The Single Scattering of Electrons in Gases†

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(Received June 11, 1945)

The single scattering of electrons was measured by means of a cloud chamber. Electrons having energies from 0.9 to 12 Mev were used in air, argon, krypton, and xenon. A total of 2173 meters of track yielded 801 deflections between 15° and 90°. The projected angles of scattering were compared with the theoretical values obtained by making a plane projection of the Mott scattering distribution. The errors inherent in the cloud chamber method and the corrections to be applied were treated in detail. From a combination of the data for all the energies and gases, the ratio of the experimental to the theoretical scattering was found to be 1.02. The angle intervals 25°–35°, 35°–45°, and 45°–55°, taken individually, show a somewhat greater ratio: 1.20, 1.43, and 1.25, respectively. Throughout the data the variations of the scattering cross section with changes in atomic number, energy, and angle were as expected within the experimental accuracy. No support whatever was found for the large discrepancies reported by several other authors.

INTRODUCTION

If cloud chamber measurements of the single scattering of electrons have been distinguished in any way, so far, it has been by the remarkable lack of agreement from one to another: variations of as much as a factor 40 have been reported for the same energy range and the same scattering gas. A review of all these experiments does not seem to be necessary at this point, since many of them are by now out of date as to apparatus and method, but in a later section of this paper a table of results will be given.

In setting out to do cloud chamber measurements, the experimenter is not at all free to choose any electron energy and any atomic number for the scattering gas he pleases, if he wishes to get usable data. A consideration of the scattering formula will assure him of this. The formula given by Mott[1] is

$$N(\theta)=\tfrac{1}{4}Nt\left(\frac{Ze^2}{mc^2\beta^2}\right)^2(1-\beta^2)\times\left[\csc^4\frac{\theta}{2}-\beta^2\csc^2\frac{\theta}{2}+\pi\beta Z\alpha\cos^2\frac{\theta}{2}\csc^3\frac{\theta}{2}\right],\quad(1)$$

where $N(\theta)$ is the average number of deflections of angle θ, per unit solid angle, which an electron will suffer in a path length t in a gas, where N is the number of atoms per cc of the gas, and where the other symbols have their usual meaning. The coefficient of the angle term depends upon the atomic number and density of the gas in the cloud chamber, and the energy of the electrons. If conditions are chosen such that this coefficient is large, a good yield of large angle deflections will be obtained, but the measurement of angle and curvature of the tracks will be rendered uncertain by the great amount of small angle scattering along the tracks. Going to the other extreme will cure the trouble arising from the small angle scattering, but will make the results statistically bad because of the poor yield of large deflections. The range in which a compromise can be made is surprisingly narrow. Our experience has shown us that satisfactory working conditions are obtained by making the coefficient such that the number of deflections greater than 15 degrees is between 0.75 and 0.25 per meter of track length. It is interesting to note that if we were to keep the number of atoms per cc of gas constant and juggle the atomic number of the gas and the energy of the electrons in such a way that the frequency of scattering in a given angle interval remained constant, we would be doing the equivalent of keeping the classical distance of closest approach the same for all gases, at a given angle of scattering. In our experiments we almost

† The material in this paper formed the major part of a doctoral thesis submitted by the first author at the University of Michigan in 1940. A preliminary report was given at the meeting of the American Physical Society at Pittsburgh, Pennsylvania, June 20–22, 1940. (Phys. Rev. **58**, 201 (1940)).
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[1] N. F. Mott, Proc. Roy. Soc. **A124**, 425 (1929).

do this, and consequently, in terms of the classical distance of approach, our results are somewhat restricted. We mention this mainly because the classical distance of approach is of some significance in considering the relative importance of the three angle terms in the Mott formula.

Offsetting the limitation just mentioned, there are some important advantages in the cloud chamber method: 1. The energy of an electron can be determined, both before and after scattering, which shows whether or not the collision was elastic. 2. With extremely small chance of error, every deflection counted can be assumed to be a single deflection, without recourse to the Wentzel[2] criterion for single scattering. 3. The greatest source of worry in other methods of measurement is eliminated, namely the chance that a very small number of slow electrons (secondaries due to collisions at slit edges, secondaries from bremsstrahlung, and electrons that have simply bounced around the chamber several times) may account for a large portion of the intensity observed at large angles, because of the extremely large cross section for scattering which these slow electrons have.

EXPERIMENTAL METHOD

Apparatus

The cloud chamber used was fully automatic, 15 cm in diameter. The details have been published in another journal.[3] A special camera arrangement was set up for this experiment, consisting of one Sept 35-mm camera seeing the chamber from directly above and another seeing the chamber in a direction 20 degrees from the vertical. Since the source of electrons was an element emitting a continuous beta-ray spectrum, it was necessary to devise a means by which a desired band of energies could be selected. The selection was made outside the chamber, in order to avoid, as far as possible, having the photographs cluttered with tracks of undesired energies. This was accomplished conveniently by the use of adjustable slits in the external part of the magnetic field, as shown in Fig. 1. After negotiating the slit system, the electrons passed into the chamber through a slot 5 cm wide and 1 cm high, which was ground in the glass side wall of the chamber, and which was covered with an aluminum or a copper foil about 0.001 inch thick. To hold the foil so that it would withstand pressures inside the chamber from vacuum up to 2 atmospheres, it was necessary to mount the foil on a heavy brass shoe about 4 inches long, having the same curvature as the chamber wall, which was fastened to the glass with Picein. The light beam which illuminated the tracks was limited, by a system of slits, to a sheet of light 2 cm thick, extending an equal distance above and below the electron entrance slot.

Three different radioactive sources were used for the electrons. For those below 1.5 Mev radiophosphorus, P^{32} was used; for those from 1.5 to 2.5 Mev a thin-walled capsule containing about 0.5 mg of radium in equilibrium with its products was used; and for those from 2.5 to 12 Mev radiolithium, Li^8 was used. The Li^8 was made by bombarding a lithium target with deuterons for about 1 second just prior to each expansion of the cloud chamber by a technique that has been described in this journal.[4,5] All three of the sources gave copious supplies of electrons, making it possible to work with a rather large distance (50 to 100 cm) between the source and the defining slits. An advantage in

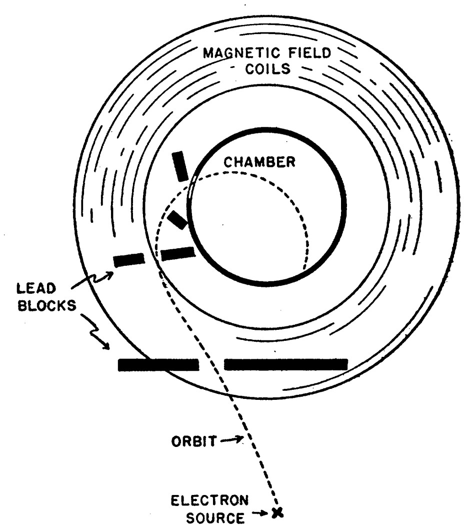

FIG. 1. System of slits used for selecting electrons within a narrow range of energy.

[2] G. Wentzel, Ann. d. Physik 69, 333 (1922).
[3] H. R. Crane, Rev. Sci. Inst. 8, 440 (1937).
[4] H. R. Crane, Phys. Rev. 52, 11 (1937).
[5] D. S. Bayley and H. R. Crane, Phys. Rev. 52, 604 (1937).

bending the electron paths in the external magnetic field, in addition to the selection of energies obtained, lay in the fact that large blocks of lead could be placed in the direct line between the source and the chamber to stop all gamma-radiation. This was of particular advantage when the radium source was used.

One liter each of xenon, krypton, and argon, 99 percent pure, were purchased for the experiments. An all glass system of "plumbing" was set up which made possible the evacuation of the chamber and the transfer of the gases back and forth between their bulbs and the chamber. An activated charcoal trap which could alternately be heated electrically and cooled with liquid air was used as a pump for transferring the gases. Each time the chamber was filled the pressure was read on a mercury barometer, and then a few cc of liquid alcohol were admitted through a needle valve. Upon transferring the rare gas back into the bulb the alcohol vapor was removed by a liquid air trap. The rare gases were transferred into the chamber and out again many times without mishap during the course of the experiments.

Selection and Measurement of Tracks

The selection of the tracks that are considered to be suitable for measurement is a matter that unavoidably involves some human judgement, with its attendant possibility of systematic error. The first requirement in combatting this is to design the mechanical set-up so that a relatively large percentage of all the tracks in the chamber are fit for measurement, and the second requirement is to make a system of rules such that the choice of tracks is as automatic as possible. The first of these requirements was met by using the slit system and pre-selection of energies already described. Confusion in the chamber was minimized by adjusting the electron intensity at all times so that an average of only three tracks appeared on each picture, and by adjusting the slits so that these had approximately the desired energy and were in the best part of the chamber. The second requirement was met by setting up the following rules of procedure for selection of the tracks for measurement.

1. The photograph taken from directly above the chamber is projected natural size onto a sheet of paper. On the paper is drawn a circle 11 cm in diameter, and this is centered in the image of the chamber, leaving a 2-cm border all around. Only the track length and only the scattering events lying within this circle are recorded. The purpose of the border is as follows: When a short piece of track lies inside the circle, the part in the border can be used as an aid in determining its curvature, but only the length inside the circle is included in the data. When a track is scattered just at the edge of the circle, the part in the border makes possible the determination of the angle of deflection. If such a circle were not used one would often meet the situation of being required to accept a scattering event just at the edge of the chamber, and of having no way of determining the angle of deflection.

2. Tracks accepted must fall within a range of curvature decided upon before beginning to measure the particular series of photographs. For each energy band used, the magnetic field is chosen so as to give the most readily measurable curvatures, usually centering around 10- or 15-cm radius. Typical limits for acceptance are 7 to 20 cm.

3. Tracks accepted must be sharp. A few old and fuzzy tracks were present in all the runs, but they amounted to only about 2 percent of the total in the usual run and perhaps 5 percent of the total in the worst runs. Most of these could have been eliminated by using a synchronized mechanical shutter in the path of the electrons, but instead we attempted to sweep out the early tracks by leaving the electrical field on until the completion of the expansion. This, we now know, was not entirely successful. Borderline cases were troublesome, in that they required a decision, and we regard these as a fertile source of error, for it is exceedingly difficult to form the same opinion of a partially fuzzy track when it has a 40-degree deflection in it as when it has no such deflection.

4. No minimum length is placed upon a track for acceptance. Any piece of track which begins at the entrance slot at the side of the chamber and which satisfies the other requirements must be accepted and its length added to the total. Most of the tracks extend across the entire chamber, but when one does not, it means that the electron passed out of the illuminated part of the chamber, which is a region 2 cm in height.

Scattering in the foil covering the entrance slot of the chamber, or, more often, one or more deflections in the gas of 5 or 10 degrees can cause the electron to pass out of the light beam when part way across the chamber.

After a track has been accepted, the procedure is to trace it with a sharp pencil, on the paper on which it is projected. This paper has the limiting circle on it, and in those cases in which it will be useful, the part of the track lying outside the circle is traced, as well as that lying inside. Only the photograph taken from directly above the chamber is projected and traced in this manner; the photograph taken from an angle is used when necessary to dispel confusion which arises when several tracks cross within a small area, when one track lies above another, etc.

The curvatures of the tracks are measured by fitting circles to them, a sheet of celluloid being used upon which is engraved a set of concentric arcs in steps of $\frac{1}{2}$-cm radius. The track length is measured with the help of a flexible rule, which is simply bent to fit each track. The angle of scattering is measured with a special pair of celluloid cards, as illustrated in an earlier publication.[6] Each of the cards has on it a set of concentric arcs, in steps of $\frac{1}{2}$-cm radius, and each has one edge which is a common radius of all the arcs. One card is right-handed and one is left-handed. In use one of the cards is placed so that one of the arcs coincides with the track and so that the radius edge cuts the track at the point of scattering. A pencil line is then drawn along the radius edge, which is perpendicular to the direction of the track at the point of scattering. The part of the track lying on the other side of the scattering point is treated the same way with the other card, and the angle between the two lines drawn along the radius edges is the angle of deflection of the electron. In a few cases the track obviously contains a minor deflection so that the segments must then be measured separately.

PROJECTION OF THE SCATTERING FORMULA

Most of the previous investigators have tried to have the conditions of measurement such that the results could be compared directly with the Mott formula, in its usual form. After some preliminary trials, we concluded that more accuracy could be obtained by measuring only the projected angle of deflection of the tracks and then projecting and integrating the scattering formula to conform to the experiment. No error is introduced by projecting the theoretical distribution, while one of the most dangerous sources of error is avoided by not having to measure the tracks and their deflections in three dimensions. As described in the section on the selection of tracks, the projected angles of deflection of all scattering events were included in the data, even those which were deflected sharply upward or downward. However, relatively little contribution is made to any given projected angle interval by tracks which have a large vertical component of scattering, because of the rapid fall-off in scattering cross section with increasing angle. Thus, for example, the number of electrons in the 35–45-degree interval in the projection is very largely composed of those actually scattered at 35–45 degrees, a point to be kept in mind when comparing the experimental and theoretical dependence of scattering cross section upon angle.

The projection of the Mott formula is not difficult, but is somewhat tedious, so only an outline of the method will be given here. By use of the substitution $mc^2\beta/e(1-\beta^2)^{\frac{1}{2}}=H\rho$ the Mott formula can be written

$$N(\theta)=\frac{NZ^2e^4}{4}\left(\frac{1}{H\rho}\right)^2\left[\frac{1}{\beta^2}\csc^4\frac{\theta}{2}-\csc^2\frac{\theta}{2}+\frac{\pi Z\alpha}{\beta}\csc^3\frac{\theta}{2}\cos^2\frac{\theta}{2}\right], \quad (2)$$

where $N(\theta)$ is the number of deflections per unit solid angle per cm track length, N is the number of atoms per cc in the gas, H is the magnetic field in gauss, and ρ is the radius of curvature of the track in cm. The coefficient in front of the bracket in (2) will be referred to as k. The resolution of the scattering angle θ into a horizontal angle ϕ and a vertical angle ψ is accomplished by the transformation $\cos\theta=\cos\phi\cos\psi$. Integrating over ψ from $-\pi/2$ to $+\pi/2$ we obtain the following result for the three terms

[6] N. L. Oleson, K. T. Chao, and H. R. Crane, Phys. Rev. **60**, 378 (1941).

of (2).

$$\frac{8k}{\beta^2 \sin^2 \phi}[1+(\pi-\phi)\cot\phi]d\phi=\frac{k}{\beta^2}f_1(\phi)d\phi,$$

$$-\frac{4k}{\cos\phi}\left[\frac{\pi-\phi}{\sin\phi}-\frac{\pi}{2}\right]d\phi=-kf_2(\phi)d\phi,$$

$$\begin{aligned}\frac{2^{\frac{3}{2}}\pi Z\alpha k}{\beta}\Bigg[1+\sum_{\text{even}}&\frac{1\cdot3\cdots2n+1}{2\cdot4\cdots2n}\cdot\frac{2\cdot4\cdots n}{1\cdot3\cdots n+1}\\ &\times(4n+1)\cos^n\phi+\frac{\pi}{2}\sum_{\text{odd}}\frac{1\cdot3\cdots2n+1}{2\cdot4\cdots2n}\\ &\times\frac{1\cdot3\cdots n}{2\cdot4\cdots n+1}(4n+1)\cos^n\phi\Bigg]d\phi\\ &=\frac{Zk}{\beta}f_3(\phi)d\phi.\end{aligned}\tag{3}$$

TABLE I. Values of function in Eq. (4).

	$f_1(\phi)$	$f_2(\phi)$	$f_3(\phi)$
15°	1410.0	39.6	1.80
20	594.0	27.9	0.960
30	177.0	16.9	0.395
40	76.5	11.7	0.201
50	39.6	8.72	0.116
60	23.6	6.80	0.0725
70	15.4	5.52	0.0475
80	10.8	4.72	0.0327
90	8.00	4.15	0.0228
Integrated values, 10° interval			
15–25°	117.0	5.02	0.180
25–35	32.6	2.99	0.0709
35–45	13.6	2 07	0.0359
45–55	7.11	1.54	0.0206
55–65	4.22	1.20	0.0127
65–75	2.74	0.972	0.00795
75–85	1.91	0.830	0.00576
85–90(×2)	1.41	0.726	0.00401
Total 15–90	179.9	14.99	0.336

Since in our experiment deflections to the left and to the right are placed in a single group, the above formula must be multiplied by 2; it becomes

$$N(\phi)d\phi=2k\left[\frac{1}{\beta^2}f_1(\phi)-f_2(\phi)+\frac{Z}{\beta}f_3(\phi)\right]d\phi.\tag{4}$$

Numerical values of $f_1(\phi)$, $f_2(\phi)$, and $f_3(\phi)$, and their integrals over 10-degree intervals in ϕ, obtained by graphical integration, are given in Table I. In using the table to compute the value of (4) to compare with any group of experimental data it is necessary to determine $\langle 1/H\rho^2\rangle_{\text{Av}}$, because of the spread in $H\rho$ within the group of data. These values are readily available, however, because the data are classified in intervals of $H\rho$ in the beginning. $1/\beta$ and $1/\beta^2$ are taken to be unity.

CORRECTIONS AND ESTIMATES OF ERRORS

If the tracks were perfect circles there would be practically no error of measurement except that brought about by the fact that the circles on the measuring card are drawn in steps of $\frac{1}{2}$ cm. This error is ±2.5 percent at 10-cm radius. A more serious source of error is the fact that small angle multiple scattering distorts the tracks from true arcs of circles. This effect makes itself felt both in the measurement of the radius of curvature and in the measurement of the angle of scattering. We shall consider first the error introduced into the measurement of the radius of curvature by the multiple scattering. There are a number of ways the error can be estimated, all of them on somewhat uncertain ground, because in the problem of deciding which of the circles on the card best fits the track, the psychology of the measurer is involved to a considerable degree. One method of treating the problem is the following.

On a magnified scale the path of the electron in a gas is composed of n circular arcs of length $s_1, s_2, \cdots s_n$, all of the same radius of curvature ρ, which is determined by the magnetic field. The angles which the arcs make with each other at the $n-1$ intersection points are the small scattering angles and will be denoted by $\phi_1, \phi_2\cdots\phi_{n-1}$. As a first approximation we may assume that the measurer judges the curvature by the total change in angle which the tangent makes with a fixed direction, divided by the total length $L=\sum_1^n s_i$ of the track. This gives

$$1/\rho_0=1/\rho+(\phi_1+\phi_2+\cdots\phi_{n-1})/L.$$

If $\rho_0=\rho+\Delta\rho$ this can be written approximately as

$$\Delta\rho/\rho=-\frac{\rho}{L}\sum_1^{n-1}\phi_i.\tag{5}$$

The probability distribution for scattering is symmetric, so $\bar{\phi}=0$ and $\langle\Delta\rho\rangle_{Av}=0$. Further, since the different scattering acts are independent of each other,

$$\langle\Delta\rho^2\rangle_{Av}/\rho^2=(\rho^2/L^2)(n-1)\langle\phi^2\rangle_{Av}. \quad (6)$$

Finally it is necessary to average over all possible number of scattering acts and ways of dividing the total length L in pieces $s_1\cdots s_n$. Since according to (6) $\langle\Delta\rho^2\rangle_{Av}$ is simply proportional to the number of scattering acts and independent of the way in which the total length L is subdivided, this averaging process gives

$$\langle\Delta\rho^2\rangle_{Av(2)}/\rho^2=(\rho^2/L^2)\nu\langle\phi^2\rangle_{Av}, \quad (7)$$

where ν is the average number of scattering acts in the track length L.

The most direct way to evaluate (7) is to make use of the fact that $\nu\langle\phi^2\rangle_{Av}$ is just

$$\int_{\phi_{\min}}^{\phi_{\max}}\phi^2N(\phi)d\phi, \quad (8)$$

where $N(\phi)$ is that defined by (4). Since $\phi_{\max}$ is small, the first term only of (4) with $f_1(\phi)=8\pi/\phi^3$ will be sufficient. The result is

$$\langle\Delta\rho^2\rangle_{Av(2)}/\rho^2=(\rho^2/L^2)\cdot 16\pi k\log(\phi_{\max}/\phi_{\min}). \quad (9)$$

$\phi_{\min}$ is the lower limit imposed upon the angle of scattering by screening, and for this we use $\lambda\!\!\!^{-}/a=(1-\beta^2)^{\frac{1}{2}}/137\beta Z^{\frac{1}{3}}$, as is done by E. J. Williams.[7] $\phi_{\max}$ in our case is the angle at which the deflections begin to be clearly recognizable as single events and for this we have taken three degrees. The value of k can of course be computed directly from the atomic number and density of the gas and the energy of the electrons, but this involves making a weighted average over all energies and different gases used. A more direct method is to make use of the relation between k and the number of deflections greater than 15° observed per unit length of track. The integral of (4) between the limits 15° and 90° can be set equal to the number of deflections observed between those limits of angle, per cm of track length, which is 0.0036 for air, 0.0026 for argon, 0.0037 for krypton, and 0.0055 for xenon. The integral of the part inside the bracket can be read from the last line of Table I. We set $\beta=1$ where it occurs inside the bracket, and we must evaluate $\log\phi_{\max}/\phi_{\min}$, for each case. Tracks for which $\rho=L$ are taken as typical ones. For these, we obtain $\langle\Delta\rho^2\rangle_{Av}/\rho^2=0.0016$, 0.0013, 0.0017, and 0.0025 for air, argon, krypton, and xenon, respectively.

As a refinement in the argument one might attempt to introduce into (5) a weighting factor to take account of the fact that deflections near the center of the track will have a greater influence upon the measured curvature than will those near either end. Equation (5) then takes the more general form

$$\Delta\rho/\rho=-(\rho/L)\sum_{1}^{n-1}\phi_iF(x_i), \quad (10)$$

where $F(x)$ is some function which is zero at either end of the track and maximum at the center, and whose integral over the length of the track is unity. The parabola $F(x)=6(x/L)(1-x/L)$ is such a function. Using this we find that $\langle\Delta\rho^2\rangle_{Av}/\rho^2$ is about 20 percent greater than the result in the above paragraph.

Assuming that the error in ρ produced by the multiple scattering can be represented by the symmetrical Gauss error function, we find the effect of that error upon the single scattering results can be calculated approximately. The theoretical single scattering intensity, computed from the measured value of

$$\sum_{i=1}^{n}l_i/(H\rho)^2$$

(where $l_1\cdots l_n$ are all the pieces of track measured) will be too high by the factor

$$\frac{\int_{-3\alpha}^{3\alpha}(\rho+x)^{-2}\exp(-x^2/\alpha^2)dx}{\int_{-3\alpha}^{3\alpha}\rho^{-2}\exp(-x^2/\alpha^2)dx}, \quad (11)$$

where $\alpha^2=2\langle\Delta\rho^2\rangle_{Av}$. The limits of integration, $\pm3\alpha$, are arbitrarily chosen to give sufficient accuracy and to avoid the singularity at $-\rho$. Evaluating (11) for the values of $\langle\Delta\rho^2\rangle_{Av}$ just calculated, we find the correction factor to be 1.01, which is agreeably small.

The other way in which small angle scattering introduces an error into the final result is through

[7] E. J. Williams, Proc. Roy. Soc. **A169**, 531 (1939).

its effect upon the measurement of the large deflections. The large deflections are measured by determining the angle between the arcs which are fitted to the tracks, at their intersection. Because of the fact that the track is not circular, the actual direction of travel of the electron is not quite the same as the direction of the tangent to the fitted circle at the point at which the deflection occurs. The question we have to answer is, to what extent, in the various parts of the track, is the wandering in direction taken into account in the process of fitting the true arc to the track? Certainly the wandering in the last millimeter of path before the point of the large deflection will have but slight effect upon the fitting of the circle, so we can say that all of the multiple scattering in this last millimeter will enter as an error in the measurement of the direction of the track at the point of the large deflection. On the other hand, it can be seen clearly by sketching a few typical cases that wandering near the middle of the track will have a much reduced effect upon the measurement of the deflection, and that wandering near the far end will have practically no effect. The problem then reduces to that of deciding upon an arbitrary weighting function, to describe the effect of small scattering angles along the track as a function of their distance from the point of the large deflection. After drawing many hypothetical cases, and remembering our experience at measuring a great many tracks, we arrived at the following as reasonable: the full value of a small deflection which occurs close to the large deflection is added to the large deflection; those 1 cm from the large deflection are added with a weighting factor $\frac{1}{2}$, and so on, giving the weighting function exp $-0.7x$ where x is the distance from the large deflection. The calculation with this function is straightforward. We set

$$\langle\Delta\phi^2\rangle_{\text{Av}}=2\int_0^L\int_{\phi\min}^{\phi\max}[\phi e^{-0.7x}]^2N(\phi)d\phi dx, \quad (12)$$

and evaluate it as we evaluated (9). The factor 2 above enters because the same error is made in the determination of the direction on each side of the point of the large deflection. $\langle\Delta\phi^2\rangle_{\text{Av}}$ comes out to be 0.0022, 0.0017, 0.0021, and 0.0035 radians^2 for air, argon, krypton, and xenon, respectively.

Assuming that $W(\Delta\phi)$ has the form of a symmetrical Gauss error function, we can easily make an approximate calculation of the amount by which the observed scattering intensity is too high. The factor is

$$F=\frac{\int_{-3\alpha}^{3\alpha}N(\phi+x)\exp(-x^2/\alpha^2)dx}{\int_{-3\alpha}^{3\alpha}N(\phi)\exp(-x^2/\alpha^2)dx}, \quad (13)$$

where $\alpha^2=2\langle\Delta\phi^2\rangle_{\text{Av}}$, and where the integration is again cut off at $\pm3\alpha$, to avoid the singularity. We have done the integration graphically, and the results are given in Table II. In the smallest angle interval (15°–25°) the size of the correction factor is too great for comfort, in view of the fact that it rests upon a somewhat arbitrary assumption. Its sensitivity to the exponent in the weighting factor is indicated by experimentally doubling and halving the exponent in the typical case of krypton, 15°–25°. This gives correction factors of 1.73, 1.35, and 1.13 for exponents -2×0.7, -0.7, and $-\frac{1}{2}\times0.7$, respectively. Fortunately the correction factor has little importance for angles greater than 25°.

The error in the knowledge of the magnetic field strength is estimated to be about ±3 percent. Since this enters into the uncertainty in $H\rho$ in just the same way as the error in ρ does, it should be handled in the way we have already outlined for the error in ρ. $\langle\Delta H^2\rangle_{\text{Av}}/H^2=0.0014$,

TABLE II. Correction factor, F [Eq. (13)].

	Air	Argon	Krypton	Xenon
15°	1.56	1.40	1.53	2.10
20	1.27	1.19	1.26	1.43
30	1.11	1.09	1.11	1.17
40	1.06	1.04	1.06	1.08
50	1.03	1.02	1.02	1.04
60	1.02	1.01	1.01	1.02
70	1.01		1.01	1.01
80				1.01
90				
15–25	1.37	1.27	1.35	1.61
25–35	1.16	1.12	1.15	1.27
35–45	1.07	1.05	1.07	1.12
45–55	1.03	1.02	1.03	1.06
55–65	1.02	1.01	1.02	1.03
65–75	1.01		1.01	1.02
75–85				1.01
85–90				

TABLE III. Comparison of theoretical and experimental results.

Gas	Energy range (Mev)	Effective energy (Mev)		15°–25°	25°–35°	35°–45°	45°–55°	55°–65°	65°–75°	75°–85°	85°–90° (×2)
Air	0.9–1.3	1.1	Theory	63.0	14.1	5.1	2.3	1.3	0.7	0.4	0.3
			Exp.	43.0	16.0	6.5	2.0	2.0	2.0	0	1.0
Air	1.3–1.6	1.4	Theory	66.0	14.7	5.3	2.4	1.4	0.8	0.5	0.3
			Exp.	45.5	22.0	6.0	2.5	2.0	0	0	0
Air	1.6–4.5	2.2	Theory	78.0	17.6	6.3	3.0	1.6	0.9	0.6	0.4
			Exp.	93.5	18.5	12.0	5.0	0.5	2.0	0	0
A	0.8–3.3	2.4	Theory	30.3	7.2	2.7	1.3	0.7	0.4	0.2	0.1
			Exp.	30.0	10.5	5.0	2.5	1.0	0	1.0	0
A	3.3–9.3	4.6	Theory	32.2	7.6	2.8	1.4	0.7	0.4	0.2	0.1
			Exp.	52.0	10.5	4.5	5.0	1.0	0	0	0
Kr	1.9–9.3	4.5	Theory	33.0	7.5	2.8	1.4	0.7	0.4	0.3	0.2
			Exp.	43.0	7.0	1.0	1.0	0	1.0	1.0	1.0
Xe	1.5–2.9	2.0	Theory	80.0	17.0	6.0	2.8	1.5	0.9	0.5	0.3
			Exp.	73.5	17.0	7.5	3.0	1.0	2.0	1.0	1.0
Xe	2.9–5.8	4.7	Theory	97.0	21.0	7.4	3.5	1.8	1.0	0.6	0.4
			Exp.	62.5	23.5	3.5	2.5	3.0	2.0	0	0
Xe	5.8–12.0	7.5	Theory	90.0	20.0	6.9	3.3	1.8	1.0	0.6	0.4
			Exp.	86.5	27.5	18.0	3.5	3.0	0	0	0
Total			Theory	569.5	126.7	45.3	21.6	11.5	6.5	3.9	2.5
			Exp.	529.5	152.5	64.0	27.0	13.5	9.0	3.0	3.0

which, when treated by an equation similar to (11), gives a correction to the scattering curve of about 1 percent.

The partial pressures of the gases in the chamber enter the calculation of the expected scattering in the first power, so any error in their measurement which has a symmetrical distribution will give zero correction, for a large number of runs. The only possible source of systematic error we know of in this connection is the following: In using the pressure of the gas in the expanded chamber for the calculation we assumed that all of the scattering occurred after the completion of the expansion. If some of the tracks were actually due to electrons which passed through the chamber before the completion of the expansion, then the above assumption is in error, and a correction will have to be made to the effective pressure of the gas. However, from the mobilities of the ions under the electric clearing field in the chamber it can easily be shown that the life of an ion track before condensation is sufficiently short (by a factor 10 to 100) to exclude the possibility that the tracks photographed were due to electrons which entered the chamber appreciably before the completion of the expansion. We seem to be safe in using the pressure of the fully expanded chamber in the calculation.

RESULTS AND CONCLUSIONS

Experimental Data

In all of the measurements a total of 2173 meters of track length was used and 801.5 deflections between 15° and 90° were found. Table III gives all of the data on scattering in collected form. The column labeled "effective energy" gives the energy corresponding to $(H\rho)_{\text{eff}}$ where $1/(H\rho)^2_{\text{eff}} = \langle 1/(H\rho)^2 \rangle_{\text{Av}}$, that is, the energy which would give the same theoretical scattering as the group of tracks under consideration. Half-integers appear in the numbers of tracks observed, because cases which fell on the dividing line between two angle intervals were split between the two intervals.

All the data are collected in Table III. We find 801.5 scattering events observed in the angle range 15° to 90°, while theory predicts 787.5.

This apparent excellent agreement means little more, however, than that the scattering in the lowest angle group checks with theory, since a preponderance of the scattering events lies in that group. The data, when broken down into angle intervals (bottom row of Table III) show that the scattering in the intervals 25°–35°, 35°–45°, and 45°–55° runs in excess of the theoretical by 20 percent, 43 percent, and 25 percent, respectively. For these intervals the statistical accuracy is not bad, so this evidence for an excess over the theoretical value is considered to be strong, though not conclusive. The data in Table III are further subdivided according to the gas used and the energy range of the electrons. None of these groups shows a behavior which is strikingly different, or perhaps even significantly different, from any other group. The statistical accuracy is still good enough to show this when the data are so subdivided, and we believe that this result is in itself important, because there is much contradictory material in the literature on this point.

The attempt was made to learn the frequency of occurrence of large energy losses, but the number of cases was found to be too small to permit a statistical study. For example, two cases showing more than 50 percent energy loss were found in all of the xenon pictures; theory predicted eight cases.

Other Experimental Work

Before asking the reader to consider any final conclusions we wish to present a summary of other work which has been done on the same problem. Table IV contains results of experiments in which individual scattering events were examined in the cloud chamber, by much the same method as our own. The total length of track and number of deflections measured are given not only to indicate the statistical accuracy, but to indicate the extent to which small angle scattering may have influenced the results.

In addition to the material given in Table IV mention should be made of the following experiments, most of which were done by methods other than the cloud chamber method. Chadwick and Mercier[8] measured the scattering of Ra E beta-rays in Al, Cu, Ag, and Au foils, using the annular ring method. Their results, obtained in the angle range 20°–40°, gave good agreement with theory. Henderson[9] later made an experiment with similar geometry and electron energy, but used gases instead of foils. The ratios of his experimental results to the theoretical predictions were 1.8, 1.4, 1.2, 1.2, and 1 for H_2, He, N_2, air, and A, respectively. Neher[10] measured the single scattering of cathode rays (145 kv) in Al, Ag, and Au foils at angles between 95° and 173°. He found that the scattering in Al was 1.32 times that predicted by the Mott theory. He did not commit himself as to whether there was agreement with theory or not in the cases of Ag and Au, because of the poorer accuracy of the theory for these elements and energies. Saunderson and Duffendack[11] studied both the single and multiple scattering of 0.2 and 1.1 Mev (Ra E) beta-rays in Al, Cu, Ag, and Au foils. They found no evidence for a large discrepancy between theory and experiment. Gautier[12] measured the scattering of 0.2 to 1.2 Mev (Ra E) beta-rays in hydrogen, air, and

TABLE IV. Comparison of results.

Element	Energy, Mev	Angle, deg.	Track length, meters	No. of deflections	Ratio exp. to theor.	Author
N	0.4–1.1	20–180	875	201	0.85	1
N	1.5–3.0	20–180	180	212	10 to 100	2
N	0.2–1.1	20–180	82	113	1.7	3
N	1.5–3.0	20–180	116	92	12	3
N	0.3–2.5	20–180	294	47	0.7	4
N	0.2–3.0	15–180	515	42	1.3	5
N	0.2–3.1	15–180	367	41	1.5	6
F	0.2–3.0	15–180	910	113	1.2	5
A	1.7–2.4	30–180	350	48	0.75	7
A	0.2–1.1	20–150	103	308	1.0	8
A	1.5–3.0	20–150	130	84	2.5	8
A	0.2–3.0	20–180	708	153	1.5	5
Kr	0.5–2.6	40–180	140	10	0.16	9
I	0.7–1.2	20–180			0.4	10
I	0.8–3.2	15–180	459	249	1.0	11
Xe	0.5–2.6	40–180	240	51	0.2	9
Xe	2.1	20–180	64	161	0.85	12
Xe	2.1	40–180	172	101	0.85	12
Hg	0.5–1.1	20–180	350	152	0.15	13

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argon, between angles of 10° and 30°. He found that the scattering varied in the expected way with atomic number, but did not determine the absolute scattering cross section.

Conclusions

After seeing Table IV one might be discouraged from attempting any concluding remarks as to whether the scattering is too high or too low. It is an author's privilege, however, to have more faith in his own results than in those of others, so on that basis the following conclusions will be drawn.

1. The highly anomalous results obtained by several other authors, and which have led them to postulate new kinds of interactions between electrons and nuclei, have not been substantiated by our experiments.

2. One of the stated objectives in most single scattering experiments has been to determine whether the experimental results show better agreement with the Mott formula, which is obtained by taking into account the spin of the electron, or with a formula derived without the spin interaction. The latter is given by the first term only of the Mott formula. It is our opinion that the combination of the inaccuracies in the cloud chamber measurements made to date (including our own) and the uncertainties due to the approximations made in the theory places such a decision just outside the range of possibility. Our own results, taken at their face value, are more favorable to the "non-spin" formula than to the "spin" formula, as one can easily see by comparing $f_1(\phi)$ in Table I with $f_1(\phi)-f_2(\phi)+Zf_3(\phi)$, but no significance is to be attached to that fact, because there are more reasonable directions in which to look for the explanation of the discrepancy.

3. The results in Table III run consistently higher than the theoretical values, by an amount which we believe to be outside our experimental error. At least part of the discrepancy may disappear when the following refinements are made in the theory: (a) In the derivation of the Mott formula the radiative force is not taken into account. Mott[13] has shown that when this is included the theoretical value of the scattering is raised by a few percent. (b) The Mott theory involves an approximation which becomes bad at large Z. Bartlett and Watson[14] have made a numerical calculation for the scattering in mercury, and the corrections to be applied to the Mott values turn out to be large: for 1.67-Mev electrons (the highest energy they give) the factor is maximum for 90 degree scattering and is 1.89. On this basis many of the results in Table IV are subject to serious revision, and our own data, for xenon at least, should be modified appreciably. (c) Bartlett and Welton[15] have treated the effect of screening in mercury, and while this correction can be neglected in our data, it should be applied to some of the values in Table IV, particularly those of Barber and Champion on mercury.

4. A point that is perhaps a secondary one is that our results show the expected behavior with variation in atomic number, angle of scattering and electron energy. A fact which was discussed in the introduction should be called to mind again, namely that the energy and atomic number were of necessity varied in such a way that the classical distance of closest approach of the electron to the nucleus remained about constant for a given angle of scattering. This may detract somewhat from the generality of the result.

ACKNOWLEDGMENTS

The authors wish to acknowledge with thanks the valuable help obtained through discussions with Professor S. A. Goudsmit and Professor G. E. Uhlenbeck. The work was made possible, financially, through a grant from the Horace H. Rackham Fund.

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