On the Inelastic Photo-Dissociation of the Deuteron

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There exists the possibility of directly verifying the theoretical prediction of the existence of excited states of the nucleons at about 45 Mev above the ground state by an inelastic photoelectric dissociation process of the deuteron with γ-rays of energies larger than the excitation energy of these states. The cross section σ' for such processes is calculated in terms of the elastic cross section σ by using the simplest form of the strong coupling theory. The quantity σ'/σ has a maximum at about 87 Mev where it reaches the approximate value of 0.07. These processes should be experimentally detectable and would furnish valuable information about the properties of nuclei.

I. INTRODUCTION

At present there exist two different forms of the meson theory of nuclear forces which can claim some chance of success, the so-called strong coupling[1] and the weak coupling[2] theory. In both theories charged and neutral mesons are assumed to interact with the heavy nucleons (neutrons and protons) in a symmetrical way which guarantees the charge independence of the nuclear forces. Neither is completely satisfactory but the difficulties which remain may well be beyond the scope of the present status of the quantum theory of fields. On the basis of the generally available experimental results on atomic nuclei it seems impossible to make a definite choice between the two theories.

The two theories distinguish themselves primarily by the different kind of approximations which are considered for the calculation of the forces between two nucleons at sufficiently large separation from each other. In the strong coupling case the forces are obtained by a development into falling powers of the coupling parameter while the weak coupling theory uses a development in rising powers of this constant. But this is not the only distinction between the two theories. It is well known that in a relativistically invariant field theory with point sources the interaction term in the Hamiltonian leads in higher approximations to divergent results. In order to discuss the validity of an approximation it is necessary to compare the first-order term with the higher order terms in the expression for the nuclear forces. Thus some kind of a model of the nucleon is necessary which leads to convergent results. One possibility is to "smear" the nucleon over a finite region in space by introducing a source function $U(\mathbf{x})$. With this method the relativistic invariance of the procedure is destroyed but all the approximations are finite, and it is found that the weak coupling is inadmissible. With the source function one has introduced essentially a new parameter "a" which we may call the effective radius of the nucleon and which may be defined as

$$\frac{1}{a}=\int\frac{U(\mathbf{x})U(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3xd^3x'.$$

This quantity is identical with the so-called spin inertia which enters in the classical theory of spinning particles.[3]

The model with the extended source is not the only possibility of a model which leads to a convergent theory. The other possibility is the so-called λ-limiting process of Wentzel and Dirac[4] which has the remarkable feature of leading to a convergent classical theory of particles interacting with a field without destroying the relativistic invariance of the theory. However in quantum mechanics some of the approximations are still divergent so that the λ-limiting process serves its

[1] G. Wentzel, Helv. Phys. Acta **13**, 269 (1940); **14**, 633 (1941); **15**, 685 (1942); **16**, 222 (1943); **16**, 551 (1943). J. R. Oppenheimer and J. Schwinger, Phys. Rev. **60**, 150 (1940). W. Pauli and S. M. Dancoff, Phys. Rev. **62**, 85 (1942). W. Pauli and S. Kusaka, Phys. Rev. **63**, 400 (1943). R. Serber and S. M. Dancoff, Phys. Rev. **63**, 143 (1943).

[2] W. Pauli, Phys. Rev. **64**, 332 (1943).

[3] H. J. Bhabha, Proc. Roy. Soc. **A178**, 314 (1941).

[4] G. Wentzel, Zeits. f. Physik **86**, 479, 635 (1933); **87**, 726 (1934). P. A. M. Dirac, Ann. de L'Inst. H. Poincaré **9**, 13 (1939).

purpose only if it is coupled with some other formalism which takes care of the remaining terms.[5] The condition for weak coupling is different from the condition in the case of the extended source, and it is found that the weak coupling condition can be fulfilled in this theory.[2]

In spite of the entirely different approach of the two methods the result for the nuclear force between two nucleons is strikingly similar in the two cases. The essential difference between the two theories is to be found in the prediction of the existence of excited states of the system "nucleons plus meson field" in the case of strong coupling. The excitation energy of these states is in the case of pseudo-scalar meson field

$$\Delta E = \epsilon\{s(s+1) - \tfrac{3}{4}\},$$

where s is a quantum number which may assume all half-integer values, $\frac{1}{2}$, $\frac{3}{2}$, $\cdots$. The parameter ϵ is related to the coupling constant f of the dimension of length and the nuclear size "a" by the formula[1]

$$\epsilon = a\hbar c/4f^2.$$

The first excited state will then be 3ϵ above the ground state of the nucleons. The ground state of the nucleon is doubly degenerate corresponding to the two states "proton" and "neutron" of the nucleon while the excited states allow higher values for the charge as well as the spin. Thus the first excited state for instance has total spin $\frac{3}{2}$ and the charge number may assume all integer values from -1 to $+2$.

For the discussion of the physical consequences of this theory the magnitude of ϵ is the decisive factor. If ϵ is large compared to the energy of a two-nucleon problem, then the force which derives from the strong coupling theory is identical with the weak coupling case apart from a numerical factor.[6] If ϵ is comparable or even smaller than the total energy in question, then the nuclear force exhibits entirely new features.

For a discussion of the nuclear forces, scattering experiments with high energies will therefore be of particular interest. Such experiments were carried out by Amaldi[7] and co-workers and by Champion and Powell.[8] It is the angular dependence of the N–P scattering which was measured by these authors up to energies as high as 14 Mev. The results show a marked preference of the scattering in the forward direction. It is interesting that the weak coupling theory in any of the symmetrical forms is at variance with this result.[9] This feature is directly related to the exchange character of the nuclear forces which causes a reversal of the sign of the potential in P-states and thus a strong backwards scattering.

On the other hand the strong coupling theory leads to the correct angular distribution for an isobar-energy $\epsilon = 15$ Mev.[10] This seems to be an indication that the strong coupling variety of the meson theory with its prediction of the isobaric states of nucleus corresponds to reality.

In view of this conclusion it seems of considerable interest to look for other experimental phenomena which would give even more direct evidence for the existence of the excited states of the nucleons. Thus for instance if scattering experiments with nucleons were carried out with an initial energy of the scattered particles larger than $2\Delta E = 6\epsilon$ in the laboratory system, the scattering will contain an inelastic part where one of the nucleons is excited. Calculations indicate that the ratio of the inelastic to the elastic cross section will be of the order 0.03 for an initial energy of the scattered nucleons of ~ 100 Mev in the laboratory system.[11]

This paper treats the theory of another possible process which could be observed and which would furnish a clear cut and simple proof of the existence of excited states of the nucleons, the inelastic photo-effect of the deuteron. If the deuteron is irradiated with 100 Mev γ-rays which can now be produced with the betatron or similar devices for accelerating electrons, then one would

[5] P. A. M. Dirac, Proc. Roy. Soc. **A180**, 1 (1942). W. Pauli, Rev. Mod. Phys. **15**, 175 (1943).

[6] W. Pauli and S. Kusaka, Phys. Rev. **63**, 400 (1943). M. Fierz and G. Wentzel, Helv. Phys. Acta **17**, 215 (1944). G. Wentzel, Helv. Phys. Acta **17**, 252 (1944).

[7] E. Amaldi, D. Bocciarelli, B. Ferretti, G. G. Trabachi, Naturwiss. **30**, 582 (1942).

[8] F. C. Champion and C. F. Powell, Proc. Roy. Soc. **183**, 64 (1944).

[9] L. Hulthén, Arkiv. f. Mat. Astron. Fys. **29**, No. 33 (1943). B. Ferretti, Nuovo Cimento **21**, No. 1, 25 (1943). J. M. Jauch, Phys. Rev. **67**, 125 (1945).

[10] G. Wentzel, Helv. Phys. Acta **18**, 430 (1945). According to a private communication of Professor Wentzel, it seems however impossible to reconcile such a low value of ϵ with the instability of the ^1S-state of the deuteron. This question needs therefore further investigation and more accurate experiments should be performed.

[11] J. L. Lopes, Ph.D. Thesis, Princeton University.

observe in addition to the normal photo-dissociation a certain fraction of dissociations with excitations of one of the nuclear particles. Because of the change in velocity, such dissociation protons would produce a considerably stronger ionization in a Wilson chamber and could therefore be in principle detected.

The excited states of the nucleon have a very short lifetime because of the emission of a magnetic dipole radiation. The electric dipole and the electric quadrupole radiation are zero in the strong coupling approximation.* The transition probability is easily calculated from the well-known formula

$$P=\frac{64}{3}\frac{\pi^4}{hc^3}\nu^3|M_{mn}|^2,$$

where ν is the frequency of the emitted radiation and M_{mn} is the matrix element of the magnetic dipole moment operator which was calculated by Pauli and Dancoff.† One obtains in this way for the lifetime

$$\tau=1/P=1.15\times10^{-18}\text{ sec.}$$

The decay radiation carries within the average the excitation energy of the nucleon. Because of the Doppler effect it may fluctuate around this value by as much as 30 percent. This fact may make the identification of an excited proton difficult. However if the neutron is excited by this process this difficulty does not appear.

2. NOTATIONS AND FORMULATION OF THE PROBLEM

We consider in the following the two-nucleon problem with the interaction operator which follows from the charge symmetrical theory with a mixed pseudoscalar and vector field. The two coupling constants and the two values of the masses shall be taken as equal in magnitude. We obtain in this way the strong coupling form of the theory which was originally proposed by Møller and Rosenfeld.[12] In the static approximation of the interaction operator (nucleons at rest) no tensor force is obtained in this theory. It is very likely that the correct nuclear interaction should contain a tensor force already in the static approximation but we disregard this force for the problem which we want to discuss here. The total value for the cross section is probably not appreciably influenced by this approximation although for some of the finer features of the photo-dissociation, as for instance the angular dependence of the emitted protons, the tensor force would be essential.[13] The approximation introduces a considerable simplification in the calculation.

In this theory a nucleon is described by the following commuting variables: the position vector $\mathbf{z}$, the spin vector $\mathbf{s}$, and the isotopic spin vector $\mathbf{t}$.[14] The commutation rules of angular momenta apply for $\mathbf{s}$ and $\mathbf{t}$. The magnitudes of these vectors together with their third component form a complete set of commuting quantities with respect to the internal degrees of freedom. In a representation in which these quantities are diagonal we have

$$\mathbf{s}^2=\mathbf{t}^2=s(s+1),\quad (s=\tfrac{1}{2},\,\tfrac{3}{2},\,\cdots),\quad s_3=m,\quad t_3=n,$$
$$-s\leqslant n,\, m\leqslant s,\quad (m,n=\pm\tfrac{1}{2},\,\pm\tfrac{3}{2},\,\cdots).$$

The physical significance of $\mathbf{s}$ is the spin while for $\mathbf{t}$ only the component t_3 has a simple physical interpretation. The charge quantum number of the nucleon is $t_3+\frac{1}{2}=n+\frac{1}{2}$. For the ground state we have $s=\frac{1}{2}$, $n=\pm\frac{1}{2}$. Thus the charge is restricted to the two values 0 (neutron) and 1 (proton).

We distinguish the two sets of variables for the two nucleon problems with indices: $\mathbf{z}_1\mathbf{s}_1, \mathbf{t}_1$; $\mathbf{z}_2\mathbf{s}_2, \mathbf{t}_2$. The quantum numbers s_1, m_1, n_1; s_2, m_2, n_2 may be used to label the internal states of the two-nucleon problem. The interaction operator between two nucleons may be written

$$V=(f\mu)^2\frac{\exp[-\mu r]}{r}\Gamma,$$

where μ is the mass of the meson, $r=|\mathbf{z}_1-\mathbf{z}_2|$ and Γ is a certain operator, operating only on the internal degrees of freedom. Γ is the generalization of the operator $(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$ of the weak

* I am indebted to Professor W. Pauli for discussion on this point.

† Reference 1, page 106.

[12] C. Møller and L. Rosenfeld, Kgl. Danske Vid. Sels. **17**, 8 (1940).

[13] Cf. for instance, W. Rarita and J. Schwinger, Phys. Rev. 59, 556 (1941).

[14] We use in the following the notation of W. Pauli and S. Kusaka (see footnote 1). The units are atomic units $\hbar$, c, and 1 cm.

coupling theory, where $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ denote the ordinary and isotopic spin matrices. In case ϵ is large compared to the average energy we can disregard the matrix elements which connect the ground state of the nucleons with its excited states. For the remaining matrix elements we have then[14]

$$\Gamma\rightarrow\tfrac{1}{9}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2).$$

The total energy of the two-nucleon system in the absence of any external perturbation will then be

$$H_{\text{nucl}}=\frac{1}{2M}p_1{}^2+\frac{1}{2M}p_2{}^2$$
$$+\epsilon\{(s_1+\tfrac{1}{2})^2+(s_2+\tfrac{1}{2})^2-2\}+(f\mu)^2\frac{\exp\left[-\mu r\right]}{r}\Gamma.$$

The first two terms represent the kinetic energy of the two nucleons, their masses being taken as equal. The third term is the isobar energy, and the fourth term is the interaction energy.

We consider now these two nucleons under the influence of a radiation field described by the operator of the vector potential

$$\mathbf{A}(\mathbf{x})=\frac{1}{\sqrt{V}}\sum_k\frac{1}{(2k^0)^{\frac{1}{2}}}$$
$$\times\{\mathbf{a}(\mathbf{k})\exp\left[i\mathbf{k}\cdot\mathbf{x}\right]+\mathbf{a}^*(\mathbf{k})\exp\left[-i\mathbf{k}\mathbf{x}\right]\}.$$

To simplify the calculations we have adopted the usual procedure of describing the field in a cubical box of volume $V=L^3$. If we impose periodic boundary conditions, the vectors $\mathbf{k}$ are restricted to the values $k_i=2\pi n_i/L$ with integer numbers n_i. Since we are dealing with a pure radiation field the vector $\mathbf{A}(\mathbf{x})$ satisfies the condition $\nabla\mathbf{A}=0$, which implies for the operators $\mathbf{a}(\mathbf{k})$

$$\mathbf{k}\cdot\mathbf{a}(\mathbf{k})=0. \tag{1}$$

The interaction of a charged particle of charge e with an external electromagnetic field is obtained by replacing in the force free Hamiltonian the momenta $\mathbf{p}$ by $\mathbf{p}-e\mathbf{A}$ where $\mathbf{A}$ is to be taken at the position $\mathbf{z}$ of the particle. This leads to terms in the Hamiltonian which are linear and quadratic in $\mathbf{A}$. The quadratic terms do not contribute anything to the photo-dissociation of the deuteron. They lead rather to scattering of photons, processes which are not considered in this paper. In the following we disregard, therefore, the quadratic terms. Since the charge of the elementary particles may be any positive or negative integer in this theory corresponding to the state of the internal degrees of freedom, we must replace the scalar quantity e by the more general charge operator defined above: $e(t_3+\frac{1}{2})$. We obtain in this way the operator for the interaction of the electromagnetic field with the nucleons the expression

$$H_{\text{int}}=-\frac{e}{M}\{(t_3{}^{(1)}+\tfrac{1}{2})\mathbf{p}_1\cdot\mathbf{A}(\mathbf{z}_1)+(t_3{}^{(2)}+\tfrac{1}{2})\mathbf{p}_2\cdot\mathbf{A}(\mathbf{z}_2)\}$$
$$=-\frac{e}{M}\frac{1}{\sqrt{V}}\sum_k\frac{1}{(2k^0)^{\frac{1}{2}}}\{(t_3{}^{(1)}+\tfrac{1}{2})\mathbf{p}_1\cdot\mathbf{a}(\mathbf{k})\exp\left[i\mathbf{k}\mathbf{z}_1\right]+(t_3{}^{(2)}+\tfrac{1}{2})\mathbf{p}_2\cdot\mathbf{a}(\mathbf{k})\exp\left[i\mathbf{k}\mathbf{z}_2\right]+\text{conj.}\},$$

where here and in the following we use the abbreviation "conj." to denote the Hermitian conjugate of the preceding expression. We remark that by virtue of (1) the order of the factors $\mathbf{p}$ and $\mathbf{A}$ is not important. For the field energy alone we have from electromagnetic theory

$$H_{\text{field}}=\sum k^0\mathbf{a}^*(\mathbf{k})\mathbf{a}(\mathbf{k}).$$

Since the field operators a, a^* satisfy the commutation rules

$$[a_r(k),a_s{}^*(k')]=\delta_{rs}\delta_{kk'},$$

the above expression for the field energy may also be written as

$$H_{\text{field}}=\sum k^0n(\mathbf{k},\lambda),$$

where $n(\mathbf{k},\lambda)$ denotes the number of photons with propagation vector $\mathbf{k}$ and polarization λ. In our problem the unperturbed state of the field will be described by one of the n's of a given $\mathbf{k}$ and λ being equal to 1 while all the others are equal to zero. Thus H_{field} simply reduces to $H_{\text{field}}=k^0$.

The total Hamiltonian for the problem of the photo-dissociation may now be written as follows

$$H=H_{\text{nucl}}+H_{\text{field}}+H_{\text{int}}. \tag{2}$$

We have disregarded in this expression for H the interaction of the field with the magnetic moment and the meson field directly. As Pais has shown, both these effects give a negligible contribution to the cross section in the energy region which we are going to consider.[15]

A considerable simplification of the problem is obtained if we transform to a new set of variables of which some are constants of motion. Now it is easily seen that $\mathbf{S}=\mathbf{s}_1+\mathbf{s}_2$ and $\mathbf{T}=\mathbf{t}_1+\mathbf{t}_2$ commute with the Hamiltonian (2) and hence are constants of motion.[16] A commuting set of integrals is obtained from these if we consider

$$\mathbf{S}^2=S(S+1);$$

$$\mathbf{T}^2=T(T+1);$$

$$S_3=M;$$

$$T_3=N.$$

S and T are integers and represent the values of the total spin and isotopic spin which for a given set of s_1, s_2 are subject to the conditions of the vector model

$$|s_1-s_2|\leqslant S,\ T\leqslant s_1+s_2. \tag{3}$$

M and N, which represent the third component of the spin and isotopic spin are restricted by

$$-S\leqslant M\leqslant S,$$

$$-T\leqslant N\leqslant T.$$

The quantum numbers S, T, M, N, s_1s_2 form again a complete set of numbers for labeling the states of the internal degrees of freedom of the nucleons. The advantage of this set is however that the operator Γ is diagonal with respect to S, T, M, N and decomposes for each set of these numbers into submatrices with respect to s_1 and s_2. The transformation of the operator Γ to this representation was carried out by Fierz.[17] It will not be necessary to write down here the complete expression for these matrix elements since we are going to use them only for a very special case.

For the following it will be convenient to introduce the coordinates of the center of mass and the relative coordinates for which we write

$$\mathbf{Z}=\tfrac{1}{2}(\mathbf{z}_1+\mathbf{z}_2),$$

$$\mathbf{z}=\mathbf{z}_1-\mathbf{z}_2.$$

The corresponding transformation of the momentum is

$$\mathbf{P}=\mathbf{p}_1+\mathbf{p}_2,\quad \mathbf{p}=\tfrac{1}{2}(\mathbf{p}_1-\mathbf{p}_2),$$

with the solutions

$$\mathbf{z}_1=\mathbf{Z}+\tfrac{1}{2}\mathbf{z},\quad \mathbf{p}_1=\tfrac{1}{2}\mathbf{P}+\mathbf{p},$$

$$\mathbf{z}_2=\mathbf{Z}-\tfrac{1}{2}\mathbf{z},\quad \mathbf{p}_2=\tfrac{1}{2}\mathbf{P}-\mathbf{p}.$$

3. PERTURBATION THEORY

The problem which we want to treat here is the interaction of electromagnetic radiation with the deuteron. It is of course necessary to make a perturbation calculation where we consider the interaction energy of radiation with the deuteron as small compared to the energy of the deuteron and the radiation alone. Such a procedure involves the exact knowledge of the unperturbed wave function, in this case the wave function of the deuteron in the ground state and in the dissociated state.

The deuteron problem in the strong coupling approximation leads to a very complicated eigenvalue problem since the operator Γ has matrix elements which connect the ground state $s_1=s_2=\frac{1}{2}$ with an infinite series of excited states.[18] Fortunately it is possible to show that for sufficiently high values of the excitation energy ΔE the influence of the states with $s_1\neq\frac{1}{2}$ or $s_2\neq\frac{1}{2}$ are negligible. The condition for this to hold was shown by Pauli and Kusaka[1] to be that $|E_D|\ll\Delta E$, where E_D is the binding energy of the deuteron. This condition is well satisfied for $\Delta E\sim 45$ Mev since $|E_D|=2.17$ Mev. We decompose therefore the operator Γ into two parts $\Gamma=\Gamma_0+\Gamma_1$, where Γ_0 contains only the diagonal elements of Γ and Γ_1 contains the off diagonal elements. These latter will be important for the calculation of the inelastic photo-effect but they are unimportant for the calculation of the unperturbed wave functions of the deuteron. The Hamiltonian of the problem may then be written in the form

$$H=H_0+H',$$

[15] A. Pais, Kgl. Danske Vid. Sels. Mat.-Fys. Medd. **20**, No. 17 (1943).

[16] See W. Pauli and S. Kusaka, reference 1, page 407.

[17] M. Fierz, Helv. Phys. Acta **17**, 181 (1944).

[18] M. Fierz and G. Wentzel, Helv. Phys. Acta **17**, 215 (1944); G. Wentzel, Helv. Phys. Acta **17**, 252 (1944).

where

$$H_0=\frac{1}{4M}P^2+\frac{1}{M}p^2+\epsilon\{(s_1+\tfrac{1}{2})^2+(s_2+\tfrac{1}{2})^2-2\}$$
$$+(f\mu)^2\frac{e^{-\mu r}}{r}\Gamma_0+k^0, \qquad (4)$$

$$H'=(f\mu)^2\frac{e^{-\mu r}}{r}\Gamma_1-\frac{e}{M}\{(t_3^{(1)}+\tfrac{1}{2})(\tfrac{1}{2}\mathbf{P}+\mathbf{p})\cdot\mathbf{A}(\mathbf{z}_1)$$
$$+(t_3^{(2)}+\tfrac{1}{2})(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\cdot\mathbf{A}(\mathbf{z}_2)\}.$$

with $\mathbf{A}(\mathbf{z}_i)$ given by

$$\mathbf{A}(\mathbf{z}_i)=\frac{1}{\sqrt{V}}\sum_k\frac{1}{(2k^0)^{\frac{1}{2}}}\{a(\mathbf{k},\lambda)\mathbf{e}_\lambda$$
$$\times\exp[i\mathbf{k}\mathbf{z}_i]+\text{conj.}\}\quad(i=1,2), \qquad (5)$$

H' is the perturbation term in our problem. The differential cross section for the photoelectric effect is given by perturbation theory to[19]

$$d\sigma=2\pi V|\mathfrak{M}|^2d\rho_E, \qquad (6)$$

where $d\rho_E$ represents the density of final states per unit energy range for the initial energy E. $\mathfrak{M}$ stands here for the matrix elements of the perturbation energy which connect the initial with the final states. It is equal to

$$\mathfrak{M}=H_{\mathrm{I}F}'=\int\Psi_{\mathrm{I}}H'\Psi_Fd\tau \qquad (7)$$

for the direct transitions and

$$\mathfrak{M}=\sum_{\mathrm{II}}\frac{H_{\mathrm{I\,II}}'H_{\mathrm{II}F}'}{E_{\mathrm{I}}-E_{\mathrm{II}}} \qquad (8)$$

for the indirect transitions. In the first case we obtain the first-order effect of the elastic cross section whereas the second case describes inelastic processes with excitation of the isobaric states. This latter case also contains a second-order correction to the elastic photo-effect which is however negligible and is disregarded here. There exists no direct (first-order) transitions which would excite the isobaric states. This is caused by the fact that the perturbation energy H' is a sum of two operators, the first of which gives only rise to transitions of the internal states of the nucleon whereas the second changes one of the numbers of the photons in the radiation field. A simultaneous change of both kinds is therefore only possible through a second-order transition of the type (8).

For the wave functions Ψ_{I}, Ψ_{II}, Ψ_F which describe the initial, intermediate, and final states we have the following expression

$$\Psi_{\mathrm{I}}=\Phi(1_{\mathbf{k},\lambda})\psi_D(\mathbf{z})\delta_{s_11/2}\delta_{s_21/2}\delta_{T,0}\delta_{S,1}$$
$$\times\delta_{M,M_0}\delta_{N,0}1/\sqrt{V}\exp[i\mathbf{K}\mathbf{Z}]. \qquad (9)$$

Here $\phi(1_{\mathbf{k},\lambda})$ denotes the wave function of the radiation field with one photon present of momentum $\mathbf{k}$ and polarization in the direction $\mathbf{e}_\lambda$. $\psi_D(\mathbf{z})$ is the wave function of the deuteron which in the ground state is a spin triplet $(S=1)$ and an isotopic spin singlet $(T=0)$. The total charge is $N+1=1$ so that $N=0$, while M, the spin component in the x_3-direction may assume the three values $M_0=0, \pm1$. Since the interaction operator is diagonal with respect to M it follows that the cross section is independent of M_0 and we may take for the initial state an arbitrary value of M_0 and calculate with it the total cross section without any further averaging. The last factor in (9) represents the wave function of the motion of the center of mass with momentum vector $\mathbf{K}$. The wave function is antisymmetrical with respect to interchange of the two particles as it is required by the exclusion principle, since the symmetry character of the spin-isotopic spin function is $(-1)^{S+T}$ and $\psi_D(-\mathbf{z})=\psi_D(\mathbf{z})$.

For the intermediate states II which in the elastic case (7) are the final states, we have two possibilities since the transition may occur to states with $T=1$ as well as $T=0$. Since the Hamiltonian (4) is symmetrical in the two particles these states will again be antisymmetrical as the initial states I. If the energy of the ejected particles is sufficiently high we may treat them as free, a procedure which is characteristic for the Born approximation. We write therefore

$$\Psi_{\mathrm{II},1,2}=\Phi(0)\left(\frac{2}{V}\right)^{\frac{1}{2}}\begin{Bmatrix}\sin\,\boldsymbol{\kappa}\mathbf{z}\delta_{T,1}\\ \cos\,\boldsymbol{\kappa}\mathbf{z}\delta_{T,0}\end{Bmatrix}$$
$$\times\delta_{S,1}\delta_{M,M_0}\delta_{N,0}\delta_{s_11/2}\delta_{s_21/2}\frac{1}{\sqrt{V}}\exp[i\mathbf{K}'\mathbf{Z}]. \qquad (10)$$

[19] Cf. W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1936), first edition, p. 121.

Here $\boldsymbol{\kappa}$ denotes the value of the relative momentum of the two particles and $\mathbf{K}'$ is the momentum of the center of mass.

The final states F have one of the quantum numbers $s=\frac{3}{2}$. Since the operator Γ is symmetrical in $s_1 s_2$ and the initial state is $s_1=s_2=\frac{1}{2}$, it is only the symmetrical combination of these two states which can occur as final state. The inequality (3) forbids any final states with $T=0$, thus only one final state is possible described by the wave function

$$\Psi_F=\Phi(0)\left(\frac{2}{V}\right)^{\frac{1}{2}} \sin \boldsymbol{\kappa}'\mathbf{z}\,\delta_{T,1}\delta_{S,1}\delta_{M,M_0}\delta_{N,0}\frac{1}{\sqrt{2}} \times\{\delta_{s_1 1/2}\delta_{s_2 3/2}+\delta_{s_1 3/2}\delta_{s_2 1/2}\}\frac{1}{\sqrt{V}}\exp[i\mathbf{K}'\mathbf{Z}]. \quad (11)$$

The energy of the unperturbed system which belongs to these states is

$$\left.\begin{aligned} E=E_{\mathrm{I}}&=\frac{1}{4M}K^2+E_D+k^0,\\ E_{\mathrm{II}}&=\frac{1}{4M}K'^2+\frac{1}{M}\kappa^2,\\ E_F&=\frac{1}{4M}K'^2+\frac{1}{M}\kappa'^2+3\epsilon. \end{aligned}\right\} \quad (12)$$

From the conservation of energy we have

$$\frac{1}{4M}K^2+E_D+k^0=\frac{1}{4M}K'^2+\frac{1}{M}\kappa'^2+3\epsilon. \quad (12')$$

For our special choice of the initial state of the radiation field we have for the matrix element of the field operators

$$\Phi(1_{\mathbf{k}\lambda})a^*(\mathbf{k}'\lambda')\Phi(0)=\delta_{\lambda\lambda'}\delta_{\mathbf{k}\mathbf{k}'} \quad (13)$$

$$\Phi(1_{\mathbf{k}\lambda})a(\mathbf{k}'\lambda')\Phi(0)=0. \quad (14)$$

From (5), (7), (9), (10), and (11) we find for the different matrix elements[20]

$$H_{\mathrm{I\,II}_1}'=\frac{e}{M}\frac{i}{\sqrt{k^0 V}}(\boldsymbol{\kappa}\cdot\mathbf{e}_\lambda)\delta_{\mathbf{K}+\mathbf{k}-\mathbf{K}'} \times\int\psi_D^*(\mathbf{z})\exp\left[-\frac{i}{2}\mathbf{k}\cdot\mathbf{z}\right]\cos\boldsymbol{\kappa}\mathbf{z}\,d^3z, \quad (15)$$

$$H_{\mathrm{I\,II}_2}'=-\frac{e}{M}\frac{i}{\sqrt{k^0 V}}(\boldsymbol{\kappa}\cdot\mathbf{e}_\lambda)\delta_{\mathbf{K}+\mathbf{k}-\mathbf{K}'} \times\int\psi_D^*(\mathbf{z})\exp\left[-\frac{i}{2}\mathbf{k}\mathbf{z}\right]\sin\boldsymbol{\kappa}\mathbf{z}\,d^3z, \quad (16)$$

$$H_{\mathrm{II}F}'=\frac{16}{9}\frac{(f\mu)^2}{V} \times\int\frac{\exp[-\mu r]}{r}\sin\boldsymbol{\kappa}\mathbf{z}\sin\boldsymbol{\kappa}'\mathbf{z}\,d^3z. \quad (17)$$

The δ-function in the first two equations expresses the conservation of momentum

$$\mathbf{K}+\mathbf{k}-\mathbf{K}'=0. \quad (18)$$

The relations (12′) and (18) which are four equations with six variables determine the energy of the ejected particles as a function of their direction. This dependence is useful if one has to distinguish experimentally between protons which are emitted in the elastic or the inelastic process. We have for the velocity v_P of the proton which is emitted in the direction θ with the incident γ-ray if the deuteron was initially at rest ($K=0$)

$$v_P=\frac{k}{2M}\left\{\cos\theta\pm\left(4\frac{|\kappa|^2}{|k|^2}-\sin^2\theta\right)^{\frac{1}{2}}\right\}, \quad (19)$$

where $|\kappa|^2$ has to be taken from the energy Eq. (12′) without or with the last term 3ϵ according to whether the process is elastic or inelastic.

4. EVALUATION OF THE CROSS SECTION

For the evaluation of the integrals in (15), (16) we assume for the wave function of the deuteron in the ground state

$$\psi_D(\mathbf{z})=(\beta/2\pi)^{\frac{1}{2}}(1/r)e^{-\beta r} \quad \text{with} \quad \beta=(M|E_D|)^{\frac{1}{2}}.$$

[20] The numerical factor in $H_{\mathrm{II},F}'$ is obtained from Fierz' expression of the matrix elements for Γ (cf. reference 17).

With this function the integrals may readily be evaluated and we have

$$\int \psi_D^*(\mathbf{z}) \exp\left[-\frac{i}{2}\mathbf{k}\cdot\mathbf{z}\right] \cos \boldsymbol{\kappa}\mathbf{z} = (8\pi\beta)^{\frac{1}{2}} \frac{\beta^2+\kappa^2+\frac{1}{4}k^2}{(\beta^2+(\boldsymbol{\kappa}+\frac{1}{2}\mathbf{k})^2)(\beta^2+(\boldsymbol{\kappa}-\frac{1}{2}\mathbf{k})^2)}, \tag{20}$$

$$\int \psi_D^*(\mathbf{z}) \exp\left[-\frac{i}{2}\mathbf{k}\cdot\mathbf{z}\right] \sin \boldsymbol{\kappa}\mathbf{z} = -i(8\pi\beta)^{\frac{1}{2}} \frac{\boldsymbol{\kappa}\cdot\mathbf{k}}{(\beta^2+(\boldsymbol{\kappa}+\frac{1}{2}\mathbf{k})^2)(\beta^2+(\boldsymbol{\kappa}-\frac{1}{2}\mathbf{k})^2)}, \tag{21}$$

$$\int \frac{\exp[-\mu r]}{r} \sin \boldsymbol{\kappa}\mathbf{z} \sin \boldsymbol{\kappa}'\mathbf{z} = \frac{8\pi\boldsymbol{\kappa}\cdot\boldsymbol{\kappa}'}{(\mu^2+(\boldsymbol{\kappa}-\boldsymbol{\kappa}')^2)(\mu^2+(\boldsymbol{\kappa}+\boldsymbol{\kappa}')^2)}. \tag{22}$$

The density function $d\rho_E$ for the final states with propagation vector $\boldsymbol{\kappa}$ situated within a solid angle $d\omega$ around a fixed direction (ϑ, φ) is given by

$$d\rho_E = \frac{1}{2}\frac{V}{(2\pi)^3}\kappa^2\frac{d\kappa}{dE}d\omega = \frac{V}{32\pi^3}M\kappa d\omega. \tag{23}$$

For the differential cross section of the elastic photoelectric effect we obtain from (6), (20), (21), and (23) for an incident radiation with momentum $\mathbf{k}$ and polarization direction $\mathbf{e}_\lambda$

$$d\sigma = \frac{1}{2\pi}\frac{e^2}{Mk^0}\beta\kappa(\mathbf{e}_\lambda\cdot\boldsymbol{\kappa})^2\left\{\frac{(\beta^2+\kappa^2+\frac{1}{4}k^2)^2+(\boldsymbol{\kappa}\cdot\mathbf{k})^2}{(\beta^2+\kappa^2+\frac{1}{4}k^2)^2-(\boldsymbol{\kappa}\cdot\mathbf{k})^2}\right\}.$$

If we average over all directions of the polarization of the incident light we obtain for

$$(\mathbf{e}_\lambda\cdot\boldsymbol{\kappa})_{\text{Av}}^2 = \kappa^2\sin^2\vartheta\frac{1}{2\pi}\int_0^{2\pi}\cos^2\varphi d\varphi = \tfrac{1}{2}\kappa^2\sin^2\vartheta.$$

For the total cross section we obtain then by integrating over all the directions of κ the formula:

$$\sigma = \frac{e^2}{Mk^3}\beta\kappa\left\{\frac{\beta^2+\kappa^2+\frac{1}{4}k^2}{\kappa k}\ln\frac{\beta^2+\kappa^2+\frac{1}{4}k^2+\kappa k}{\beta^2+\kappa^2+\frac{1}{4}k^2-\kappa k}-2\right\}. \tag{24}$$

For $\kappa k \ll \beta^2+\kappa^2+\frac{1}{4}k^2$ this expression goes over into the formula of Bethe and Peierls,[21]

$$\sigma = \frac{2}{3}\frac{e^2}{Mk}\frac{\beta\kappa^3}{(\beta^2+\kappa^2+\frac{1}{4}k^2)^2}. \tag{25}$$

These two expressions (24) and (25) are only then appreciably different from each other if $k/M \sim 1$ for 100-Mev γ-rays $k/M \sim \frac{1}{9}$. For these energies the error due to the simplified wave function of the deuteron is probably still larger. It must be remarked also that for $k \sim M$ the relativistic effects which we did not consider here become important. The inelastic cross section is obtained from (6) and (8).

[21] H. A. Bethe and R. Peierls, Proc. Roy. Soc. **A148**, 146 (1945). In comparing our formula with theirs, one has to remember that the unit of charge is so chosen that $e^2 = 4\pi/137$.

For $\mathfrak{M}$ we obtain with the help of (20) and (22)

$$\mathfrak{M}=\frac{16}{9}(f\mu)^2\frac{(8\pi)^{\frac{1}{2}}}{(k^0)^{\frac{1}{2}}V}i\sqrt{\beta}\int(\boldsymbol{\kappa}\cdot\mathbf{e}_\lambda)\frac{\beta^2+\kappa^2+\frac{1}{4}k^2}{(\beta^2+\kappa^2+\frac{1}{4}k^2)^2-(\boldsymbol{\kappa}\cdot\mathbf{k})^2}$$

$$\times\frac{\boldsymbol{\kappa}\cdot\boldsymbol{\kappa}'}{(\mu^2+\kappa^2+\kappa'^2)^2-4\boldsymbol{\kappa}\cdot\boldsymbol{\kappa}'}\,\frac{1}{E_D+k^0-\dfrac{1}{4M}k^2-\dfrac{1}{M}\kappa^2}d^3\kappa. \quad (26)$$

The integration must be carried out over all values of the wave vector for the intermediate states. The last factor has a pole for

$$\kappa=(Mk^0-\beta^2-\tfrac{1}{4}k^2)^{\frac{1}{2}}.$$

The value of the integral would thus become undetermined unless some boundary conditions for the wave functions are introduced. The condition which is here necessary requires that in the intermediate states for large distances from the scatterer only out-going spherical waves shall exist. As is shown in the appendix this condition leads for the integration over the magnitude of κ to the rule that the pole must be avoided by displacing the path of integration around the pole into the lower half of the complex κ-plane.

We carry out first the integration over the angles. Here a simplification is possible if the energy of the γ-radiation is not too large; more precisely, if

$$\kappa k/(\beta^2+\kappa^2+\tfrac{1}{4}k^2)\ll 1. \quad (27)$$

This is the same condition which allowed us to replace (24) by (25).[22] In this case we may neglect the variation of the denominators and we obtain for $\mathfrak{M}$ the simplified expression:

$$\mathfrak{M}=\frac{16}{9}(f\mu)^2\frac{e}{V}\left(\frac{\beta}{k}\right)^{\frac{1}{2}}\frac{(8\pi)^{\frac{1}{2}}}{(2\pi)^3}i\int d^3\kappa\,\frac{(\boldsymbol{\kappa}\cdot\mathbf{e}_\lambda)(\boldsymbol{\kappa}\cdot\boldsymbol{\kappa}')}{(\beta^2+\kappa^2+\frac{1}{4}k^2)(\mu^2+\kappa^2+\kappa'^2)^2(kM-\beta^2-\frac{1}{4}k^2-\kappa^2)}.$$

The integration over the angles is now easily carried out and the result is

$$\mathfrak{M}=\frac{16}{9}(f\mu)^2\frac{e}{V}\left(\frac{\beta}{k}\right)^{\frac{1}{2}}\frac{(8\pi)^{\frac{1}{2}}}{(2\pi)^3}i(\boldsymbol{\kappa}'\mathbf{e}_\lambda)I,$$

with

$$I=\int_0^\infty\frac{\kappa^4d\kappa}{(A^2+\kappa^2)(B^2+\kappa^2)^2(C^2-\kappa^2)},$$

$$A^2=\beta^2+\tfrac{1}{4}k^2,$$

$$B^2=\mu^2+\kappa'^2,$$

$$C^2=kM-\beta^2-\tfrac{1}{4}k^2.$$

Since the integrand is an even function in κ, we may extend the path of integration from $-\infty$ to $+\infty$. The pole $\kappa=-C$ on the negative real axis will then be avoided by displacing the path of integration into the positive imaginary half-plane. Since the integrand goes to zero like κ^{-4} for large κ we can complete the path of integration by a large half-circle in the positive imaginary half-plane to a closed path and then use the theorem of the residues. With this method we obtain

$$I=i\pi\{\mathrm{Res}\,(iA)+\mathrm{Res}\,(iB)+\mathrm{Res}\,(C)\}.$$

[22] This approximation corresponds to the neglection of multipole transitions. For 100-Mev γ-rays the error due to this approximation is about 20 percent. Neglecting the tensor force introduces an uncertainty which is probably larger than this error.

A detailed numerical discussion shows that the first two terms may be neglected for γ-rays with energies $\sim$100 Mev.[23] We find thus for the integral the approximate value

$$I = i\frac{\pi}{2}\frac{C^3}{(A^2+C^2)(B^2+C^2)^2}.$$

The cross section for the inelastic photo-effect becomes then

$$\sigma' = 0.053(f\mu)^4 e^2 \frac{\beta}{k^3}\frac{\kappa'^3}{M}\frac{(kM-\beta^2-\frac{1}{4}k^2)^3}{(\mu^2+\kappa'^2+kM-\beta^2-\frac{1}{4}k^2)^4}.$$

The ratio of the inelastic to the elastic cross section is

$$\frac{\sigma'}{\sigma} = 0.08(f\mu)^4 M^2 \frac{(Mk-\beta^2-\frac{1}{4}k^2)^{\frac{3}{2}}(Mk-\beta^2-\frac{1}{4}k^2-3\epsilon M)^{\frac{3}{2}}}{(\mu^2+2Mk-2\beta^2-\frac{1}{2}k^2-3\epsilon M)^4}.$$

For $k\sim 100$ Mev we may neglect the term $\frac{1}{4}k^2$ in all these brackets and we have, writing $x = Mk - \beta^2 - 3\epsilon M$,

$$\sigma'/\sigma = 0.08(f\mu)^4 M^2 F(x),$$

$$F(x) = \frac{x^{\frac{3}{2}}(x+3\epsilon M)^{\frac{3}{2}}}{(\mu^2+3\epsilon M+2x)^4}.$$

F has a maximum at $x = \frac{1}{2}\{3\mu^2 - 6\epsilon M + [(3\mu^2-6\epsilon M)^2 + 18\epsilon M(\mu^2+3\epsilon M)]^{\frac{1}{2}}\}$ which corresponds to an energy of the incident γ-rays of 87 Mev. For this value of the energy we have

$$\sigma'/\sigma \sim 0.125 \times (f\mu)^4.$$

For $(f\mu)^4$ Pauli and Kusaka[6] find the value 0.14 while Hulthen[24] obtains $\sim$0.58. Hulthen's value is calculated without the tensor force which furnishes additional attractive potential in the ground state of the deuteron. This is the reason for the somewhat higher value. Since we have calculated without the tensor force throughout, Hulthen's value is probably the better. The correct value would probably be somewhere between these two. With Hulthen's value we find for σ'/σ near the maximum:

$$\sigma'/\sigma = 7.2 \text{ percent.} \tag{28}$$

5. CONCLUSION

It is seen from (28) that the ratio of the number of inelastic to the number of elastic photoelectric dissociations of the deuteron is near the maximum about 7 percent. The maximum is reached for an initial energy of the γ-rays of 87 Mev if we assume the first excited states of the nucleons at 45 Mev. It is conceivable that a systematic search for the inelastic dissociation process might lead to the discovery of the excited states of the nucleons.

APPENDIX

We use the stationary state method of perturbation theory and write for the problem

$$(H+H')\psi = E\psi.$$

E is the total energy, H is the unperturbed Hamiltonian given by (ψ), and H' is the perturbation energy. The states are labeled by k, κ, s where k stands for the state of the radiation field, κ describes the relative motion of the nucleons, and s contains the quantum numbers for the internal states of the nucleons. We write then for the wave function the development

$$\psi = \psi_0 + \psi_1 + \psi_2 + \cdots,$$

where the terms are arranged in rising order of smallness. For these wave functions we obtain then the following set of equations:

$$H\psi_0 = E\psi_0,$$
$$H'\psi_1 = (E-H)\psi_0,$$
$$H'\psi_2 = (E-H)\psi_1, \cdots.$$

Let the state of the unperturbed system be denoted by $\psi_0(k_0\kappa_0 s_0)$. Since H is diagonal in the variables $k_0\kappa_0 s_0$, this function will be a δ-function in these variables representing a photon with momentum and polarization, characterized by the letter k_0, a deuteron in the ground state with center of gravity at rest (κ_0) and the internal degrees of freedom s_0 of two nucleons in the ground state.

[23] I am indebted to Mr. E. Gross for some help with the evaluation of these integrals.

[24] L. Hulthen, Arkiv för mat. astr. och fysik **A28**, No. 5 (1942).

From the second equation we obtain then

$$\psi_1(k'\kappa's')=(k'\kappa's'|H'|k_0\kappa_0s_0)\left\{\frac{1}{E-E_{\rm I}}-i\pi\delta(E-E_{\rm I})\right\},$$

where $E_{\rm I}$ is the energy which belongs to the state $(k'\kappa's')$. The second term is so chosen that this solution when transformed to X-space represents only outgoing spherical waves provided that we define an integration over the pole $E=E_{\rm I}$ always by its principle value.[25] The second-order solution which is of interest in our problem becomes then

$$\psi_2(k,\kappa,s)=\frac{1}{E-E_F}\sum_{k'\kappa's'}(k,\kappa,s|H'|k'\kappa's')(k'\kappa's'|H'|k_0\kappa_0s_0)\times\left\{\frac{1}{E-E_{\rm I}}-i\pi\delta(E-E_{\rm I})\right\}.$$

The summation on the right-hand side is equivalent with an integration without δ-term in such a way that the path of integration is displaced in the complex κ-plane around the pole in the negative imaginary half-plane. This was the procedure used in the text.

[25] Compare P. A. M. Dirac, The Principles of Quantum Mechanics, Clarendon Press, Oxford, second ed., p. 195ff.

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On the Heavy-Electron Pair Theory in the Limit of Strong Coupling

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The pair theory of Marshak and Weisskopf was investigated assuming strong coupling. The strong coupling criterion is $A=(Nf/\mu)>5$ where f = coupling constant, μ = heavy electron mass, $N=\int U^2(x)d^3x$, $U(x)$ = source function of the nucleon. $N\cong 1/\pi a^3$, where a = source radius (all in units where $\hbar=c=1$). With this condition, the magnetic moment turns out to be of order A^{-1}, and of magnitude too small to account for the observed anomalies. The leading term in the potential of the force between two nucleons (for $r_{AB}>2a$) is independent of the spin orientations, of the coupling constants, and of the type of coupling (as long as no derivatives of the heavy-electron field quantities occur in the coupling term). This potential is identical with the one calculated by Jauch and Houriet, but was not considered by Nelson and Oppenheimer. The next term in the potential is of order A^{-3}. It corresponds to a superposition of a $(\Sigma_A\cdot\Sigma_B)$ term, a tensor force term, and an ordinary force. Being of order A^{-3} it is too small, however, to fit the experimental results.

1. INTRODUCTION

THIS paper deals with the strong coupling version of the meson theory proposed by Marshak and Weisskopf.[1] In this theory the meson field quantity is assumed to be a complex spinor. The associated quanta (mesons) are then heavy electrons in all respects. They satisfy Dirac's equation in the absence of interactions with nucleons. The negative energy states are taken into account by the usual hole-theory approach. The interaction term with the nucleon is assumed to be quadratic in the meson field quantities. This identifies the theory as a "pair theory." The name refers to the fact that in the weak coupling approximation the mesons are emitted and reabsorbed by the nucleons in pairs. The particular interaction term proposed by Marshak and Weisskopf is of the form

$$\bar{H}_i=f\psi^+(z)P_s\psi(z). \quad (1.1)$$

Here $\bar{H}_i$ is the interaction term in the Hamiltonian, f is a coupling constant, ψ is the meson field quantity (an operator), a cross denotes its Hermitean conjugate, z is the position of the nucleon in question, and P_s is one of several operators which are possible if we want the theory to be relativistically invariant. Marshak points out that it is possible, in a weak coupling theory, to eliminate all but one of them by a consideration of the deuteron problem.

It is true, however, that a weak coupling theory based on an interaction term of the form (1.1) can never explain the anomalous magnetic moments of the proton and neutron. Since $\bar{H}_i$ contains both ψ^+ and ψ, the isotopic spin of the

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[1] R. E. Marshak, Phys. Rev. 57, 1101 (1940); R. E. Marshak and V. F. Weisskopf, Phys. Rev. 59, 130 (1941).