PHYSICAL REVIEW VOLUME 69, NUMBERS 7 AND 8 APRIL 1 AND 15, 1946

Cascade Showers and Mesotron-Produced Secondaries in Lead

SALWA NASSAR* AND W. E. HAZEN
Department of Physics, University of California, Berkeley, California
(Received October 17, 1945)

A study has been made of cascade showers in lead and also of electron secondaries accompanying mesotrons in lead. The cloud chamber that was used for the observations contained four lead plates and was placed in an 1100-gauss magnetic field. In the case of showers, comparison with theory was made in terms of: (a) the average shower curve for 11 showers with initiating energies of about 200 Mev; (b) the energy distribution near the shower maximum of 17 showers; (c) the ratio of height to area of the shower curve of 44 showers; and (d) the fluctuation in number of particles in the showers of (a). The theory agrees with the experiment for (b) and (d) but not for (a) and (c). It is likely that errors arising from known physical approximations in the theory account for the discrepancies. In the case of electron secondaries accompanying mesotrons, a study was made of the number of electrons with energy greater than E relative to the number of mesotrons. For small values of E, the theoretical values are too large by a factor of two, but as E increases the discrepancy becomes smaller.

I. CASCADE SHOWERS

COSMIC-RAY experiments have demonstrated the general validity of the Bethe-Heitler radiation and pair-production cross sections for electrons and photons of energies up to at least 10^9–10^{11} ev. These experiments, however, have been quite general in character and chiefly have demonstrated that there is not a complete breakdown in the theory at high energies. The multiplication cross sections have been used in the development of equations describing the course of cascade showers and hence the penetrating power of electrons and photons. The calculations involve physical and mathematical approximations that, until recently, have limited the accuracy of the results to somewhat better than order of magnitude. Nevertheless, the rough theory has sufficed for the interpretation of cosmic-ray phenomena in a qualitative way. It is now important to obtain experimental data that will allow a quantitative verification of shower theory in order that there be no limitation in its usefulness as a tool in unraveling more complex interrelations between cosmic rays. For example, the studies of bursts, of the origin of the soft component, of Auger showers, etc., all involve shower theory.

Problems such as Auger showers and the development of the soft component in the atmosphere involve cascade showers in air. Since it is relatively difficult to make a detailed study of showers in elements of low atomic number, we must depend on measurements in substances such as lead. The energies involved in some problems of air showers are much greater than in the showers studied in lead, but, since the critical energy in air is 15 times as great as in lead, the results in lead correspond to energies 15 times greater in air, i.e., energies up to 10^{10} ev in the case of the present results. As mentioned in an earlier article,[1] there are no good experimental results that allow a really detailed verification of shower theory. On the other hand, the theoretical calculations have recently been improved[2,3] to the extent that the predictions are probably accurate to 20–30 percent, even in the case of moderately low energies in lead. The present experiment is an improvement over the earlier experiment[1] in that electron energies have been measured, and the showers occurred under more carefully controlled conditions. More extensive comparisons with theory have thus been made possible. The initiating energies were still not determined directly.

Experimental Procedure

Four lead plates, each 0.7 cm thick, were placed in a cloud chamber that was 30 cm in

* Now at the American University of Beirut, Lebanon.

[1] W. E. Hazen, Phys. Rev. **66**, 254 (1944).
[2] H. J. Bhabha and S. K. Chakrabarty, Proc. Ind. Acad. Sci. **A15**, 464 (1942).
[3] H. J. Bhabha and S. K. Chakrabarty, Proc. Roy. Soc. **A181**, 267 (1943).

diameter and 45 cm in depth. The plates, which were separated by 5.3 cm, were tilted so that the camera viewed them nearly edge on. For aid in illumination, the lead plates were covered with thin aluminum foil. The chamber was placed in a magnetic field of 1100 gauss, which was produced by a pair of coils. In the photographed region the field was uniform to within 15 percent. The cloud-chamber expansion was controlled by a three-counter telescope placed above the chamber. The glass counter cylinders were painted with an "Aquadag" cathode in order to reduce their absorbing power and thus to minimize the possibility of starting a shower in the immediate vicinity of the chamber. During most of the investigation, there was a thin sheet-iron roof over the apparatus, and during a small portion of the investigation a thin sheet of lead was placed above the top counter with the hope that some of the photons would materialize into shower-producing particles.

For illumination two pairs of discharge tubes were used, one on each side. The beams of light were directed forward at an angle of 45°. Each pair of tubes was connected across a bank of condensers of 150 μf charged to 2200 volts. Later it was found more advantageous to use 150 μf across each discharge tube; longer service was thus obtained, and the capillaries were kept free of dark deposits. A stereoscopic camera was used for the photography.

It was desirable to obtain narrow tracks in order to minimize the uncertainty in measuring their curvatures. A very deep chamber requires additional precautions in order to obtain narrow tracks. For example, the diaphragm has a large distance to travel and therefore requires a larger driving force than a shallow chamber. The track width finally obtained was 0.14 cm in the chamber. From comparisons with other chambers, it was concluded that the limitation on the expansion speed, as the chamber was finally operated, was the rate of air flow through the black velvet. The curvature of the high energy tracks was determined from plots of coordinates that were obtained from traveling microscope readings.[4]

[4] C. D. Anderson, Phys. Rev. **44**, 406 (1933).

Results

A particular shower is completely described when the initiating energy is given and when the numbers of positrons, electrons, and photons and their energy distributions are stated as a function of depth of absorber. In practice, theoretical results have been obtained for (1) the *average* number of electrons (plus and minus) and the average number of photons as a function of initiating energy and depth, (2) the energy distributions at various depths for the "average" shower, and (3) the expected mean square deviation from the average. Previous experimental results that can be compared with any of the above theoretical predictions are almost completely lacking.

The present results consist in a comparison with the theory of (a) the average number of particles (from eleven showers with approximately the same total number of particles) as a function of depth, (b) the energy distribution of the particles near the maximum for 17 of the larger showers, (c) the number of particles at the maximum *versus* the total number in four or five shower units, and (d) the mean square deviations for the showers of (a). Where no ambiguity is likely to result, (a) will be called a "shower curve," (b) the "energy distribution," (c) the "shape of the shower curve," and (d) the "fluctuation."

The following notation will be used in the discussion of the cascade showers:

E_0 = energy of initiating particle,
t = depth in showers units (=0.52 cm in lead),

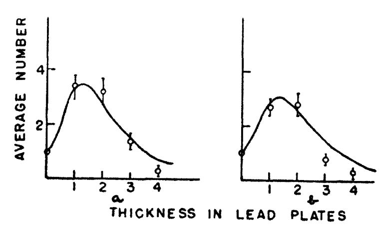

FIG. 1. Shower curves for a small initiating energy. The experimental points were obtained as the averages of eleven showers that contained a total of eight to ten particles. (a) (Left) $N(E_0, 0, t)$ *versus* depth. The curve is for $E_0 = 140$ Mev (see Fig. 7), according to the results of B–C. (b) $N(E_0, \beta/2, t)$ *versus* depth. The curve is for $E_0 = 230$ Mev (see Fig. 7), according to the equations of B–C. The curve of (a) gives a false impression of agreement with the data. A curve for larger E_0 (say 230 Mev) should be used but B–C do not give figures for this initiating energy.

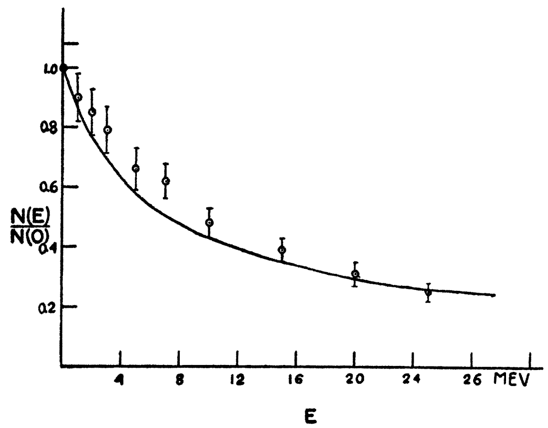

FIG. 2. Energy distribution among the electrons at the observed maxima of 17 showers with initiating energies of 200–1000 Mev. $N(E)/N(0)$ is the relative number of electrons with energy greater than E. The curve was obtained from equations of B–C.

E = energy of shower particle,
β = critical energy (=6.9 Mev in lead) (following reference 3),
$N(E_0, E, t)$ = average number of particles with energy $>E$ at a depth t in a shower initiated by a particle of energy E_0.

(a) *Shower Curve*

Among 1500 photographs, there were 53 with cascade showers. Eleven of the 53 showers contained a total of eight to ten particles and therefore were judged to have been initiated by electrons of approximately the same energy. This particular group was chosen because it was in the most densely populated region of the size distribution. Four of the eleven showers had a total of eight particles each, one had nine, and the remainder had ten each. An average shower, compounded from the above group of eleven actual showers, is indicated in Fig. 1. The points represent simply the average number of particles under each of the lead plates, and the vertical lines indicate the standard probable errors. The plot (Fig. 1b) for $N(E_0, \beta/2, t)$ was made in order to eliminate uncertainties deriving from approximations in the theory that are very poor at the lowest energies.

The curve of Fig. 1a was obtained from the equations of Bhabha and Chakrabarty[2] (hereinafter referred to as B–C). Saddle points were determined from Eq. (26) and the values of $N(E_0, \beta/2, t)$ from Eq. (27) of reference 2. Many of the functions of the introduced variable s that enter into the calculations are given in the tables of Rossi and Greisen[5] with somewhat different notation, e.g., s in reference 2 becomes $s+1$ in reference 5. The value of N thus obtained is actually the first term of a series, but the second term is quite small except for large values of t, values that are not considered here. The curve of Fig. 1a was obtained from the tables of values for N and from the equations for position and height of the maximum given by B–C.

As a result of the following considerations, showers with the same total number of particles are considered to have approximately the same initiating energies. The entire original energy is ultimately dissipated by particles in collision losses and hence

$$E_0 = \int_0^\infty N(E_0, 0, t)(dE'/dt)dt, \qquad (1)$$

where the rate of energy loss by particles $(dE'/dt) \simeq \text{constant} = \beta$. The area A under a

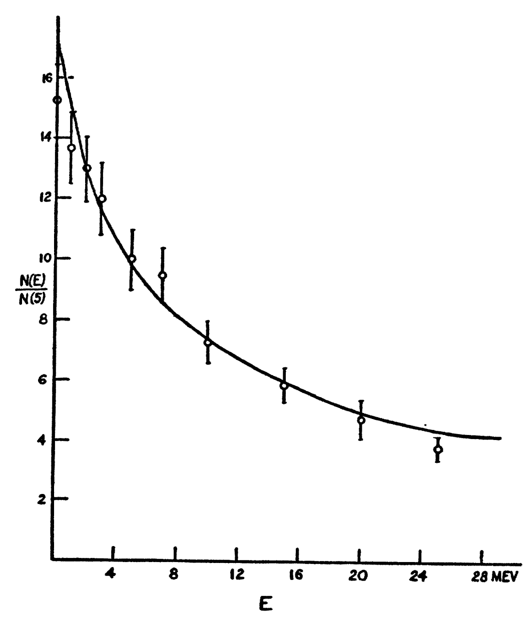

FIG. 3. Energy distribution of Fig. 2 replotted in terms of the number of electrons with energy greater than E relative to the number with energy greater than 5 Mev. Thus the effects of approximations in the theory that are poor at low energies are eliminated.

[5] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941).

shower curve is found to be approximately

$$A \simeq E_0/\beta \simeq \int_0^\infty N(E_0, 0, t)dt. \qquad (2)$$

Thus the total number of observed particles, which is a summation and approximates the integral (2), is a reasonably satisfactory way of determining the initiating energy. In the experiment, the summation extended to only the fourth or fifth radiation unit, but contributions at greater depths would be small and their variations still smaller.

(b) *Energy Distribution*

Among the 53 showers, 17 were found suitable for a study of the energy distribution near the shower maximum. Each of the 17 showers contained 6 to 20 particles at the observed maximum and thus was probably initiated by an electron with energy 200 to 1000 Mev. The observed energy distribution is given in the form of an integral spectrum in Fig. 2. The vertical lines, which represent probable errors, do not include uncertainty in energy determinations but simply indicate the statistical uncertainty. Since approximations in the theory are very poor at extremely low energies, the data have been replotted in Fig. 3 in units of number of particles with energy greater than 5 Mev.

The theoretical curves were obtained from the equations of B–C.[2] The calculations were made by the method employed by B–C in obtaining their Eq. (33). Since the spectral distributions for energies up to 30 Mev are nearly the same for initiating energies of 380 to 1000 Mev, the procedure of including showers within a wide range of energies is justified.

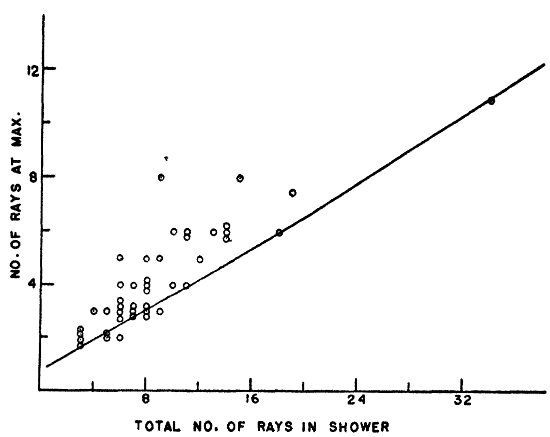

FIG. 4. Shapes of the shower curves. Number of particles at the observed maxima *versus* the total number of rays in the shower for *four* lead plates.

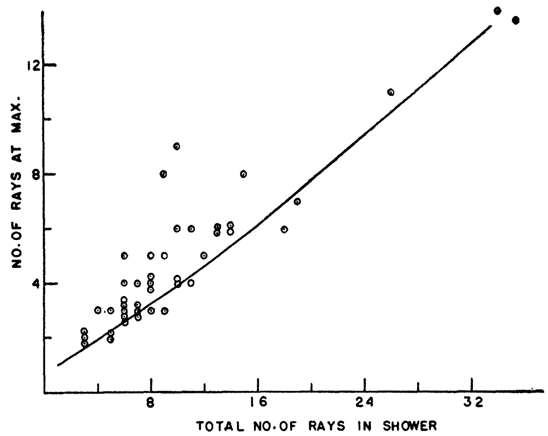

FIG. 5. Shapes of the shower curves. The total number is for *three* lead plates.

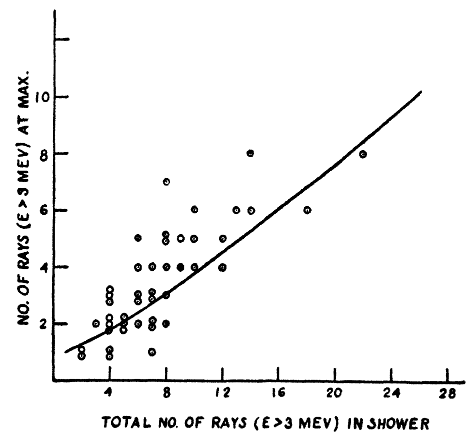

FIG. 6. Shapes of the shower curves with particles of $E>3.5$ Mev excluded. The total number is for three lead plates.

(c) *Shape of the Shower Curves*

The method of analysis that includes the greatest number of showers is that of comparing the number of rays at the maximum with the total number of rays in the shower. There were 44 showers that are suitable for this type of analysis. Since the illumination in the bottom section of the chamber, below the fourth lead plate, was not always as intense as in other regions of the chamber, the data are plotted both for three and

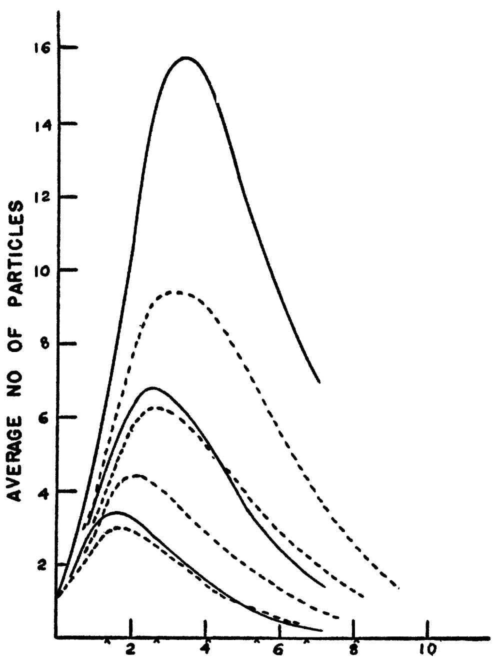

FIG. 7. Theoretical shower curves for electron initiated showers. The arrows on the scale of abscissae indicate depths in number of lead plates. The solid curves are from figures given by B–C for number of particles of all energies and are for initiating energies of 1000, 380, and 140 Mev. The dotted curves were obtained from equations of B–C and are for number of particles with $E>3.5$ Mev. The initiating energies are 1000, 620, 380, and 230 Mev.

also for four lead plates. As in (a) and (b), the results are also given with particles of low energy ($E<\beta/2$) excluded. The experimental results are represented by the circles in Figs. 4–6.

The theoretical curves were obtained from the shower curves of Fig. 7. The shower curves were calculated from the equations of B–C in the manner described under (a). The ordinates of the curves in Figs. 4–6 are simply the heights of the curves of Fig. 7, and the abscissae are the sums of the number of particles at depths of 0, 1, 2, 3, and 4 lead plates (Fig. 4) or 0, 1, 2, and 3 lead plates (Figs. 5 and 6). Since the abscissae of Figs. 4–6 represent the area under a shower curve and the ordinates the height, the plots are simply a measure of the shape of the shower curves.

(d) *Fluctuations*

The eleven showers of (a), each of which probably represents roughly the same initiating energy, were analyzed for magnitude of fluctuations at depths of 1 and 2 lead plates. The magnitude of the fluctuations, as usual, is measured by $\langle (N-\bar{N})^2\rangle_{\text{Av}}$. The experimental results are presented in tabular form in Table I.

If the shower particles were completely independent, we should expect a distribution in observed values of N that approached a Poisson distribution for a large number of observations and consequently $\langle (N-\bar{N})^2\rangle_{\text{Av}}=\bar{N}$. But since the particles are in fact related, we certainly expect larger fluctuations. The simplest model (Furry model) that takes this fact into account predicts $\langle (N-\bar{N})^2\rangle_{\text{Av}}=\bar{N}(\bar{N}-1)$. Scott and Uhlenbeck[6] have included ionization loss in the theory of fluctuations and obtain results that indicate an expected root mean square deviation a few times $\bar{N}$. The values of $\bar{N}$ and $\bar{N}(\bar{N}-1)$ are included in Table I for the sake of comparison.

Comparisons of theory and experiment lead to the conclusions stated in the following paragraphs.

(a) The shower curves agree moderately well with experimental data (Fig. 1), but there is an indication that the theoretical curve does not decrease with sufficient rapidity after the maximum. Exclusion of low-energy electrons does not result in better agreement (Fig. 1b).

(b) The energy distribution (Fig. 2) differs significantly from the experimental results, but in this case exclusion of low energy particles (Fig. 3) does result in satisfactory agreement.

(c) The results show that the ratio of height to area given by the theory is too small (Figs. 4 and 5). When low energy electrons are omitted (Fig. 6), there is less discrepancy, but the discrepancy still exists.

(d) The observed fluctuations are such that the mean square deviation is roughly $1\frac{1}{2}$ times the

TABLE I. Distribution in size, at two depths, for eleven showers initiated by approximately the same energy.

		Below 1st plate	Below 2nd plate
(a)	$\bar{N}$	3.4	3.2
(b)	$\bar{N}(\bar{N}-1)$	8.2	7.0
(c)	$\langle (N-\bar{N})^2\rangle_{\text{Av}}$	4.3	5.2

(a) The observed average number of particles ($\bar{N}$) and hence the expected value of $\langle (N-\bar{N})^2\rangle_{\text{Av}}$ according to Poisson distribution.
(b) Expected value for $\langle (N-\bar{N})^2\rangle_{\text{Av}}$ according to the Furry model.
(c) Observed value for $\langle (N-\bar{N})^2\rangle_{\text{Av}}$.

[6] W. T. Scott and G. E. Uhlenbeck, Phys. Rev. **62**, 497 (1942).

Poisson value, whereas Scott and Uhlenbeck[6] have predicted a mean square deviation 2 to 3 times the Poisson value for somewhat larger showers. The disagreement probably need not be considered as significant in view of the approximations in the calculations, the smallness of the sample of showers, and the small size of the showers.

Discussion

The main experimental uncertainty is in the determination of energies and results from uncertainties in the measurement of the curvature and of the angle of the particle with the magnetic field. The uncertainty in curvature increases with energy and is such that the error is of the same magnitude as the curvature itself for energies of 35 Mev in the case of the best tracks. The angle of inclination of the high energy rays in a shower is generally so small that errors in its measurement are unimportant. Aside from the study of the energy distribution (b), there is no particular interest in the energy of particles except near 3.5 Mev. At low energies there is little uncertainty in the curvature measurements, but the angle with H is frequently large and uncertainties in the angle become more important. The estimated error for low energies is 15 percent.

The inaccuracies of the theoretical calculations originate in mathematical and physical approximations. In the work of B–C, errors due to the former are probably unimportant insofar as comparison with the present experimental results is concerned. The important physical approximations are the assumptions that (1) the collision loss by particles is independent of energy, (2) the shower is one-dimensional, (3) the multiplication cross sections are those for complete screening, and (4) the Compton effect is negligible.

(1) In reality, the collision loss increases rapidly for electrons with $E<1$ Mev and increases slowly (roughly as $\log E$) for $E>10$ Mev. (2) The lateral spread of the shower is determined almost entirely by scattering of the electrons.[5] In the case of lead, scattering becomes important at 3 Mev and serious at 1 Mev. The result is a decrease in the actual penetration of the electrons in the direction of shower propagation. (3) When incomplete screening is taken into account, the pair production cross sections are reduced significantly at lower energies (10–200 Mev for lead).[7] (4) In lead, the Compton cross section becomes equal to the pair production cross section for a photon energy of about 5 Mev. At the energies involved in our shower studies, the Compton effect results in the production of low energy electrons and the consequent dissipation of the energy of the photon at the rate of 1–2 Mev per cm in the case of lead.[3]

The principal conclusion that can be made is that the shape of the theoretical curves is wrong. From (c) we can conclude that the theoretical curves should be more sharply peaked for all initiating energies up to 1000 Mev, and from (a) we can add that the error is probably on the side of the peak toward greater thicknesses.

The most important experimental uncertainty, energy measurement, can have no effect on the results when particles of all energies are included. The most useful comparison with theory is made, however, by excluding the particles of low energy, because the effect of certain approximations in the theory can thus be eliminated: namely, the neglect of scattering (2) and increased collision loss (1) at low energies, which, if not neglected, virtually eliminate the contribution of electrons to a shower when their energies fall below one Mev. When we consider the cases in which low energy electrons are excluded, the experimental uncertainty in energy measurement would tend to increase the included number of electrons since the population is greater at the lower energies. The effect is smaller at the maximum of the shower curve, where the average energy is high, than at greater depths, where the average energy decreases. The predicted consequence of the experimental uncertainties would therefore be a broadening of the shower curves when low energy particles are excluded. Thus a correction would result in a *greater* discrepancy.

Let us consider the qualitative effect of the other physical approximations used in the theory. If we take into account increased collision loss at higher energies (1), the value of dE'/dt becomes larger for larger energies. The area under

[7] H. C. Corben, Phys. Rev. **60**, 435 (1941).

a given portion of the shower curve,

$$A = E_0/(dE'/dt)_{Av} = \int_{t_1}^{t_2} N(E_0, 0, t)dt,$$

is decreased more for small t than for large t because the average energy of the particles and consequently $(dE'/dt)_{Av}$ are greater for small t. Thus a correction would lead to a broader shower curve and greater divergence from the experimental results.

When we take incomplete screening into account (3), the multiplication cross sections are smaller, particularly the pair-production cross section at lower energies. The result is a decrease in the number of particles for thickness $t < 3t_{max}$[7] and, from conservation of energy, an increase in the number of particles for greater thicknesses. Corben[7] has shown that the maximum of a shower with $E_0 = 1000$ Mev is only 13.5 particles as contrasted with 16 particles[2] when complete screening is assumed. Thus the theoretical curve is again broadened when the theory is improved. On the other hand, when only electrons with $E > \beta/2$ are considered, there is a partially compensating effect which may be seen from the following consideration. The area under a shower curve ($N(E_0, \beta/2, t)$ *versus* t) is

$$\int N(E_0, \beta/2, t)dt = E_0/\beta - \int [N(E_0, 0, t) - N(E_0, \beta/2, t)]dt, \quad (3)$$

where the latter integral is the track length generated by electrons with $0 < E < \beta/2$.

The latter integral represents the track length generated by all particles while their energies are less than $\beta/2$. When incomplete screening is taken into account, the number of individual particles is reduced and hence the latter integral is reduced. Since E_0/β is unchanged, the former integral is increased. Thus the area under a shower curve in which low energy electrons are excluded is increased, and, since the number of electrons that fall below the energy limit $\beta/2$ is greater where the average energy is smaller, we should expect the increase in area to occur predominately in the early stages, i.e., near the maximum.

Finally, the neglect of the Compton effect ascribes a significantly greater penetrability to the lower energy photons, since the photons actually dissipate a portion of their energy in the production of low energy Compton recoils. The result is an increase in the height to area ratio of a shower curve. When low energy electrons are excluded, the effect is enhanced: the second integral in Eq. (3) is increased, since inclusion of the Compton effect predicts more particles, and the area under the shower curve is decreased, the main decrease occurring at greater depths because the average energy is less.

It is difficult to estimate the magnitude of the changes that would be produced by the modifications discussed in the above paragraphs. It seems likely, however, that improvements in the physical approximations of the theory might result in satisfactory agreement with the experimental results. At present, it is only possible to say that the theoretical multiplication cross sections have been approximately verified for energies up to 10^9 ev. The methods herein employed amount to a comparison between multiplication processes and collision losses. The latter have been confirmed experimentally and hence a satisfactory verification of the shower theory would constitute a confirmation of the multiplication cross sections.

II. MESOTRON-PRODUCED SECONDARIES

The electron component of cosmic rays in air apparently originates almost entirely as a secondary effect of mesotrons,[8] a secondary effect that occurs in the lower regions of the atmosphere insofar as the majority of the electron secondaries observed at sea level are concerned. Since both mesotron decay and close electron collisions play important roles in the production of secondaries, it is of importance to study one or the other individually in order to separate the two effects. Decay electrons can be practically eliminated by observing the secondaries under dense materials. Cloud-chamber observations alone have given unambiguous results, but there is poor agreement among many of the findings.[9] Likely explanations for the discrepancies are given in reference 9. In order adequately to compare results with the theory, it is necessary to know at least the

[8] K. Greisen, Phys. Rev. **63**, 323 (1943); B. Lombardo and W. E. Hazen, Phys. Rev. **68**, 74 (1945).
[9] W. E. Hazen, Phys. Rev. **64**, 7 (1943).

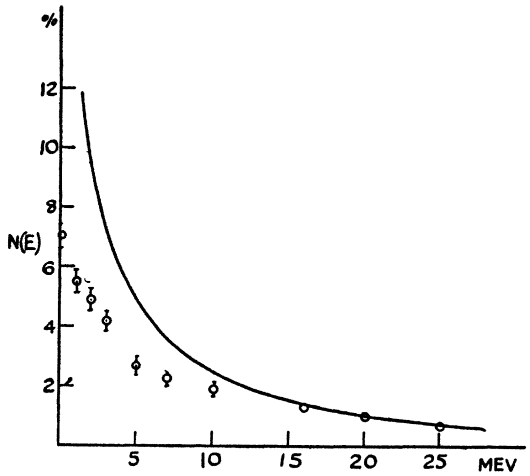

FIG. 8. Number of secondary electrons in equilibrium with mesotrons in lead. $N(E)$ is the number of electrons with energy greater than E. The curve is from equations of Tamm and Belenky.

distribution in energy of the secondaries at the lower energies. Wilson and Neddermeyer and Anderson[10] have previously studied secondary production with a magnetic field. Neddermeyer and Anderson used their data individually as arguments against the possibility of protons as the main constituent of the hard component. Wilson used his data statistically as an indication of a mesotron mass of a few hundred electron masses.

In the present work the same group of pictures that was used in the study of showers was analyzed for number and energy distribution of electron secondaries accompanying the mesotrons in lead.

Results and Discussion

Only those pictures that showed counter-controlled mesotron tracks with the three center segments clearly defined were chosen. There was thus considerable assurance that very few secondaries would escape observation. The observed number of secondaries for various lower energy limits are indicated in the integral spectrum plotted in Fig. 8. The total number of mesotron traversals of a lead plate was 1674. Tamm and Belenky[11] have obtained an expression for the number of secondaries to be expected in lead. In making a numerical evaluation, T–B used a mesotron energy spectrum that was appropriate for determining the number of secondaries in air, and they used a mesotron mass of $160m_e$. In the present experiment, the observed secondaries are all produced within the apparatus, and hence it is the sea-level mesotron spectrum that should be used for the calculations. In addition, present experimental evidence favors a mesotron mass somewhat greater than $160m_e$. Consequently the theoretical curve of Fig. 8 has been obtained from Eq. (6.1) of T–B, but with a value of $\bar{E}_m$ (the maximum transferable energy averaged over the mesotron spectrum) determined with the use of Blackett's sea level spectrum[12] and a mesotron mass of $200m_e$. Williams[13] has calculated the relative number of electron secondaries to be expected in lead with a result of 8 to 13 percent with energy greater than 3 Mev. The lower limit of 8 percent is slightly above the curve obtained from T–B.

The theoretical curve is in marked disagreement with the data even at moderately high energies, energies where scattering is not serious. Wilson, on the other hand, observed a value of 4.8 percent for the number of electrons with $E>10$ Mev in equilibrium with mesotrons in gold. When the theory of T–B is used in order to change 4.8 percent to a corresponding value for lead, the result is 5.0 percent. This point would lie above the curve of Fig. 8. Wilson does not mention the type of counter control that was employed, but it is likely that it consisted of counters both above and below the chamber. Evidence for this conclusion is given by the fact that his data indicate an extremely high proportion of events with high energy secondaries and with two or more secondaries. A counter below the chamber would result in a selectivity in favor of mesotrons accompanied by high energy secondaries. Unless this fact were taken into account in selecting the photographs to be studied, it might result in an apparent increase by a factor as large as two, depending on the magnetic field strength, for secondaries with $E>3\times10^7$ ev.

The accuracy of the theoretical curve is not

[10] J. G. Wilson, Nature **142**, 73 (1938); S. H. Neddermeyer and C. D. Anderson, Rev. Mod. Phys. **11**, 191 (1939).
[11] Ig. Tamm and S. Belenky, J. Phys. Acad. U. S. S. R. **1**, 177 (1939).
[12] P. M. S. Blackett, Proc. Roy. Soc. **A159**, 1 (1937).
[13] E. J. Williams, Proc. Camb. Phil. Soc. **36**, 183 (1940).

known and is difficult to estimate since it involves shower theory. Williams' calculations indicate an uncertainty of about 30 percent for his results but unfortunately the suggested range of possible values extends upward from the curve. His calculations would agree moderately well with those of T–B if the more exact cross section for production of knock-on electrons were used.

Large changes in mesotron mass would be required in order to remove the discrepancy, e.g., an increase in mesotron mass from 200 to $250m_e$ results in only a 12 percent decrease in the predicted number of secondaries. Large changes in the mesotron spectrum also produce only small alterations in the expected number of secondaries.

We have greatly appreciated instructive conversations with Dr. David Bohm concerning the shower theory.

PHYSICAL REVIEW VOLUME 69, NUMBERS 7 AND 8 APRIL 1 AND 15, 1946

Disintegration Schemes of Radioactive Substances IX. Mn^{52} and V^{48}

WENDELL C. PEACOCK AND MARTIN DEUTSCH
Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received December 29, 1945)

The radiations emitted in the decay of $_{25}Mn^{52}$ (6.5 day) and of $_{23}V^{48}$ have been investigated by spectrometer and coincidence methods. Mn^{52} decays with the emission of positrons of maximum energy 0.582 ± 0.015 Mev, followed by three gamma-rays in cascade, of energies 0.734 ± 0.015 Mev, 0.940 ± 0.02 Mev, and 1.46 ± 0.03 Mev, respectively. These energies are the multiples 3, 4, and 6 of 0.240 Mev, within the experimental errors. The orbital electron capture by Mn^{52} leads to the same excited state of Cr^{52} as the positron emission with which it competes. The positrons of V^{48} are emitted with a maximum energy of 0.716 ± 0.015 Mev, with successive emission of two gamma-rays of 1.33 ± 0.03 Mev and 0.980 ± 0.02 Mev energy. These gamma-ray energies are in the ratio 4:3. From the disintegration schemes one finds the mass differences between neutral atoms: $Mn^{52}-Cr^{52}=5.10\pm0.15\times10^{-3}$ a.m.u. and $V^{48}-Ti^{48}=4.37\pm0.12\times10^{-3}$ a.m.u. Some evidence is also presented concerning the disintegration of V^{52} and of Sc^{48}. The energy levels of Cr^{52} and Ti^{48} identified in the radioactive processes are compared with those found by other methods. The disintegration energies of several nuclei studied are examined with a view to their dependence on atomic weight. It is shown that beta-ray theory explains consistently the lifetimes, the shapes of the positron spectra, and the ratio of electron capture to positron emission if one assumes tensor interaction, angular momentum change $\Delta I=0$ or $\Delta I=\pm1$ (not $0\rightarrow0$) without parity change in the case of Mn^{52} and with parity change in the case of V^{48}.

INTRODUCTION

In paper VII of this series[1] we called attention to some apparent similarities between the energy levels of certain nuclei in the same region of the periodic table. It seems desirable to study the excited states of several nuclei which might be expected to differ in some regular fashion in their internal structure. The most desirable method for such a study would be—in many respects—the measurement of the energies of secondary heavy particles from transmutation or inelastic scattering processes. When this method is developed to the ultimate precision of which it is capable, it should prove to be as powerful a tool for the analysis of energy levels in light nuclei as the study of spontaneously emitted alpha-particles has been for the levels of very heavy nuclei. However, the contemporary techniques are such that the resolution obtained is less than some known level spacings, even in rather light nuclei, such as Fe^{56} (paper VI). Therefore a study of the radiations emitted in radioactive disintegrations appears to be the most hopeful approach at present. This method has yielded reliable and rather precise information concerning the energy levels of a number of nuclei, but its extension to the study of any selected sequence or group of nuclei presents certain inherent difficulties. Each of the nuclei to be investigated must be the product of one or, preferably, two or three radioactive processes of sufficiently long period to permit thorough study, and involving several excited states. Furthermore there must be some practical method to produce the radioactive species in-

[1] Reference to the first five papers will be found in paper VI: Phys. Rev. **64**, 321 (1943); papers VII and VIII appeared in Phys. Rev. **65**, 211 (1944) and **68**, 193 (1945).