
ThA-ThB	9.1_2	8.9	6.904	6.90	4.95	5.0	19.5	1.53	1.49
*ThC-ThC″	7.5_7	7.0	6.1607	6.20	4.65×10^{-5}	6.7×10^{-5}	25.4	1.28	1.18
Pa-Ac	8.1_8	8.6	5.24[d]	5.16	6.7×10^{-13}[i]	6.9×10^{-13}	27.2	1.34	1.41
*An-AcA	8.7_0	8.5	6.953	6.95	1.22×10^{-1}	1.77×10^{-1}	21.5	1.45	1.42
AcA-AcB	8.9_9	8.8	7.508	7.51	3.47×10^{2}	3.5×10^{2}	19.3	1.51	1.48
*AcC-AcC″	7.9_0	7.3	6.739	6.74	4.48×10^{-3}	5.3×10^{-3}	24.3	1.33	1.23

[a] Feather, see reference 4.
[b] From values of range given by Sizoo, Physica **4**, 791 (1937) and graphs of range against energy given by M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 266 (1937). Value then corrected to include recoil energy, since the tabulated value is total energy.
[c] A. O. Nier, Phys. Rev. **55**, 150 (1939).
[d] Obtained as in (*b*), using range given by Mme. Curie, reference 4.
[e] Hernegger, Akad. Wiss. Wien **143**, 367 (1934).
[f] W. Y. Chang, Phys. Rev. **69**, 60 (1946).
[g] Obtained as in (*b*), using range given by G. H. Henderson and G. C. Laurence, Phys. Rev. **52**, 46 (1937).
[h] 1.7×10^{-8} is given by Feather reference 4 and by Stranathan, reference 4.
[i] Stranathan, reference 4.