The Doppler shift of the light emitted from a distant point may be attributed in a conventional manner to the alteration in the emission time of a signal with the coordinate system, since

$$\Delta t' = (1-\alpha_{14}t)\Delta t,$$

where $t=r/c$ is now the travel time of the signal from the source to the observer.

The structure of the conformal group shows that this type of transformation is introduced as soon as one expands the Lorentz group by adjoining to it the transformations to accelerated axes, owing to the non-commutativity of the velocity and acceleration transformations. This procedure in turn rests on the hypothesis that the invariance of the Maxwell field equations is a fundamental physical requirement.

While a more complete analysis of the physical interpretation of the full group of transformations will be required before all of its implications can be appreciated, the simple treatment given here does seem to strengthen Milne's idea of an approach to the red-shift phenomenon without an explicit appeal to the relativistic theory of gravitation.

[1] E. L. Hill, Phys. Rev. 67, 360 (1945).

The Pair Production of Light Mesons by Electrons

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In a recent series of seminars given at the Massachusetts Institute of Technology, M. Schein discussed certain experiments in cosmic rays which seemed to indicate the existence of mesons of low mass of about 20 electron masses. The development of the betatron has made available a source of electrons of sufficiently great energy to make the laboratory production of these mesons feasible.

The mesons may originate either in the meson field or in the electric field associated with a nucleus. In this note we shall compute the second effect only, inasmuch as the only unknown constant involved is the meson mass whereas the first effect would also involve the unknown interaction between nucleons and light mesons. The method of virtual quanta, as given for example by E. J. Williams,[1] is used. In this procedure an electron of energy E may be replaced by a photon field such that the number of photons with an energy between ϵ and $\epsilon+d\epsilon$ is:

$$N(\epsilon)d\epsilon=\frac{2\alpha}{\pi}\left(\ln\frac{E}{\epsilon}\right)\frac{d\epsilon}{\epsilon}, \quad (1)$$

$$\alpha=e^2/\hbar c.$$

These photons will under the influence of the Coulomb field of a nucleus produce meson pairs. The cross section for this process for "scalar" particles[2] of spin 0 and for particles[3] of spin $\hbar/2$ is

$$Q(\epsilon)=Z^2\alpha\left(\frac{e^2}{\mu c^2}\right)^2\left[A\ \ln\frac{2\epsilon}{\mu c^2}-B\right], \quad (2)$$

where Z is the charge of the nucleus, μ = meson mass, $A=16/9$, $B=104/27$ for spin 0, and $A=28/9$, $B=218/27$ for spin $\hbar/2$. The cross section for the total process, the pair production of mesons by electrons becomes:

$$\sigma=\int^E N(\epsilon)Q(\epsilon)d\epsilon\sim\frac{A}{3\pi}(\alpha Z)^2\left(\frac{e^2}{\mu c^2}\right)^2\left(\ln\frac{E}{\mu c^2}\right)^3. \quad (3)$$

A similar formula has been derived[4] for the case of the production of an electron-positron pair by an electron. It differs from (3) with $A=28/9$ only in the replacement of the meson mass by the electron mass. Formula (3) is valid only when both the energy of the incident electrons and the emitted mesons is high compared to their rest mass. Moreover the energy lost by the electron should be small compared to its initial energy.

When the incident electron energy is 100 Mev, the meson mass is 20 electron masses, and the nucleus $Pb(Z=82)$ then for scalar mesons, $\sigma\sim 2\times 10^{-28}$ cm² while for spin $\hbar/2$ mesons $\sigma\sim 3\times 10^{-28}$ cm².

It should be emphasized that formula (3) underestimates the total cross section since only one of the possible processes in which mesons can be produced is calculated. In addition, if the mesons were of the vector type, i.e., of spin $\hbar$, one would judge by analogy with the known meson bremstrahlung formulae[5] that the cross section for the pair production of vector mesons is greater than that given by (3).

[1] E. J. Williams, Kgl. Danske Vid. Sels. Math.-Fys. Medd. 13, 4 (1935).
[2] W. Pauli and V. Weisskopf, Helv. Phys. Acta. 7, 709 (1934).
[3] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, Oxford, 1936), p. 200.
[4] See reference 3, p. 203.
[5] R. F. Christy and S. Kusaka, Phys. Rev. 59, 405 (1941).

Method of Increasing Betatron Energy

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Conventional betatrons used to date have followed the conservative designs similar to those described previously.[1,2] In these betatrons the flux density at the orbit of the electron is limited to a value less than half of the flux density which can be used in the center of the orbit. These betatrons have customarily been operated with approximately 4000 gauss at the orbit and 11,000 or 12,000 gauss in the iron within the orbit. This distribution of flux was necessary to have an equilibrium orbit at radius, r, since the flux, ϕ, within the orbit must satisfy $\phi-\phi_0=2\pi r^2H$. ϕ_0 is the flux linking the orbit when the field at the orbit is zero. With the conventional machine, ϕ_0 is zero.

The purpose of this letter is to point out that the flux, ϕ_0, need not be zero when the electrons are injected; it can be biased reversely so that the flux through the center of the orbit has a large negative value initially and changes to a large positive value during the acceleration process. Thus, since the maximum flux density in the center of the orbit can be made to change by twice the conventional amount, the field at the orbit and hence the final energy of the electrons can be twice as great with a biasing flux. Geometrical factors due to the introduction of biasing coils for the biasing system will not allow a full factor of 2 to be achieved in a machine of a given pole diameter.

The biasing can most easily be achieved when used in conjunction with a closed central gap in the betatron. To close the magnet gap in the center of the orbit a back-wound coil about this center piece of iron but within the orbit is necessary. This can be connected in series with the main coils, and it must so reduce the ampere turns about the low reluctance central piece of iron that the flux condition $\phi-\phi_0=2\pi r^2H$ is not exceeded. If the back-winding contains fewer turns than the total on the main coils, then a slight air gap left in the center may be used to adjust the flux condition.

This back-winding can reduce the energy which must be stored in the magnetic field and in the condensers to one-third of that in a non-back-wound betatron. The low reluctance of the central flux path requires few ampere turns of direct current for flux biasing. The biasing current may flow through the central back-wound coil, through both back-wound and all or part of the main coils, or through a separate coil around the poles. A choke coil generally is needed in the direct current circuit to block an alternating current except in case the right fraction of the main coil is in series with the back-wound coil to cancel the a.c. voltage.

A serious inadequacy of these methods, if applied to a betatron having a minimum of focusing region and a maximum closed gap area for the central flux, is that the orbit will vary in radius during acceleration as a result of non-linearity of the magnetic characteristics of iron. Much experience has shown the variation to be serious, and decreasing or eliminating the central air gap will make the variation worse; thus steps must be taken to make the flux condition hold. This can be accomplished by connecting in parallel two coils, one about the central flux and one about the complete pole having fewer turns than the coil about the central flux. The turn ratio must be chosen so that if the proper flux condition holds, then the voltage across the two coils is the same and hence they could be connected in parallel. Any deviation from the proper flux distribution then causes a circulating current which tends to restore the proper flux condition. The error in the flux forcing system is a result of the resistive voltage drop in the coils. This error is proportional to $\int_0^t IRdt$. Thus the resistance of the coils and the current through the coils must be held to a minimum. It is calculated that this method of flux forcing will hold the variation in radius to approximately one percent for the design we are interested in using.

Early in 1942 considerations for back-winding and flux biasing a betatron of approximately 250 Mev were begun. While the details of constructing such a betatron were being worked out this summer, information from Germany indicated that a similar plan was being drawn up there for a 200 Mev betatron, including back-winding, biasing, and the use of magnetic focusing lenses along the orbit. Apparently no flux forcing was contemplated.

[1] D. W. Kerst, Phys. Rev. **60**, 47–53 (July 1, 1941).
[2] D. W. Kerst, Rev. Sci. Inst. **13**, 387–394 (Sept., 1942).