should be its "stablest" member, the disintegration energies of its two radioactive neighbors being about equal. Heavier members of the sequence will tend to have too high a charge for greatest stability so that the disintegration energy of the negatron-emitting neighbor will tend to be less as the mass increases, reaching zero at the end of the stable sequence. On the other hand the disintegration energy of the positron-emitting neighbor should increase as the mass increases. The reverse situation should hold on the low mass side of the "stablest" member of the sequence.

Figure 8 shows a schematic representation of the disintegration energies. It appears that the stablest mass number is 52 or 56. For heavier mass numbers the negatron energies fall off smoothly, reaching zero just beyond the known limit of stability. Less evidence is available concerning the positron-emitters on the light side of the maximum. One might expect their disintegration energies to follow the dotted line shown in Fig. 8. This curve indicates that the positrons emitted by Cl³⁶ should be very soft. Also the electron capture process by K⁴⁰ which leads to an excited state of A⁴⁰ at about 2 Mev should have a very low transition energy, so that no positrons could be emitted. This seems to be in agreement with experimental evidence.

Unfortunately no experimental points are available beyond the point of greatest stability. The tentative disintegration scheme for V^{52} , shown in Fig. 7, would indicate a bending-over of the negatron branch, contrary to expectation. Some irregularities in the dependence of the disintegration energy on mass number may be introduced by the fact that the disintegrating state is not necessarily the ground state of the radioactive nucleus. Of the sixteen possible radioactive neighbors of the eight stable nuclei, eleven have been definitely identified.18 Three of these, Sc44, Mn52, and Co60 have been found in two isomeric states. One might expect others to exist in low-lying, relatively long-lived states, and it is not known whether the radioactive states actually observed correspond to the higher or lower member of an isomeric pair.

We wish to thank Professor Robley D. Evans for his continued interest in these studies; Professor M. S. Livingston and the cyclotron crew for making the bombardments; and Professor John W. Irvine, Jr. for help with chemical procedures.

¹⁸ G. T. Seaborg, Rev. Mod. Phys. 16, 1 (1944).

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Experimental Test of Beta-Ray Theory for the Positron Emitters Na²², V⁴⁸, Mn⁵², Co⁵⁸

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The positron branching ratio $\lambda_+/(\lambda_c+\lambda_+)$ has been measured by a coincidence counting method for four isotopes in which radioactive decay involves competition between positron beta-rays and orbital electron capture. The results are ${}_{11}Na^{22}:1.00\pm0.05$, ${}_{23}V^{48}: 0.58\pm0.04$, ${}_{26}Mn^{52}:0.35\pm0.02$, ${}_{27}Co^{58}:0.145\pm0.005$. It is shown that these values, as well as the observed shapes of the beta-ray spectra, and the observed lifetimes are in agreement with predictions of the beta-ray theory for nuclear angular momentum changes $\Delta I=0$ or $\Delta I=\pm1$. The type of interaction remains uncertain, as does the parity change, except that the scalar interaction gives consistent results if one assumes the parity change in every case except Mn^{52} .

INTRODUCTION

A NUMBER of attempts have recently been made to refine the theory of beta-decay and

to verify it experimentally.¹ In general the conclusion is drawn that the available experimental data can be explained by the Fermi theory which assumes an interaction between the nucleons and the electron-neutrino field depending

¹ E. J. Konopinski, Rev. Mod. Phys. 15, 209 (1943).

^{*} Based in part upon a thesis submitted by Wilfred M. Good to the faculty of the Massachusetts Institute of Technology in partial fulfillment of the requirements for the degree of Doctor of Philosophy in 1944.



only on the position of the particles. It seems preferable, though, to replace the original vector form of the interaction proposed by Fermi by some other form, particularly the tensor form, of combining the wave functions. This has the effect of changing the matrix elements and the energy distribution for particular changes in angular momentum and parity combinations. The behavior of "allowed" transitions is the same for all forms of the Fermi theory. The so-called K-U theory, in which the interaction depends on the derivatives of the wave functions, is ruled out by experiment. The experimental evidence available to test the theory in detail is still inadequate.

Two types of information that are usually considered are the energy distribution of the beta-particles, and the relationship between the half-life and the maximum energy of the particles. Lawsons careful measurement² on the beta-ray spectrum of In¹¹⁴ has shown excellent agreement with theory in the case of what is undoubtedly an "allowed" transition. This is powerful experimental support for the Fermi theory, but it does not distinguish between the several forms of the interaction. Of the forbidden spectra, only RaE and possibly P32 have been studied with sufficient precision to allow comparison with theory. The results, although not conclusive, indicate that either the tensor form or Fermi's original polar vector interaction yields fair agreement.3

There is only one beta-disintegration for which the angular momenta of the initial and final states are known with certainty, namely K40. The disintegrations of Rb⁸⁷, Lu¹⁷⁶, and Be¹⁰ which are sometimes considered in this connection involve other states besides the ground states of the nuclei concerned, since gamma-rays are emitted. The observed lifetime and maximum beta-ray energy of K40 are consistent with either the tensor or the polar-vector interactions, although the tensor interaction seems favored.

In the case of positron emitters there is a third possibility of testing the theory, namely a determination of the relative values of the probabilities of positron emission, λ_+ , and orbital electron capture, λ_c . In favorable cases, such an experiment involves a minimum of difficulties and uncertainties of technique. It permits a differentiation between various forms of the theory to about the same extent as a careful measurement of the beta-ray energy distribution. This can be understood qualitatively if we consider positron emission in a formal manner as capture of electrons from negative energy states. Thus we compare the integral of the capture probability for the continuum of electron negative energy states with the integral—or rather the sum-over the discrete bound atomic states. On the other hand a measurement of the beta-ray spectrum compares the probabilities for various intervals of the continuum.

We have measured the ratio, λ_{+}/λ_{c} , of the positron emission and electron capture decay constants for four radioactive substances. This quantity together with the total decay constant (half-life), the beta-ray energy distribution, and the maximum positron energy, comprise a set of data for comparison with theory.

The disintegration schemes of the substances discussed in this paper were investigated in this laboratory. Co⁵⁸ has been reported by Deutsch and Elliott,⁴ Mn⁵² and V⁴⁸ by Peacock and Deutsch.⁵ The results on Na²² have not been reported previously. The gamma-ray spectrum of Na²² showed only one gamma-ray of 1.30 ± 0.03 Mev energy besides the annihilation radiation.

² J. L. Lawson and J. M. Cork, Phys. Rev. 57, 982

<sup>(1940).
*</sup>E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. 60, 308 (1941); Hereafter referred to as K-U II.

⁴ M. Deutsch and L. G. Elliott, Phys. Rev. 65, 211

^{(1944).} ⁶ W. C. Peacock and M. Deutsch, Phys. Rev. 69, 306 (1946).

This is in agreement with the result of Oppenheimer and Tomlinson.⁶ A Fermi plot of the positron spectrum is shown in Fig. 1. The maximum energy is 0.575 Mev. Coincidence measurements proved that there is one 1.30 Mey gamma-ray accompanying each positron.7 Details of the methods employed have been published in several papers dealing with the study of disintegration schemes. Figure 2 summarizes the disintegration schemes of the four substances including the branching ratios reported in this paper. It should be mentioned that there is no positive proof for the absence of orbital electron capture transitions of V⁴⁸ to the intermediate excited state of Ti⁴⁸ (such as we have in the case of Mn⁵²) but the absence of high energy positrons is a rather convincing argument.

EXPERIMENTAL METHOD

The method used to determine the ratio $\lambda_{\pm}/\lambda^{c}$ of the partial decay constants consists essentially in measuring what fraction of all the gamma-rays emitted is due to annihilation radiation. It has been shown previously that in every case discussed in this paper, except V48 and probably for it also, all of the disintegrations, whether by positron emission or by electron capture, lead to the same excited state of the product nucleus. That is, the positron spectra and the "capture spectra" are simple. Therefore we limit our discussion of the method to this case, although it can be extended to more complicated decay schemes.

Consider a sample of the substance under investigation surrounded by enough material to stop all of the positrons in a small space. At some distance from the source there is placed a gamma-ray counter whose efficiency is known as a function of gamma-ray energy. The method for determining this relationship has been discussed elsewhere.8 Let the efficiency for detecting an annihilation quantum be e_a and the total efficiency for detecting the nuclear gamma rays emitted in a disintegration (on the average, if

there is gamma-ray branching) be Σe_{γ} . Let the disintegration rate of the source be N_0 per minute. Then the counting rate will be

$$N = N_0(\Sigma e_\gamma + 2e_a R), \tag{1}$$

where *R* is the branching ratio for positron decay!

$$R = \lambda_+ / (\lambda_c + \lambda_+). \tag{2}$$

The counting rate due to annihilation quanta only is

$$N_a = 2N_0 Re_a. \tag{3}$$

From (1) and (3) we find

where

$$R = k \Sigma e_{\gamma} / 2 e_a (1-k), \qquad (4)$$

$$k = N_a / N. \tag{5}$$

To determine k we make use of the fact that the two annihilation quanta are known to be emitted exactly in opposite directions.9 Thus consider the arrangement shown in Fig. 3 in which the two similar gamma-ray counters are connected to a coincidence circuit and may be placed either in collinear position (A) or at right angles with respect to the source (B). Then, except for some minor corrections which are discussed below, the difference in the coincidence counting rates in the two positions is due entirely to annihilation



FIG. 2. Disintegration schemes of the substances investigated.

⁹ R. Beringer and C. G. Montgomery, Phys. Rev. 61, 222 (1942).

⁶ F. Oppenheimer and E. P. Tomlinson, Phys. Rev. 56, 858 (1939).

⁷ Since the first draft of this paper was written, a concordant result has been published by H. Maier-Leibnitz, Zeits. f. Physik 122, 233 (1944).
⁸ A. Roberts, L. G. Elliott, J. R. Downing, W. C. Peacock, and M. Deutsch, Phys. Rev. 64, 268 (1943).

(8)

quanta. Call this difference the *net* coincidence rate C_a . Then the net coincidence rate per annihilation quantum recorded in one of the counters is

$$C_a/N_a = \eta_a. \tag{6}$$

We call η_a the *intrinsic* efficiency for detecting an annihilation quantum

$$\eta_a = e_a/\omega, \tag{7}$$

where ω is the solid angle subtended by one of the counters at the source. With a source emitting nuclear gamma-rays as well as annihilation radiation, the net coincidence rate per recorded gamma-ray count (due to either nuclear or annihilation radiation) is

 $C_a/N=k\eta_a,$

$$k = (C_a/N)/\eta_a. \tag{8'}$$

 C_a/N is a directly observed quantity. In order to determine η_a directly we need a positron emitter without nuclear gamma-rays so 'that (8') reduces to (6). Such a substance is Cu⁶⁴. We have made a careful search for nuclear



FIG. 3. Arrangement of counters for the determination of the number of positrons.

gamma-rays of Cu⁶⁴, using the magnetic lens beta-ray spectrometer and confirm the results of other observers¹⁰ that none are emitted. Thus k=1 for Cu⁶⁴ and η_a can be determined directly.

The gamma-ray counters used in the arrangement in Fig. 3 were seamless copper tubing, 2.50 cm in diameter and 12 cm long, plated with 130 grams of bismuth. These counters were filled with $\frac{1}{2}$ percent water vapor and helium to atmospheric pressure. Figure 4 shows η_a as a function of the distance between source and front end of the cathode. The variation can be readily interpreted by considering the angle of incidence of the gamma-rays on the cathode. The distance actually used in these experiments was 11 cm. The dependence of e_{γ} on energy for these counters and this geometry was found by comparison with the standard Pt counters used in this laboratory⁸ and is shown in Fig. 5. A small correction for absorption in the source was applied where necessary.

As mentioned above, certain small corrections must be applied in determining the net coincidence rate C_a . In general we have four causes for coincidences in position A: (1) Annihilation pairs of quanta, (2) coincidences of nuclear gamma-rays with each other or with annihilation quanta, (3) cosmic-ray coincidences, (4) chance coincidences.

In position B coincidences occur due to all but the first of these causes. The rate of chance coincidences is certainly unaffected by the position. The cosmic-ray coincidence rate is slightly higher in position B, but this effect was made negligible by the strength of the sources used. Effect (2) is expected to be independent of position according to all reliable experimental evidence available. Even if there should be an angular correlation of nuclear gamma-rays the effect on our experiment can be made negligible by making the source-counter distance large enough, i.e., by making the ratio $\eta_a/\Sigma e_{\gamma}$ sufficiently large. The cosmic-ray coincidence rate is also reduced by this procedure. Thus it is advantageous to make the distance between source and counters as large as possible with reasonable source strength. A typical set of results, for Co⁵⁸ is shown in Table I.

or

¹⁰ A. W. Tyler, Phys. Rev. 56, 135 (1939).



FIG. 4. Intrinsic efficiency of the gamma-ray counters for annihilation radiation vs. distance from source.



of gamma-ray energy.

EXPERIMENTAL RESULTS

In Table II we summarize the experimental results for the partial decay constants. In the last two columns we give the total decay constants and the disintegration energies including the rest energy of the positron. The Fermi plot for Na²² is given in Fig. 1 and the Fermi plots for V⁴⁸, Mn⁵², Co⁵⁸ are given in references 4 and 5.

COMPARISON WITH THEORY

For our purpose the tensor formulation of the Fermi theory as given in K-U II³ seems better

suited than e.g., the spherical harmonic formulation of Marshak.¹¹ We shall follow the notation of K-U II except where otherwise indicated, and the reader is referred to that paper for the meaning of symbols, selection rules, etc. In this notation one writes for the emission probability of a positron with momentum p, due to any one matrix element M (with the appropriate selection rules)

$$P(p)dp = |M|^{2}(G^{2}/2\pi^{3})C(-Z') \times F(-Z', p)p^{2}K^{2}dp, \quad (9)$$

where Z' is the atomic number of the *product* nucleus, F is the "Fermi function," C the appropriate "correction factor," and G is a universal constant. The total decay probability by positron emission is then

$$\lambda_{+} = |M|^{2} (G^{2}/2\pi^{3}) B_{+}, \qquad (10)$$

$$B_{+} = \int_{0}^{r_{0}} C(-Z') F(-Z', p) p^{2} K^{2} dp, \quad (10')$$

where $p_0 = (W_0^2 - 1)^{\frac{1}{2}}$ is the maximum momentum of the particles and $K = (W_0 - W)$ is the neutrino energy. The correction factors C are given in K-U II, except for the inclusion of the matrix element IMI² which we write as a separate factor in (9) and (10).

¹¹ R. E. Marshak, Phys. Rev. 61, 431 (1942).

TABLE I. Typical experimental values.

Net counting rate A	1660 ± 30 counts per minute
Net counting rate B	1690 ± 30 counts per minute
Coincidence rate A	$8.50 \pm .24$ counts per minute
Coincidence rate B	$0.42 \pm .02$ counts per minute
Chance coincidences	$0.29 \pm .03$ counts per minute
Cosmic-ray pos. A	<0.05 counts per minute
Cosmic-ray pos. B	<0.06 counts per minute

We can write similar formulas for orbital electron capture¹²

$$\lambda_c = |M|^2 (G^2/2\pi^3) B_c, \qquad (11)$$

where M and G have the same values as in (10) for the same transition

$$B_c = (\pi/2)(W_o + W_n)^2 \Sigma_n (N_n/2n^2) C^{(n)}(Z_n) F_c, (11')$$

$$F_{c} = (1/2)(2\alpha Z_{n})^{3-(\alpha Z_{n})^{2}} \rho^{-(\alpha Z_{n})^{2}} \exp((-2\alpha Z_{n}\rho)).$$
(12)

 B_c represents a summation over the various shells with radial quantum numbers n. N_n is the number of electrons actually present in the nth shell. The summation over the various subshells with orbital angular momentum quantum numbers l is made by the correction factors $C^{(n)}$ which have the same form as those for the continuous spectrum given by K-U II, except that the neutrino energy K now has the value (W_0+W_n) and the radial wave functions of the bound atomic electrons are used instead of those for the continuum. Thus

$$L_{l^{(n)}} = (F_{c})^{-1} \frac{g_{l^{(n)}}^{2} + f_{-l-2^{(n)}}^{2}}{\rho^{2l}},$$
(13a)

$$M_{l^{(n)}} = (F_{c})^{-1} \frac{f_{l^{(n)}}^{2} + g_{-l-2^{(n)}}^{2}}{e^{\rho^{2l+2}}},$$
(13b)

$$N_{l}^{(n)} = (F_{c})^{-1} \frac{f_{l}^{(n)}g_{l}^{(n)} + f_{-l-2}^{(n)}g_{-l-2}^{(n)}}{\rho^{2l+1}}.$$
 (13c)

 $\rho = 0.0038 \mathrm{A}^{\frac{1}{2}}$ is the nuclear radius. Z_n is the effective charge of the *initial* nucleus for the *n*th shell: $Z_K = Z - 0.30$, $Z_L = Z - 4.15$, etc.¹³ $W_n = [1 - (\alpha Z_n/n)^2/2]$. If the capture probabilities for the separate shells or subshells are wanted, only the terms involving the corre-

sponding wave functions should be included in Eqs. (11) and (13). These wave functions in approximation good to terms of the order $(\alpha Z)^2$ have the following values at the surface of the nucleus.¹⁴ For the shells K, $L_{\rm I}$, $L_{\rm III}$, $M_{\rm I}$, $M_{\rm III}$, $M_{\rm V}$, etc.

$$\frac{g_{l^{(n)^{2}}}}{\rho^{2l}} = \frac{(n+l)!}{2n(n-l-1)![(2l+1)!]^{2}} \times (2\alpha Z_{n}/n)^{2l+3-(\alpha Z_{n})^{2}/l+1} \rho^{-(\alpha Z_{n})^{2}/l+1}}$$

$$\times \exp\left(-2\alpha Z_n \rho/n\right), \quad (14a)$$

$$\frac{f_{l^{(n)}}^{2}}{\rho^{2l+2}} = (\alpha Z_{n}/2\rho)^{2}(1/l+1)^{2}\frac{g_{l^{(n)}}^{2}}{\rho^{2l}}$$
(14b)

and for the shells L_{II} , M_{II} , M_{IV} , etc.

$$\frac{f_{-l-2}^{(n)^2}}{\rho^{2l}} = \rho^2 \frac{n^2 - (l+1)^2}{n^2} \frac{f_{l}^{(n)^2}}{\rho^{2l+2}},$$
(14c)

$$\frac{g_{-l-2}^{(n)^2}}{\rho^{2l+2}} = \left[(\alpha Z_n/2l)^2 + 2(\alpha Z_n/2\rho) \right] \frac{f_{-l-2}^{(n)^2}}{\rho^{2l}}.$$
 (14d)

In most cases up to second forbidden transitions the single term with n = 1 (*K*-capture) contributes over 90 percent of the total capture probability unless the transition energy is very low.

In Table III we have collected the values of B_+ and B_c calculated from Eqs. (10') and (11') for the four transitions investigated, and for the matrix elements of interest. Since the factors multiplying the integral or sum represented by B are the same in Eqs. (10) and (11), the ratio $B_+/(B_c+B_+)$ can be compared directly with the corresponding value of $\lambda_+/(\lambda_c+\lambda_+)$ in Table II. Keeping in mind the selection rules for the various matrix elements, as given in K-U II, it appears immediately that only for transitions with changes in the nuclear angular momentum of $\Delta I=0$ or $\Delta I=\pm 1$ does the theory predict a ratio of the partial decay constants reasonably close to the observed value. The "allowed" term

TABLE II. Experimental partial decay constants.

Nucleus	$\Sigma e_{\gamma}/e_a$	k	$\lambda_+/(\lambda_c + \lambda_+)$	$\lambda_{e} + \lambda_{+}$	W_0	
Na ²²	1.54	0.570	$\begin{array}{c} 1.00 \ \pm 0.05 \\ 0.58 \ \pm 0.04 \\ 0.35 \ \pm 0.02 \\ 0.145 \ \pm 0.005 \end{array}$	8.7 ×10 ⁷ sec. ⁻¹	2.12 mc ²	
V ⁴⁸	2.86	0.289		1.4 ×10 ⁶ sec. ⁻¹	2.42 mc ²	
Mn ⁵²	4.08	0.146		5.6 ×10 ⁵ sec. ⁻¹	2.15 mc ²	
Co ⁵⁸	1.18	0.202		5.6 ×10 ⁶ sec. ⁻¹	1.92 mc ²	

¹⁴ E. L. Hill and R. Landshoff, Rev. Mod. Phys. 10, 106 (1938).

¹² After these calculations were completed it came to our attention that a similar treatment was given by E. Greuling, Thesis Indiana University, 1942.

¹³ E. Greuling, Phys. Rev. 61, 568 (1942).

Inter-	Matrix		Na ²²	B+		V48	B+		Mn ⁵²	B+		Co ⁵⁸	B+	Spec- trum
action	elements	B+	Be	B_++B_c	B+	Be	B_++B_c	B+	Be	$\overline{B_+ + B_c}$	B+	Be	B_++B_c	Fig. 6
Tensor Scalar Vector Axial v.	$\int \sigma, \int \alpha$ $\int 1$ $\int 1, \int \alpha$ $\int \sigma, \int \gamma$	0.37	0,0295	0.93	0.54	0.43	0.56	0.205	0.36	0.36	0.075	0.410	0.155	Ъ
Tensor Tensor Scalar Vector Axial v. Axial v. Vector	$\int \sigma \cdot r, \ \int \alpha \cdot r$ $\int \sigma \times r$ $\int r$ $\int \sigma \\ \int \sigma \times r$ $\int \sigma \times r$ $\int \alpha \times r$ $\int \alpha \times r$	4.67 3.50	0.70 0.262	0.87 0.94	19.5 16.0 19.8 16.3 16.0 19.6	24.0 11.3 25.2 12.2 11.1 24.2	0.45 0.59 0.44 0.57 0.59 0.45	6.75 5.7 6.90 5.95 5.7 6.8	17.8 9.0 18.5 9.6 8.8 18.0	0.28 0.39 0.27 0.38 0.39 0.27	3.30 2.92	26.1 15.2	0.112 0.161	a b a b a
Tensor Vector	$\Sigma B_{ij}, \Sigma A_{ij}$ ΣA_{ij}	0.05	0.025	0.67	0.11	0.42	0.21	0.031	0.29	0.097	0.008	0.27	0.029	c
Axiai v.	ZDij													
Tensor	ΣT_{ij}							0.5	8.5	0.055				c
Tensor	ΣSijk							7×10⁻⁴	0.03	0.022				d

TABLE III. Types of spectra and values of the theoretical qualities B_+ , B_c for different interactions and matrix elements.

—identical of all interactions—yields theoretical ratios in satisfactory agreement with observation in all cases investigated. However, as we shall see presently, there is strong evidence from the lifetimes that at least some of the transitions considered are "forbidden."

In the last column of Table III we indicate the theoretically predicted shape of the positron spectrum as calculated from Eq. (9). The letters refer to the curves in Fig. 6 which show the Fermi plots predicted by theory for transitions of Mn⁵² involving various matrix elements. This method of presenting the theoretical spectra, i.e., a plot of $(CK^2)^{\frac{1}{2}}$ vs. W has the advantage of being readily compared with the experimental results as they are usually shown. The trend of the predicted spectra for the other transitions is very similar to that for Mn⁵². Matrix elements marked with the same letter in the last column of Table III do not necessarily give rise to identical spectra but rather the predicted distributions are indistinguishable by present techniques. The observed spectra as shown in Fig. 1 and references 4 and 5 are definitely of type bexcept possibly in the case of Co⁵⁸ where the experimental uncertainty is too great to allow a distinction between types a and b. This again

shows that transitions with $\Delta I = \pm 2$ or more are excluded. Furthermore the single "first forbidden" term $\int r$ of the scalar interaction predicts too small a relative probability of positron emission and a spectrum of type a and is therefore excluded by experiment. All other interactions involve several terms in the first forbidden approximation and by proper adjustment of the several matrix elements agreement with experiment can be obtained for transitions with $\Delta I = 0$ or $\Delta I = \pm 1$. Since other evidence¹ seems to favor the tensor interaction it is of particular interest to see whether a reasonable estimate of the several matrix elements yields the observed ratio of the partial decay constants and the observed shape of the spectrum. For $0 \rightarrow 0$ transitions to which only the matrix element $\int \sigma \cdot r$ contributes, the predicted relative probability of positron emission is too small in all cases. For other transitions with $\Delta I = 0$ we have a combination of four matrix elements. Greuling¹³ has made an estimate of the magnitudes of these matrix elements. For three of them he finds $|\int \sigma \cdot r|^2$ $\approx |\int \sigma \times r|^2 \approx |\Sigma B_{ij}|^2 \approx 2\rho^2$. The magnitude of $|\int \alpha|^2$ is rather more uncertain. It appears that it should be about ten times as large as the others. Using these estimates we can calculate the



FIG. 6. Fermi plots predicted by theory. See Table III.

predicted ratio $\lambda_+/(\lambda_c+\lambda_+)$ by summing, respectively, the four terms $|M|^2B_+$ and $|M|^2B_c$. These sums should be proportional to the λ 's. For $\Delta I = \pm 1$ only three matrix elements $\int a_{i}$ $\int \sigma \times r$, and ΣB_{ij} contribute to the decay probability. In Table IV we summarize the predicted values of $\lambda_+/(\lambda_c+\lambda_+)$, together with the observed values. It will be seen that the values for $\Delta I = \pm 1$ are, generally, in better agreement with experiment than those for $\Delta I = 0$. The predicted spectra, in both cases are of type b. We can use Greuling's estimates of the matrix elements to calculate the predicted lifetimes for $\Delta I = 0$ or $\Delta I = \pm 1$. For this it is necessary to make some assumptions concerning the lifetime for allowed transitions, for which the matrix element is conventionally taken as unity. Equations (10) and (11) apply strictly only to transitions within the same supermultiplet, i.e., practically only to very light elements because of the greater complexity of heavy particle states in heavier nuclei. We shall, therefore, follow the empirical approach of Konopinski¹ which essentially replaces $(G^2/2\pi^3)$ in Eqs. (10) and (11) by a different constant for each region of the periodic table, determined by the half-lives of the most nearly "allowed" transitions in that region. In Table IV we summarize the predicted half-lives, obtained by this

	Allowed	Calcu Fi $\Delta I = 0$	Observed			
Na ²² V ⁴⁸ Mn ⁵² Co ⁵⁸	0.93 0.56 0.36 0.155	0.90 0.51 0.32 0.133	0.93 0.575 0.375 0.158	0.67 0.21 0.097 0.029	$\begin{array}{rrrr} 1.00 & \pm 0.05 \\ 0.58 & \pm 0.04 \\ 0.35 & \pm 0.02 \\ 0.145 \pm 0.00 \end{array}$	$\left\{ \lambda_{+}/(\lambda_{e}+\lambda_{+}) \right\}$
Na ²² V ⁴⁸ Mn ⁵² Co ⁵⁸	2.5 hr. 14 hr. 1 d. 1 d.	20 d. 35 d. 70 d. 40 d.	30 d. 70 d. 130 d. 70 d.	30 yr. 10 yr. 25 yr. 23 yr.	3 yr. 16 d. 6.5 d. 65 d.	}Half-life

TABLE IV. Observed and calculated values of half-lives and positron branching ratios for the tensor interaction.

method, together with the observed values. Mn⁵² is probably an allowed transition because it is easier to understand that the lifetime belengthened by unfavorable heavy particle wave functions than that it be shortened by a factor of ten compared with expectation. V48 and Co58 may well be first forbidden transitions (i.e., parity change) with $\Delta I = \pm 1$. Since the evidence of partial decay constants and positron energy distribution excludes $\Delta I = 2$ for Na²² we must assume that ΔI is 0 or ± 1 with particularly unfavorable nuclear configurations. It is true, of course, that by admitting a reduction of the decay probability by a factor of 30 in the case of Na²², we also admit the possibility that all of the other three transitions may be allowed, with unfavorable heavy particle configurations.

We may summarize the evidence by stating that the angular momentum change in all four transitions appears to be one or zero. The form of the interaction and the parity change are not certain.

It is obvious that more, highly forbidden transitions should be studied with a view to deciding between the several forms of interaction. A program of such studies was interrupted by the pressure of war work which also delayed publication of these results.

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