been raised as to just how much the cadmium shield may be expected to cut out. Now the equilibrium energy of neutrons in nitrogen may be defined as that energy at which the capture and scattering cross sections are equal. Taking the values cited in previous work, this energy is about 0.1 ev. If we take the cadmium cutoff as about 0.3 or 0.4 ev, the reduction effected by the cadmium shield ought to be about in the ratio of the square roots of the energies, i.e., between 1.7 and 2 for the cadmium values mentioned. Inspection of Fig. 1 shows that the cadmium curve lies approximately half-way between the boron curve and the unshielded one.

* Cornell University, Ithaca, New York.
** Los Alamos, New Mexico.

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The Structure of Cosmic-Ray Air Showers

KONRAD L. KINGSHILL AND LLOYD G. LEWIS
Ryerson Physical Laboratory, The University of Chicago, Chicago, Illinois
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Bursts of ionization occurring in each of two thin-walled unshielded ionization chambers were measured at Chicago (elevation 190 meters), with the same apparatus used at Echo Lake (elevation 3100 meters). A comparison of the size-frequency distribution curve for bursts occurring in a single chamber at Chicago with the corresponding curve observed at Echo Lake shows that the altitude dependence for bursts containing more than 50 particles is similar to the altitude dependence of large air showers measured with Geiger-Müller counters. In contrast to this, the altitude dependence for the largest bursts is much greater. The ratio of the coincident burst rate to the single chamber burst rate is as low at Chicago as at Echo Lake. If one assumes the validity of the cascade theory of showers and of the theory of multiple scattering of electrons in air, one must conclude from these data that showers exhibiting these narrow regions of high particle density cannot originate close to the top of the atmosphere. The data presented in this paper, together with the data of Lapp and of Carmichael, show that the slope of the size-frequency distribution curve for bursts in a single chamber is dependent upon the chamber wall.

MEASUREMENTS with two thin-walled unshielded ionization chambers at Echo Lake, Colorado,[1] revealed the presence of many relatively narrow cosmic-ray air showers. These showers were more numerous and had a much smaller lateral extension of their high particle density region than is anticipated[2,3] on the basis of cascade theory applied to primary electrons. This result at Echo Lake makes the sea-level rates for the single bursts and for coincident bursts of particular interest, since the comparison of the two sets of data might give a clue to the vertical structure of air showers.

APPARATUS

The ionization chambers and the associated d.c. amplifiers[1] were used under exactly the same conditions as at Echo Lake. The galvanometers, however, were adjusted for a higher current sensitivity, so that bursts containing more than 30 particles could be measured. This higher sensitivity could be used at Chicago since the accidental coincidence rate is negligible for bursts considerably smaller than 80 particles. The precision with which the occurrence of a coincidence could be determined was increased by changing the rate of travel of the galvanometer film from 75 mm/hr. to 300 mm/hr. This made it possible to determine a burst coincidence to within 0.1 second (the collection time of the ions was 0.4 second).[1]

The whole apparatus was placed in the same portable houses used in the burst investigations at Echo Lake. It should be pointed out here that, in addition to the thin roof described previously, the walls and the floors of these

[1] L. G. Lewis, Phys. Rev. **67**, 228 (1945).
[2] L. Wolfenstein, Phys. Rev. **67**, 238 (1945).
[3] H. Euler, Zeits. f. Physik **116**, 73 (1940).

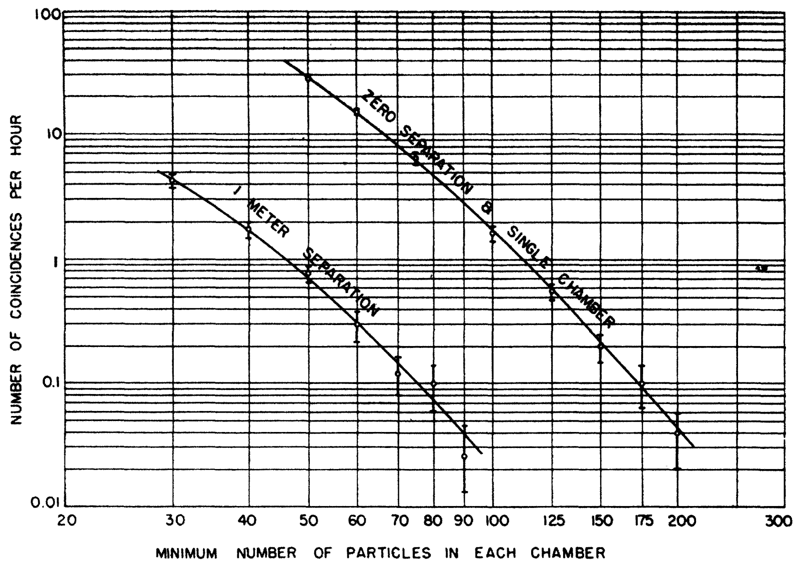

FIG. 1. The frequency of coincident bursts of more than a given number of particles in each chamber is plotted as a function of that number of particles (in a double logarithmic scale), for zero separation and for one meter separation of the ionization chambers. The zero-separation frequency is numerically equal to the single chamber rate.

portable houses consisted of sheets of plywood. The walls had an outer sheet of three-eighths of an inch and an inner sheet of one-quarter of an inch. The three-inch space between the inner and outer sheets was filled with loosely packed shredded redwood bark. This provided good heat insulation without bringing any heavy atoms or much total mass in the vicinity of the chambers. In order to avoid mesotron initiated showers or multiplication of electrons, the portable houses were located on an open space of the campus of the University of Chicago, with no buildings in the immediate neighborhood. The mechanical thermostats used at Echo Lake for controlling the temperature in the houses were replaced by vacuum tube thermostats capable of holding the inside temperature within 0.1°C. Each thermostat consisted essentially of a Wheatstone bridge. Two of the resistors were tungsten filament wires and the other two were Advance wires. An a.c. potential of low voltage was put across the bridge: when the bridge unbalance was changed due to a change in temperature the resulting signal change was amplified and actuated a relay controlling the current to the electrical heaters.

The calibration of the burst apparatus was carried out in the same way as at Echo Lake. The number of particles per burst was computed by using the value of 100 ion pairs per centimeter path for a pressure of argon of one atmosphere, determined for Echo Lake.[1] Test runs with the galvanometers disconnected from the ionization chamber circuits showed only very small fluctuations, probably caused by external vibrations or by Brownian motion. A run was performed with chamber *I* filled with argon as usual and with chamber *II* evacuated to a pressure of a few mm Hg. Bursts were recorded in chamber *I* unaccompanied by any fluctuations in chamber *II*, demonstrating that there was no coupling between the amplifiers of the two chambers. To make certain that the bursts recorded on the film were caused by particles entering from the outside, one chamber was covered with $1\frac{1}{2}$ inches of lead and the other was left unshielded. The shielded chamber recorded more than twice as many bursts containing more than 50 particles.

RESULTS AND DISCUSSION

A total of 2000 cosmic-ray bursts was recorded at Chicago. Over the entire period of time the absolute burst rates for the two chambers were the same within statistical error. For this reason it was possible to include data from both chambers in the cumulative size-frequency distribution curve of Fig. 1, labeled "zero separation and single chamber rate." This curve shows that the size-frequency distribution at Chicago for single bursts does not follow a power law with a constant exponent. This is in contrast to the computation of Wolfenstein,[2] who concluded that, if the showers are originated at the top of the atmosphere by primary electrons which have a power law spectrum, the size-frequency distribution curve for bursts must also be a power law with exponent smaller than that of the primary spectrum. The average slope of the curve given in Fig. 1 is approximately 4.6, which cannot be explained by a primary power law spectrum with exponent 1.8. It will be pointed out later in this paper that the slope of the size-frequency distribution curve is dependent on the ionization chamber construction.

The ratio of the single chamber burst rates at Echo Lake to those at Chicago is given in Fig. 2. In this graph the ratio of the frequency of bursts greater than a given size at Echo Lake to the frequency of the corresponding bursts at Chicago is plotted as a function of the burst size. Logarithmic scales are used on both coordinates. The points can be approximated by a straight line giving a power law relation between the ratio of the frequencies (R) and the minimum number of particles per burst (N).

$$R = KN^{\beta},$$

where K is a constant and $\beta = 2.7 \pm 0.3$. The point at $N > 50$ was obtained by extrapolating the Echo Lake data. This point, therefore, cannot be considered as accurate as the other points.

The theoretical results computed by Wolfenstein gave an increase of burst frequency of 5.7 for densities greater than 1000 per square meter (97 particle bursts) and an increase of 4 for densities greater than 2000 per square meter (193 particle bursts). These values are very much smaller than those given in Fig. 2. In addition, the slope of the calculated curve is negative whereas the slope of the observed curve is positive.

The altitude dependence of extensive cosmic-ray air showers was investigated by Hilberry,[4] who used a set of counters in fourfold coincidence. He found that the counting rate at Echo Lake was ten times higher than that at Chicago. The ratio computed from the cascade theory for his counter geometry was 15. In more recent experiments, Rogozinski[5] observed 7 as the ratio of the counting rates of coincidence counters for these two stations. All these results seem to show general agreement with the value given in Fig. 2 for the smallest size bursts. The slope of the curve in Fig. 2 is sufficient to explain why a relatively small variation in the counter geometry or in the counter shielding would result in the relatively large variation in the observed altitude dependence, because the geometry and the shielding of the counter arrangement select that particular size of shower which produces the majority of the counter coincidences.

The data on burst coincidences were taken only with a chamber separation of 1 meter. Present circumstances do not permit a more extensive investigation. The curve of Fig. 1 labeled "one meter separation" represents the frequency of coincident bursts of more than a given number of particles in each chamber as a function of that number of particles, for a chamber separation of one meter. The small percentage of coincident bursts in comparison with the single bursts is shown by a comparison of this curve with the "zero separation" curve,

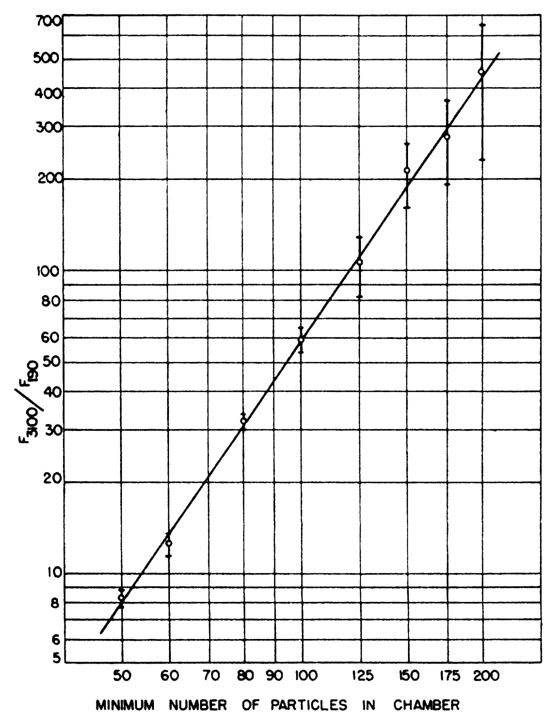

FIG. 2. The ratio of the frequency of occurrence of bursts with more than a given number of particles at Echo Lake (F_{3100}), to the frequency of occurrence of the corresponding bursts at Chicago (F_{190}), is plotted as a function of that number of particles.

[4] N. Hilberry, Phys. Rev. 60, 1 (1941).
[5] A. Rogozinski, Phys. Rev. 65, 291 (1944).

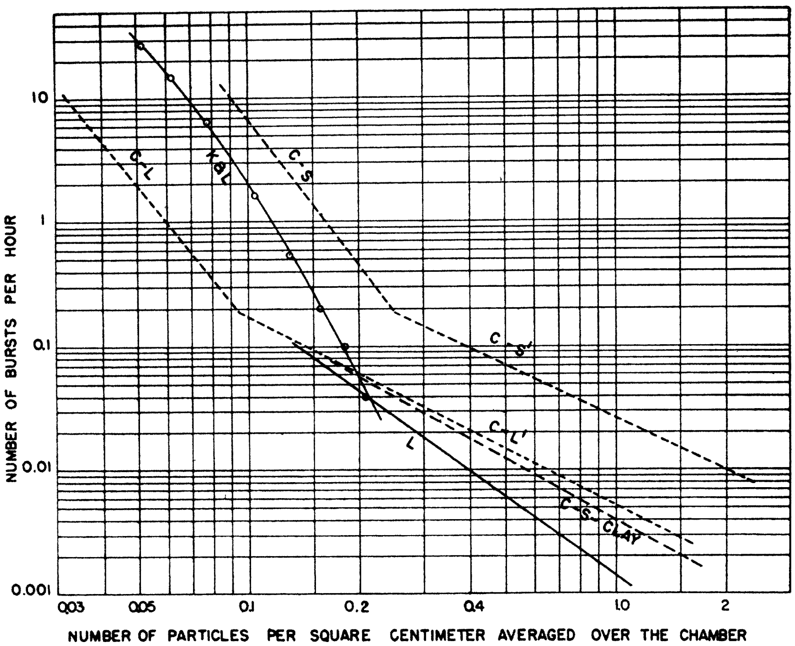

FIG. 3. Size-frequency distribution curves for various chambers are replotted on this graph to show the frequency of occurrence of bursts with greater than a given average particle density as a function of the particle density. All the data are reduced to a specific ionization of 100 ion pairs per cm. path length in argon of atmospheric pressure. The curve $C-S$ represents Carmichael's results taken with his small ionization chamber described in Table I; the curve $C-L$ represents those taken with Carmichael's large chamber; the curve L represents Lapp's data; and the curve *K&L*, the data presented in this paper. The curve $C-S$-clay represents the results obtained with Carmichael's small chamber shielded by 30 meters of clay. Carmichael's original curves differ slightly from the straight lines used in this figure.

Fig. 1. It is indicated that the ratio of coincident burst frequency to single burst frequency decreases with increasing burst size. This is similar to the result found at Echo Lake. Deflections on the photographic trace corresponding to 10 particles could be measured but were difficult to distinguish from statistical fluctuations: hence burst coincidence measurements were not extended below 30 particles. The measured frequency of the 30-particle burst coincidences is probably slightly too high because of the statistical fluctuations.[6]

A comparison of burst coincidence rates at the two altitudes was possible only for bursts of more than 80 particles in each chamber. At Echo Lake the coincidence rate was 14.5 per hour and at Chicago 0.1. The ratio of 145:1 between the two elevations is similar to that observed for single bursts of more than 120 particles (cf. Fig. 2), suggesting that single bursts of 120 particles and coincident bursts of 80 particles are caused by a similar type of air shower. The lack of burst coincidences, for a chamber separation of one meter, shown in Fig. 1, demonstrates that at Chicago the majority of the bursts of more than 50 particles must originate from showers having a small lateral spread of their high density region.

It is possible that one or more heavily ionizing particles with sufficient energy may traverse the chamber, giving rise to a burst of ionization comparable to that produced by more than 50 cosmic-ray particles. The nature of the present experiment, however, excludes the possibility that any appreciable fraction of the large bursts and burst coincidences in the unshielded ionization chambers can originate from such heavily ionizing particles. This is supported by the following facts: only protons of a relatively narrow energy range (about 10^7 ev) would be capable of traversing our chamber and still have velocities sufficiently small to produce a number of ion-pairs equivalent to a burst of more than 50 particles. Such protons could possibly enter the

TABLE I. Comparison of ionization chambers.

	Carmichael large chamber	Carmichael small chamber	Carnegie Model C	Kingshill and Lewis
Shape	cylinder	cylinder	sphere	sphere
Volume	175 liters	1 liter	19.7 liters	22.4 liters
Cross-sectional area	1960 cm²	~65 cm²	1000 cm²	965 cm²
Filling	115 cm Hg. argon	not specified	50 atmos. argon	100 cm Hg argon
Wall thickness:	3 mm steel	1.2 cm Al	1.25 cm steel	0.03 cm steel
in radiation units	0.167	0.12	0.69	0.017

[6] L. G. Lewis and R. Hayden, Phys. Rev. **65**, 346A (1944).

ionization chambers from the air, but this is an extremely rare event at elevations close to sea level. Protons may also be ejected from the walls of the chamber in a process of nuclear evaporation. This possibility is ruled out in the case of large bursts by the direct experiment of Lapp,[7] who measured bursts in a high pressure chamber of 1.25-cm wall thickness in coincidence with distant Geiger-Müller counters, and found that whenever a burst of more than 100 particles occurred in the ionization chamber the distant counters were tripped simultaneously. Most of the protons produced in nuclear evaporations would not possess enough energy to penetrate deep into Lapp's chamber, which was filled with 50 atmospheres of argon. On the other hand, the fact that Lapp's data can be correlated satisfactorily with the results of the present experiment is strong additional evidence that most of the large bursts in unshielded ionization chambers are not caused by the passage of nuclear particles through the chamber.

Lapp[7] and Carmichael[8,9] have reported size-frequency distribution curves for bursts occurring in single unshielded ionization chambers. Lapp used a Carnegie Model C chamber in coincidence with several sets of Geiger-Müller counters. This chamber was spherical and had about the same volume as the chambers used in the present investigation. Carmichael used two cylindrical chambers, one with a volume of 1 liter and the other with a volume of 175 liters. The specifications of the several chambers mentioned in this discussion are given in Table I.

The data of Lapp[7] and of Carmichael[8,9] are replotted on the graph of Fig. 3. In all the curves of Fig. 3 a specific ionization of 100 ion-pairs per cm path length per atmosphere of argon was used. The ordinate used in Fig. 3 is the frequency of occurrence of bursts with more than a given average particle density, and the abscissa the average particle density in the ionization chamber. These coordinates were chosen for burst data on air showers, since the information desired is the rate of occurrence of a shower of a given total number of particles.

[7] R. E. Lapp, Phys. Rev. **64**, 129 (1943).
[8] H. Carmichael and Chang-Ning Chan, Nature **144**, 325 (1939).
[9] H. Carmichael, Proc. Roy. Soc. **A154**, 223 (1936).

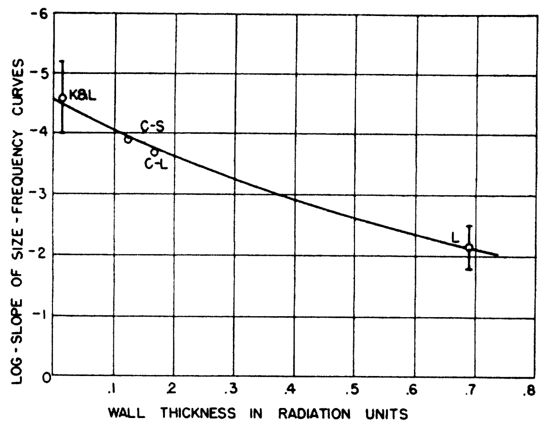

FIG. 4. This graph represents the logarithmic slope of the size-frequency distribution curves taken with the various ionization chambers described in Table I as a function of the wall thickness of the chambers measured in radiation units. The point $C-S$ gives the slope for Carmichael's small chamber; $C-L$ for Carmichael's large chamber; L for Lapp's chamber; and $K\&L$ for the chamber used in this experiment. It was not possible to determine the error limits for Carmichael's data.

Two facts concerning Carmichael's data should be mentioned before drawing any conclusions from the curves of Fig. 3. (1) Carmichael himself draws attention to the fact that the bursts of less than 100 ionizing rays were difficult to distinguish from the normal statistical fluctuations. This obviously means that his curves are not very accurate for the smaller size bursts. (2) The location of Carmichael's chambers was not very favorable for measuring particle densities in large air showers. His description of this location is as follows: "Above the ionization chamber, as shown in Fig. 1, is a strong wooden platform on which heavy material can be placed. The roof of the building is 8 feet above this platform, flat, made of concrete and supported by heavy iron girders. Directly above the ionization chamber, however, is a skylight, 6 feet wide." The walls of the building were probably in keeping with the roof and, therefore, were also heavy.

The portions $C-S'$ and $C-L'$ of Carmichael's size-frequency distribution curves given in Fig. 3 are very nearly parallel to the size-frequency distribution curve for bursts occurring under 30 meters of clay (Fig. 3, curve $C-S$-clay). Bursts occurring under this much shielding are caused by mesotron initiated showers. Therefore, the similarity in slope of these curves, and the

TABLE II.

	Wall thickness in radiation units	Slope of size-frequency distribution curves of Fig. 3
Kingshill and Lewis	0.017	−4.6 (approx.)
Carmichael (small)	0.12	−3.9 (approx.)
Carmichael (large)	0.167	−3.7 (approx.)
Lapp	0.69	−2.15±0.35

amounts of heavy material in the neighborhood of the supposedly unshielded ionization chambers, makes it probable that the portions of Carmichael's curves labeled $C-S'$ and $C-L'$ do not represent bursts due only to air showers, but represent bursts caused predominantly by mesotron initiated showers. This conclusion is strengthened by the data given in curve *K&L*. This curve covers a range of frequencies which includes the one corresponding to the discontinuities in the slopes of Carmichael's curves $C-L$ and $C-S$, yet does not exhibit such a discontinuity. For these reasons, we limit our discussion to those parts of Carmichael's curves ($C-S$ and $C-L$) which are most probably due to air showers alone.

Lapp's size-frequency distribution curve in Fig. 4 is not subject to the above criticism, since his chamber was located under the thin glass roof of the greenhouse on top of the Botany Building at Chicago. In addition, four Geiger-Müller counter coincidence sets were used in order to make sure that the recorded bursts of ionization were caused by air showers.

In spite of the fact that it is extremely difficult to make quantitative comparisons between bursts measured in cylindrical and in spherical ionization chambers, certain relations can be found between the various curves of Fig. 3. The most noticeable is that Carmichael's small chamber recorded the highest average particle density for bursts of a given frequency ($C-S$), whereas his large chamber recorded the lowest ($C-L$). The chambers used in this investigation had a cross-sectional area intermediate between Carmichael's small and large chambers, and recorded an intermediate value of the average particle density at a given frequency (*K&L*). This is precisely what one should expect from the comparison of single and coincident bursts represented in Fig. 1.

A second relation can be described as follows: Lapp's size-frequency distribution curve of Fig. 3, which has been shown to be due to large air showers, has a very different slope from that of the curve found in this experiment. This difference is presumably due to the fact that Lapp's chamber had a considerably heavier wall. It follows, then, since his chamber and the one used in this experiment were both spherical and had substantially the same diameter, that the average energy of the electrons in the showers producing the bursts at the point of intersection of the two curves *K&L* and *L* is such that the increase in the number of shower particles by multiplication in the wall of Lapp's chamber is approximately compensated for by a decrease of their number through absorption. For bursts of particle density less than at this point of intersection, the absorption effect apparently predominates, whereas, for bursts of particle density greater than that at the intersection point, the opposite is the case. The four curves in Fig. 3 seem to be in agreement with such an explanation, since their slopes are such that the curve corresponding to the chamber with the thinnest wall has the steepest slope (*K&L*); the curve for the next thinnest wall has the next steepest slope ($C-S$); and so on. This is demonstrated clearly by the figures given in Table II, and by the graph in Fig. 4. An exponential curve is drawn through the points of Fig. 4 (however, a linear curve could be drawn as well).

These comparisons of the three sets of experimental data lead to the following two conclusions: First, the density distribution function for air showers near sea level is such that measurements near the center of the showers should be made with very small ionization chambers (of the order of a few liters volume); second, that a wall thickness of even 0.1 of a radiation unit may alter the size-frequency distribution curve of bursts originating from air showers. Wall thicknesses usually described as "thin" are therefore not sufficiently thin for such measurements. Spherical chambers in general are much more suitable for burst investigations since a comparison between experiment and theory is possible.

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