Reciprocal Electric Force

F. W. Warburton
University of Kentucky, Lexington, Kentucky
December 20, 1945

THE force of one electrical charge on another may be obtained from an assumed potential energy W expressed in ascending powers of $1/c$ and the relative velocity $\mathbf{u}=\mathbf{v}-\mathbf{v}'$ of the charges e and e',

$$W=\frac{ee'}{r}\left[1+A\frac{u^2}{c^2}+B\frac{(\mathbf{u}\cdot\mathbf{r})^2}{c^2r^2}+C\frac{u^2(\mathbf{u}\cdot\mathbf{r})}{c^3r}+D\frac{(\mathbf{u}\cdot\mathbf{r})^3}{c^3r^3}+\cdots\right], \quad (1)$$

with coefficients $A, B, C, D, \cdots$ to be determined by experiment. When W is inserted in the Lagrangian equation of motion, the reciprocal force of e' on e is readily found to be

$$\begin{aligned}\mathbf{F}=ee'\Bigg[&\frac{\mathbf{r}}{r^3}+(1-A)\frac{\mathbf{r}u^2}{c^2r^3}-3(\tfrac{1}{2}-A)\frac{\mathbf{r}(\mathbf{u}\cdot\mathbf{r})^2}{c^2r^5}\\ &-2A\frac{\mathbf{u}(\mathbf{u}\cdot\mathbf{r})}{c^2r^3}+2(\tfrac{1}{2}-A)\frac{\mathbf{r}(\mathbf{g}\cdot\mathbf{r})}{c^2r^3}+2A\frac{\mathbf{g}}{c^2r}\\ &+2C\left\{\frac{\mathbf{u}u^2}{c^3r^2}-2\frac{\mathbf{u}(\mathbf{u}\cdot\mathbf{r})^2}{c^3r^4}+\frac{\mathbf{u}(\mathbf{g}\cdot\mathbf{r})}{c^3r^2}+\frac{\mathbf{g}(\mathbf{u}\cdot\mathbf{r})}{c^3r^2}+\frac{\mathbf{r}(\mathbf{u}\cdot\mathbf{g})}{c^3r^2}\right\}\\ &+2D\left\{3\frac{\mathbf{r}u^2(\mathbf{u}\cdot\mathbf{r})}{c^3r^4}-4\frac{\mathbf{r}(\mathbf{u}\cdot\mathbf{r})^3}{c^3r^6}+3\frac{\mathbf{r}(\mathbf{u}\cdot\mathbf{r})(\mathbf{g}\cdot\mathbf{r})}{c^3r^4}\right\}+\cdots\Bigg], \quad (2)\end{aligned}$$

where $\mathbf{g}=\mathbf{f}-\mathbf{f}'$ is the relative acceleration of the charges e and e', and B has been determined in terms of A by applying the equation to the pairs of charges making up dipole elements of two currents and integrating for closed circuits.

Neglecting terms in $1/c^3$, we find Eq. (2) reduces to the Weber force if $A=0$, reduces to the Riemann force if $A=\frac{1}{2}$, omits an undesirable term in u^2 if $A=1$, and except for the acceleration terms gives the Ritz formula which is based on ballistic emission. For relative velocity $\mathbf{u}$ and relative acceleration $\mathbf{g}$ directed along r, it reduces to

$$F=ee'\left[\frac{1}{r^2}-\frac{u^2}{2c^2r^2}+\frac{g}{c^2r}+\cdots\right]. \quad (3)$$

As ee'/c^2r is small compared to the mass m of the electron e, Eq. (3) becomes approximately $F=(ee'/r^2)(1-u^2/c^2)^{\frac{1}{2}}=mg$, and the "apparent" force $F'=ee'/r^2=mg/(1-u^2/c^2)^{\frac{1}{2}}$ gives the apparent increase in mass, $m'=m/(1-u^2/c^2)^{\frac{1}{2}}$. The acceleration term, $F_g=ee'g/c^2r=W_sg/c^2=m_sg$ yields an electrostatic mass-energy relation $W_s=m_sc^2$ for the group of charges comprising a body, to which an external charge gives an average acceleration g transmitted from one charge to another in the group by their mutual electrostatic forces. And there is the possibility that nuclear attractions may be expressed as a type of electrostatic van der Waal or resonance force, whence nuclear energy may be expected to fall into this type of mass-energy equivalence.

Equation (2) may be used to describe the change in magnetization of a rod by the passage of longitudinal current through the rod. For the idealized case of a long filament of current i' acting on a circular electronic orbit of radius ρ, let the z axis be parallel to the filament and pass through the center of any orbit, and let the magnetic axis of the orbit lie in the xz plane making an angle α with z. Let β be the angle between the x axis and b, the distance from the orbit to the current filament, and let ϕ give the angular position of the electron e in the orbit. The factor $n'e'v'/c$ is replaced by $i'dz$ and each term of Eq. (2) integrated over the long filament. The term expressing the resulting large periodic force proportional to the central acceleration v^2/ρ of the orbital electron

$$\begin{aligned}-(2C+3D)\frac{ei'v^2\pi}{c^2\rho}[&\mathbf{i}\cos\phi\sin\alpha\cos\beta+\mathbf{j}\cos\phi\sin\alpha\sin\beta\\ &-\mathbf{k}(\cos\phi\cos\alpha\cos\beta+\sin\phi\sin\beta)]\end{aligned}$$

can evidently account for the change in magnetization in the rod known as "shock effect." Also the average torque of the conduction current i' on the orbit in the direction of increasing α, becomes

$$\begin{aligned}L_\alpha=&\frac{ev}{c}\frac{2i'\rho}{b}\cos\alpha\sin\beta\\ &+(C+3D/2)\frac{ev^2i'\pi}{c^2}(\cos^2\alpha-\sin^2\alpha)\cos\beta\\ &+(C+3D/2)\frac{ev}{c^2}\frac{di'}{dt}\frac{\pi}{b}\rho\cos\alpha\sin\beta. \quad (4)\end{aligned}$$

The first term in Eq. (4) gives the standard torque producing circular flux in the rod due to the longitudinal current. $\cos^2\alpha-\sin^2\alpha$ in the large second term is positive for orbital alignment along z or $-z$. Ordinarily $\cos\beta$ vanishes by symmetry. It is suggested that a very slight dissymmetry due to the longitudinal wedge cut in the specimen to reduce the circular flux, would give a minute average value of $\cos\beta$ and account for the effect Perkins[1] observed in nickel which reverses with reversal of current in the rod.

The symmetric reciprocal force provides a direct and unified description of electromagnetism, while the conventional treatment with its necessary relativity corrections built to fitt he classical non-relative, non-reciprocal magnetic force, $e\mathbf{v}\times e'\mathbf{v}'\times\mathbf{r}/c^2r^3$, maintains a force, $e\mathbf{v}\times\mathbf{H}/c$, which is not complete but which requires "other" quantities of the same magnitude depicted as rate of change in field momentum and as relativistic increase in mass, and which denies action of e' on e proportional to acceleration of e. Equation (2) permits ballistic emission from an oscillating dipole source, whose mix-up-in-phase at angles other than normal to the dipole (where emission speed is c) provides a physical reason for the mathematical expression for the transverse nature of light, which is not present in conventional theory. It conforms to operational theory in that it assigns meaning only at the source and at the absorber where the measurements are made, but it does not preclude description of reaction on the source and action on the absorber by some mechanism which may be developed later to describe transmission from e' to e.

[1] Henry A. Perkins, Phys. Rev. **66**, 21 (1944).