Neutron Scattering in Ortho- and Parahydrogen and the Range of Nuclear Forces*

C. S. WU, L. J. RAINWATER, W. W. HAVENS, JR., AND J. R. DUNNING
Department of Physics, Columbia University, New York, New York
February 15, 1946

USING the results of Alvarez and Pitzer[1] on the neutron scattering cross section of ortho- and parahydrogen [$\sigma_{para}=(5.2\pm0.6)\times10^{-24}$ cm² and $\sigma_{ortho}=(100\pm3)\times10^{-24}$ cm²]. Schwinger[2] has calculated the range of the neutron proton forces and found this to be either zero or 8×10^{-13} cm. Both of these values are theoretically unreasonable. Schwinger also showed that the value of the neutron scattering cross section of the free proton calculated from these values was incompatible with the experimental value.

We wish to point out that, if a reasonable range of the neutron proton force is assumed to be 2.8×10^{-13} cm² and the more recently measured value of 21×10^{-24} cm² [3] is taken for the neutron scattering cross section of the free proton, the calculated value of the cross section of parahydrogen is not very different from that measured by Alvarez. However, the measured value of the scattering cross section of orthohydrogen cannot be reconciled with the calculated value.

In contemplating a repetition of the experiments of Alvarez and Pitzer with the Columbia University neutron velocity spectrometer the theoretically expected cross sections were calculated. The two equations derived by Schwinger[2] for the scattering cross section of ortho- and parahydrogen under Alvarez's experimental conditions were used. These equations are:

$$\sigma_{para}=\sigma_{0\to0}=6.473(3a_1+a_0)^2$$
$$\sigma_{ortho}=\sigma_{1\to1}+\sigma_{1\to0}=6.291[(3a_1+a_0)^2+2(a_1-a_0)^2]+1.447(a_1-a_0)^2,$$

where a_0 and a_1 are the singlet and triplet scattering amplitudes which are related to the corresponding cross sections by $\sigma_0=4\pi a_0^2$ and $\sigma_1=4\pi a_1^2$. The slow neutron cross section of the free proton can also be expressed in terms of a_1 and a_0 by

$$\sigma_{fp}=\pi[3a_1^2+a_0^2]=\tfrac{1}{4}\pi[(3a_1+a_0)^2+3(a_1-a_0)^2].$$

The value of a_1 can be expressed in terms of the energy of the triplet state (which is accurately known) and the range of the nuclear forces. The corresponding a_0 can be obtained from the equation for the cross section of the free proton. Recently the value of the cross section of the free proton was remeasured using the Columbia University neutron velocity spectrometer and found to be $(21\pm1)\times10^{-24}$ cm² which is in good agreement with the value obtained by Hanstein.[3]

The variation of the calculated cross section of both ortho- and parahydrogen with the range of the neutron proton forces is shown in Fig. 1 for three different values of the scattering cross section of the free proton which constitute the outside limit of error on the measurement of this value. If the value of the para scattering cross section obtained by Alvarez is used, the range of the nuclear forces will be $(1.7\pm0.8)\times10^{-13}$ cm from the curves which is not unreasonable.

From the point of view of remeasuring the values of ortho- and parahydrogen scattering cross section it is of interest to use the generally accepted value of the range of the nuclear forces as 2.8×10^{-13} cm² and find the expected value of the ortho- and parascattering cross sections. Using the scattering cross section of the free proton as 21×10^{-24} cm², the expected values are $\sigma_{para}=3.4\times10^{-24}$ cm² and $\sigma_{ortho}=125\times10^{-24}$ cm².

It does not seem unreasonable that the value obtained by Alvarez for the cross section of orthohydrogen is too low or that the value obtained for parahydrogen is too high. It seems quite possible that in the measurement of the ortho cross section the ratio of the para- and orthohydrogen in the cell was no longer that of normal hydrogen which is 1 to 3, respectively. At these extremely low temperatures the walls of the container might act as a catalyst for conversion of ortho- to parahydrogen. Also special

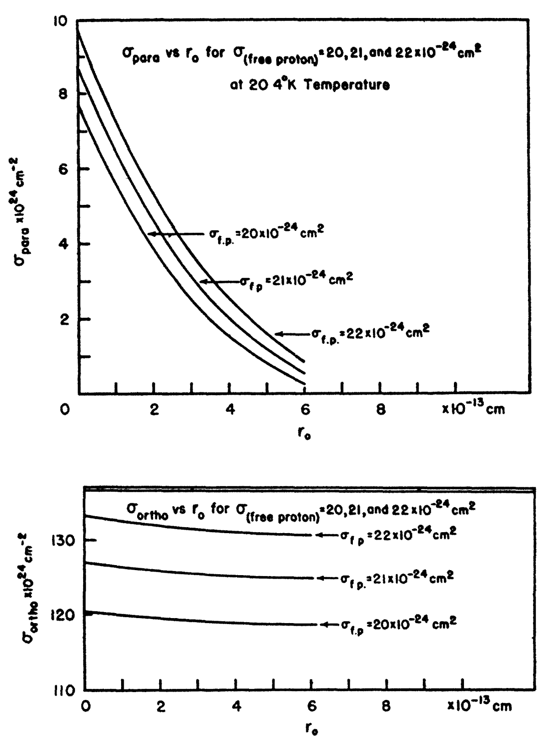

FIG. 1.

precautions might not have been taken to insure the normal composition of the gas before it was introduced into the cell.

An extremely small increase in the amount of orthohydrogen in the measurement of the parahydrogen cross section would account for the larger measured value of the cross section than the calculated value. Any improvement in the experimental technique in the performance of this experiment should tend to lower the measured value of the para cross section slightly and should increase the measured value of the ortho cross section considerably.

* Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University, New York, New York.

[1] L. W. Alvarez and K. S. Pitzer, Phys. Rev. **55**, 596 (1939); Phys. Rev. **58**, 1003 (1940).
[2] J. Schwinger, Phys. Rev. **58**, 1004 (1940).
[3] H. B. Hanstein, Phys. Rev. **59**, 489 (1941).

On the Abundance of Nuclei in the Universe

C. Lattes and G. Wataghin
Department of Physics, University of São Paulo, Brazil
February 11, 1946

THE correlation between the abundance of various nuclei in the universe and their mass defects, pointed out by several authors, gives us the first indication on the thermodynamical character of processes which determined the actual distribution of the isotopes. C. F. v. Weizsaecker,[1] S. Chandrasekhar and L. Heinrich[2] studied the problem and found by different methods the required temperature (with results of the expected order of magnitude: $kT \sim 10^6$ ev), but some disagreements between theoretical and observed abundances remained unexplained. The purpose of the present remark is to point out the advantages and the limitations of a statistical method, suggested some years ago by one of us,[3] and to give results of some preliminary calculations which may help to decide whether the nuclei were formed in a nearly thermal equilibrium state of matter of high temperature and density.

Obviously, if $kT \sim 10^6$ ev, many nuclear processes: as excitation of higher energy-states and higher spins, photo-effect, neutron evaporation and capture, β-processes and fission, become essential. Thus a non-negligible fraction of nuclei will not react in their normal states, and that is the chief reason of the difficulties of the Weizsaecker's method. Mesons production at this temperature can be neglected.

In view of the importance of β-processes under the conditions of our problem, it will be necessary to introduce explicitly the neutrinos into the statistics. Let N_s, n_{es}, n_{ps}, n_{ns}, n_{as}, n_{ZAs} be the numbers (per cm³) of photons, electrons, positrons, neutrinos and antineutrinos and nuclei of charge Z and mass-number A, which belong to the momentum interval p_s, p_s+dp_s. We assume the validity of the laws of conservation of charge

$$(\Sigma_s[n_{ps}-n_{es}+Zn_{ZAs}]=\text{const.}),$$

of energy, of the total number of nucleons

$$(\Sigma A n_{ZAs}=\text{const.})$$

and of

$$\Sigma_s[n_{ps}-n_{as}-n_{es}+n_{ns}]=\text{const.}$$

or

$$\Sigma_s[n_{ps}-n_{es}+n_{as}-n_{ns}+Zn_{ZAs}-(A-Z)n_{ZAs}]=\text{const.} \quad (1)$$

We obtain in the usual way the formula for n_{ZAs} already published[3] with a somewhat different meaning of the parameter α, because of (1). Integrating n_{ZAs} in the momentum space (summing with respect to s), one can show that the term ± 1 can be neglected and one obtains, with a good approximation the following value for the abundance

$$n_{ZAs}=1.735\times 10^{35}\times(2\zeta+1)\times\left[\frac{kT}{mc^2}\times\frac{E_{ZA}}{M_H c^2}\right]^{\frac{3}{2}} \times\exp\left[-\alpha Z+\gamma A-\frac{E_{ZA}}{kT}\right]. \quad (2)$$

Here E_{ZA} is the total energy of a nucleus at rest in the reference system of the centrum of gravity of the assembly

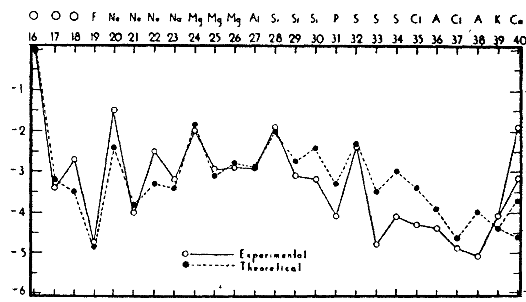

Fig. 1. Comparison of present theory with measured abundances.

(including the excitation-energy acquired in an inelastic collision). In the present preliminary survey, we shall substitute E_{ZA} with $M_{ZA}c^2$ where M_{ZA} is the rest mass of the nucleus in the fundamental state. In this way we obtain:

$$\log\frac{n_{ZA}}{n_{Z'A'}}\cong\log\frac{2\zeta+1}{2\zeta'+1}+\frac{3}{2}\log\frac{M_{ZA}}{M_{Z'A'}} +D\times(A-A')-B(M_{ZA}-M_{Z'A'})-a(Z-Z'). \quad (3)$$

Here the parameter values

$$a=\alpha\log e;\quad B=(c^2/kT)\log e;\quad D=\gamma\log e$$

can be determined from 3 values of $\log(n_{ZA}/n_{Z'A'})$ known experimentally. Because of the uncertainty of the experimental data and of the limitations of the suggested method, we have chosen by trial approximated values of these parameters and we obtained as a preliminary result the theoretical abundances (for between 8 and 20) indicated in Fig. 1, with: $a=0.1$, $D=525$, $D-B=-0.85$.

We have not considered the excited states and the higher spin states. We have not taken into account the changes in the relative abundance subsequent to the equilibrium and due to the radioactivity, photo-effect, evaporation, and fission processes at temperatures below the equilibrium when the main nuclear reactions are already slowed or stopped. The general satisfactory accord between the theoretical and observed abundances, shown in Fig. 1, leads us to the conclusion that nuclei were formed under conditions not far from the conditions of a thermal equilibrium. Further results and a detailed account of the calculations will be published elsewhere.

[1] C. F. v. Weizsaecker, Physik. Zeits. **39**, 633 (1938).
[2] S. Chandrasekhar and L. Heinrich, Astrophys. J. **95**, 288 (1942).
[3] G. Wataghin, Comptes rendus **203**, 909, (1935); Phys. Rev. **66**, 149 (1944).