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On the Scattering of Slow Mesons

M. S. Sinha
Cosmic Ray Research Unit, Indian Institute of Science, Bangalore, India

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Photographs of mesons of total energy lying between 1.55×10^8 and 2.55×10^8 ev have been obtained by a special arrangement of counters controlling the expansion of the cloud chamber. The scattering of these particles, both multiple and single, has been studied in two thicknesses of lead (2 cm and 4 cm). It is found that though the $\sqrt{t}$ law of Williams holds good for the average angle of scattering, its absolute value is only about 50 percent of the theoretical value expected from William's formula. The Gaussian distribution of the number of particles with angle of scattering is found to be approximately true. A cross section for the non-Coulombian nuclear scattering has been calculated. The value of this is found to be 1.84×10^{-26} per nucleon for mesons of mean total energy 2×10^8 ev. This agrees satisfactorily with that calculated by Bhabha and Weinberg and Ma for transversely polarized mesons of this energy, but does not agree with that found experimentally by Shutt and Code. Our value is about twenty-five times the value obtained by Shutt. The too low value of Shutt and others may be attributed to the presence of high energy mesons (for which the nuclear scattering cross section is very small) in large numbers so as to mask the effect of low energy mesons. One case of a high energy proton being singly scattered has also been obtained.

INTRODUCTION

EXPERIMENTS on the scattering of mesotrons by plates of copper, lead, and gold have been done by Wilson[1] and also by Code,[2] the latter using a 3.5-cm tungsten plate as the scattering material. In these experiments the energy of the mesons photographed varies from 2×10^8 to about 2×10^9 ev. Both these experiments confirm the Gaussian distribution of the number of particles as a function of the product of the energy E and the scattering angle θ, and they also confirm within experimental errors the theoretical mean value of $E\theta$ as expected from Williams[3] formula. On the other hand the experiments of Fowler and Oppenheimer[4-6] *et al.* on the scattering of electrons of mean energy between 6 and 11 Mev in lead give only about 50 percent of the mean angle expected theoretically from Williams' formula. Vargus has measured the scattering angle of some 55 particles of energy below 5×10^8 ev and finds the mean value of $E\theta$ 40 percent higher than the theoretical value. He is not, however, sure whether this result is outside experimental error.

In all these experiments, however, the method of measuring the energy of the particle is by measuring the curvature of its path in a magnetic field, and the scattering angle is measured by setting a cross-wire tangentially to the track of the particle below and above the scattering plate. Wilson[1] says that the accuracy of the measurements cannot be stated with certainty. In fact there is a systematic error due to particles (positive or negative) being bent either in the same direction or in a direction opposite to the direction of scattering. The presence of a magnetic field automatically excludes some low energy particles which are curved so much as not to reach the counter below the chamber. Besides, particles scattered through large angles are helped by the magnetic field to go out of the illuminated depth of the chamber. In fact, the magnetic field always sets a lower limit to the maximum scattering angle that can be observed in the chamber.

Recently Shutt[7] has measured the scattering of a large number of particles in 1 and 5 cm of lead, where he has not used a magnetic field. He does not know the energy of the particles photographed, not even their energy distribution, very

[1] J. G. Wilson, Proc. Roy. Soc. **A174**, 73 (1940).
[2] F. L. Code, Phys. Rev. **59**, 229 (1941).
[3] E. J. Williams, Proc. Roy. Soc. **A169**, 531 (1939).
[4] W. A. Fowler and J. R. Oppenheimer, Phys. Rev. **54**, 22 (1938).
[5] N. L. Oleson, K. T. Chao, J. Halpern, and H. R. Crane, Phys. Rev. **56**, 482 (1939).
[6] C. W. Sheppard and W. A. Fowler, Phys. Rev. **57**, 273 (1939).
[7] R. P. Shutt, Phys. Rev. **61**, 6 (1942).

accurately as he uses a 16-cm lead plate above the chamber, and this shifts the energy-spectrum by a considerable amount. Shutt has not tried to confirm Williams' formula. He has assumed Williams' equation and has calculated by an ingenious method the average nuclear scattering cross section of the mesons photographed. His value seems to be too low for a meson of spin 1, but agrees fairly well with the average value of Marshak and Weisskoff[8] if we assume Hartree's energy distribution. In view of the theoretical calculations of Bhabha,[9] Weinberg,[10] and Ma[11] all of which show a maximum of the cross section for a certain energy and then a falling off on both sides, I have tried to measure the scattering of mesons of different energy intervals in the way described below. The contents of this paper are concerned only with low energy mesons.

EXPERIMENTAL

The counter arrangement that controls the expansions of the chamber is shown in Fig. 1. Counter-pairs C_1, C_2, C_3 are in coincidence and the three counters A (which are in parallel) are in anti-coincidence with $C_1C_2C_3$. We can place lead of any thickness up to 6 cm between C_3 and A and also up to 25 cm between C_2 and C_3. In the present experiment, the thickness of lead between C_3 and A is 6 cm, and there is no lead between C_2 and C_3. A 2-cm lead plate was used as the scattering material in the first part of the experiment and a plate 4 cm thick in the second part. The 9-cm lead plate placed above the top counters is used only to cut out the soft component. From the arrangement it is obvious that mesons of range more than 2 cm but less than 8 cm are photographed in the first series, and those having a range between 4 and 10 cm are photographed in the second series. The lead plate above the chamber excludes electrons and slows down some high energy mesons. From the range energy relation of mesons as given by Rossi,[12] the kinetic energies of the mesons photographed in the first series lie between 0.55×10^8 ev and 1.3×10^8 ev, and one meson of the second series lies between 0.8×10^8 ev and 1.55×10^8 ev. The energy interval in the two cases does not differ much. By placing larger amounts of lead between C_2 and C_3, we can make the energy interval shift to higher values. That is, if the thickness of lead between C_2 and C_3 is x cm, the particles photographed will have an energy corresponding to a range greater than x but less than $x+6$ cm. It is the intention to use this method to measure the scattering of mesons of higher energies also.

Before starting the experiment, the rate of counts $C_1C_2C_3$ without the anti-counters was first recorded. This gives the rate of mesotrons of all energies striking the scatterer. Next the efficiency of the anti-counters was tested in the following way. No lead was placed between the counters C_3

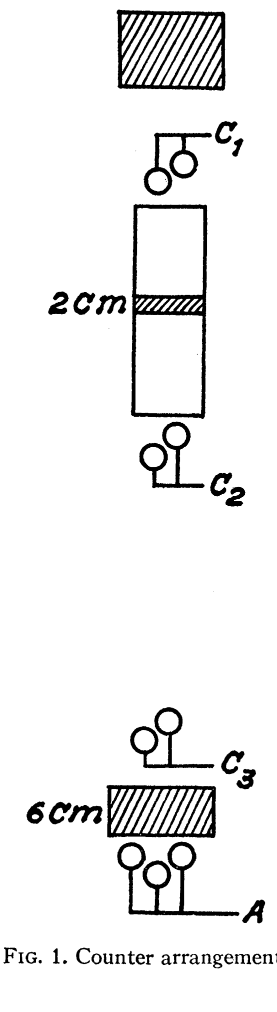


FIG. 1. Counter arrangement.

[8] R. E. Marshall and V. K. Weisskoff, Phys. Rev. 59, 130 (1941).
[9] H. J. Bhabha, Proc. Roy. Soc. A178, 314 (1941); Phys. Rev. 59, 100 (1941).
[10] J. W. Weinberg, Phys. Rev. 59, 776 (1941).
[11] S. T. Ma and C. F. Hsuh, Proc. Camb. Phil. Soc. 40, 171 (1944).
[12] B. Rossi and K. Greisen, Rev. Mod. Phys. 13, 249 (1941).

TABLE I. Counting rates.

Counting arrangement	Rate of counts per hour	Thickness of lead between C_3 and A	Thickness of scatterer
$C_1C_2C_3$	28.2 ± 0.75	0 cm	2 cm
$C_1C_2C_3-A$	0.25 ± 0.03	0 cm	2 cm
$C_1C_2C_3-A$	1.45 ± 0.07	6 cm	2 cm
$C_1C_2C_3-A$	1.58 ± 0.08	6 cm	4 cm

and A and the rate of counts with the anti-counters on (i.e., $C_1C_2C_3-A$) was found to be about one in four hours which is less than one percent of the total rate. The 6-cm lead plate was now placed between C_3 and A and the rate of coincidence $C_1C_2C_3-A$ was noted for series one. This rate comes to 5.1 percent of the total rate. The rate slightly increases in the second series when the scatterer inside the chamber is 4 cm (see Table I).

It will be seen that since we do not use a magnetic field it is easier to measure the angle between the tracks of the particles above and below the scattering material, and at the same time we know that the energy of the particles lies within certain limits. The scattering angle was measured by fixing each negative between two clean glass plates on the stage of a very low power (magnification 2) microscope. This stage could be rotated. A small mirror was fixed to the periphery of this stage, and the angle of rotation of this stage was measured by a telescope and scale arrangement. As the position of the tracks in the negatives was different from different photographs, the two glass plates holding the negative between them were bodily moved until the point of intersection of the track above and below the lead plate was on the axis of the microscope. At this position the negative was rigidly fixed, and then by rotating the stage, a very fine cross-wire (which was sufficiently long to cover the whole negative) in the eyepiece was made to coincide first with the track above and then with the track below the scatterer. Half the deflection obtained in the scale divided by the distance of the scale from the mirror gave the actual angle of scattering. The accuracy with which each setting could be done was about 10′, and the mean of several settings should be accurate to 0.1°.

The defect of the above method lies principally in the fact that we do not know accurately the energy of each particle, but only know that it lies in a certain energy interval. Hence the comparison with the theory could only be made in a way slightly different from Wilson and Code.[2]

COMPARISON WITH THEORY

The theoretical development of electrical scattering of mesons and electrons is mainly owing to Williams.[3] He has taken into account screening of extra-nuclear electrons, and the arithmetic mean deflection for a finite nucleus is given by

$$J=(19.5-3.1\log_{10} z)^{\frac{1}{2}}\delta, \tag{1}$$

where

$$\delta=\frac{2(Nt)^{\frac{1}{2}}Ze^2}{Mc^2\beta^2\xi}. \tag{2}$$

$Mc^2\xi$ represents the total energy of the incident particle, for $\xi=1/(1-\beta^2)^{\frac{1}{2}}$. If E denotes the total energy, then dividing both sides of (1) by mc^2, we get upon substituting for δ

$$\frac{J}{2mc^2}=\frac{(19.5-3.1\log_{10} Z)^{\frac{1}{2}}(Nt)^{\frac{1}{2}}Ze^2}{E\beta^2\cdot mc^2}.$$

Substituting for e^2/mc^2 (classical radius of the electron) and for N (number of atoms per cc) and Z, we have

$$E\beta^2=0.906(\sqrt{t}/\theta)\times10^9 \text{ ev},$$

where θ is the average deflection in degrees.

Now the mean kinetic energy of the mesons that we have photographed is 10^8 ev, and hence the mean total energy is 2×10^8. The mean value of β^2 that we can take is, therefore, given by

$$2\cdot10^8=\mu c^2/(1-\beta^2)^{\frac{1}{2}}=10^8/(1-\beta^2)^{\frac{1}{2}},$$

or

$$\beta^2=3/4.$$

Substituting for β^2 we have finally

$$E=1.208(\sqrt{t}/\theta)\times10^9 \text{ ev}. \tag{3}$$

We shall now see how our results fit in with this theory. All the photographs were carefully analyzed, and only those tracks which have been recorded caused by the particles being stopped in the lead plate below were taken for measurement. Particles which made such large angles with the vertical that they were outside the solid angle covered by the anti-counters were excluded from measurement. Angles of scattering of individual

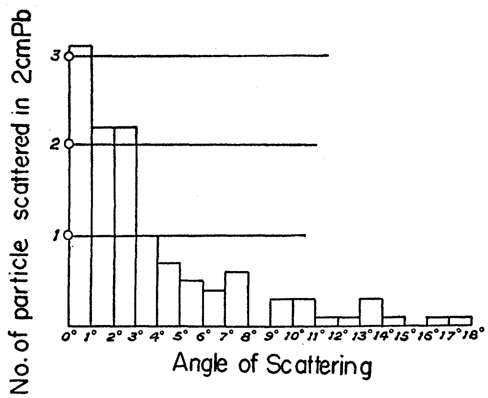


FIG. 2. Scattering in lead.

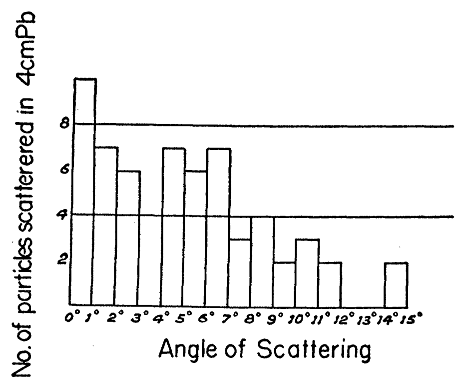


FIG. 3. Scattering in lead.

tracks being measured were classified into groups of 0° to 1°, 1° to 2°, 2° to 3°, and so on. Figure 2 shows the distribution of particles with scattering angle for $t=2$ cm, and Fig. 3 shows the same for $t=4$ cm.

The mean deflection for $t=2$ cm is $4.14°\pm.08$,
for $t=4$ cm is $5.67°\pm.10$.

Equation (3) shows that if E remains constant, which is very nearly true in our case, $\theta/\sqrt{t}$ should be constant for the two thicknesses. The values of $\theta/\sqrt{t}$ for $t=2$ and 4 cm are 2.93 and 2.83, respectively. The difference in these values is about 3 percent only for an increase in $\sqrt{t}$ by a factor 1.4. Considering that for $t=4$, the value of E is a little greater than for $t=2$, we can say that the $\sqrt{t}$ law is well confirmed.

Let us now see how the mean deflection agrees with the energy. If we put $\theta=4.14°$ and $t=2$ cm in Eq. (3), we get the value of E to be 4.13×10^8 ev.

The maximum value of E possible for the 2 cm series is, however, only 2.3×10^8 ev, the minimum being 1.55×10^8 ev. If we take the energy spectrum in this interval to be smooth, the mean value of the total energy E of the particles registered is actually 1.93×10^8 ev which is only 47 percent of the value found out from William's formula. Consequently we may say that the average deflection given by Williams equations is 53 percent too large. This is not in agreement with Wilson[1] or Code,[2] who have verified Williams' equation for high energy mesons. Fowler and Oppenheimer[4–6] *et al.*, have, however, found almost the same amount of deviation for electrons of mean energy between 6 and 11 Mev, as we have found for mesons of low energy. Since the mean angle of multiple scattering for high energy mesons satisfies Williams' equation whereas the low energy mesons show a marked deviation from the theory, the cause of the discrepancy seems to depend on the energy of the mesons concerned. We have seen that the $\sqrt{t}$ law is confirmed satisfactorily, and hence the statistical effect of all the nuclei taken together agree with the theory. It therefore appears that the electrical scattering of each individual nucleus is to be modified in such a way as to give a smaller mean angle for lower energies than the theory.

NUCLEAR SCATTERING

We can see from Fig. 2 and Fig. 3 that the distribution of particles with angle of scattering shows a greater amount of deviation from Gaussian distribution for $t=4$ cm, than for $t=2$ cm. This is to be expected since the nuclear scattering cross section is proportional to the thickness of the scatterer, whereas multiple scattering varies as $\sqrt{t}$. Undoubtedly, the percentage of single scattering, which is generally large angle scattering, is more in Fig. 3 than in Fig. 2. Shutt has developed a method of calculating the average nuclear scattering cross section from the scattering data for two thicknesses of lead. The main idea is the following. If $I(\theta, t)d\theta$ represents the multiple scattering intensity between θ and $\theta+d\theta$, expressed as the percent of the total number of traversals, then instead of calculating the distribution with respect to θ we transform θ to a new variable u given by

$$u=\theta/\sqrt{t}.$$

We then have

$$f(u)=I(\theta,\, t)(\partial\theta/\partial u)=I(\theta,\, t)\sqrt{t},$$

where $f(u)du$ is the number of particles in the range between u and $u+du$. If the scattering is entirely multiple, the $f(u)$ values for all angles would be identical for the two thicknesses of lead except, however, for statistical fluctuations. Following Shutt we have calculated the $f(u)$ numbers, in the range of u from 2 to 12 for both the thicknesses, and they are given in Table II. Summing up the $f(u)$ numbers we find that instead of an equality the value for the 4 cm thickness is greater than that for 2 cm by

$$\delta=116.8-56.4=50.4.$$

The method used by Shutt for calculating the number of singly scattered particles is not strictly correct. The difference between the figures in the second and third columns of Table II shows that multiple scattering due to the electric charge of the nucleus is not the only scattering that prevails. We therefore proceed as follows. We assume that $I(\theta, t)$ represents the sum of multiply and singly scattered particles I_m and I_s, respectively, so that

$$I(\theta,\, t)=I_m+I_s.$$

I_s is directly proportional to the thickness of the scatterer, and consequently we can write

$$I_s=kt,$$

where k gives the percentage singly scattered in 1 cm of lead.[13] That is

$$I(\theta,\, t)=I_m+kt,$$

or

$$f(u)=I(\theta,\, t)\sqrt{t}=(I_m+Kt)\sqrt{t}=I_m(\sqrt{t})+kt^{\frac{3}{2}}.$$

Since for multiple scattering $I_m\sqrt{t}$ remains constant for different values of t, δ the difference between the values of $f(u)$ for $t=4$ and $t=2$ will be given by

$$\delta=K(4^{\frac{3}{2}}-2^{\frac{3}{2}})=5.172K.$$

The experimental value of δ is 50.4, which gives

$$K=9.73\pm 2.4 \text{ percent}.$$

The statistical error has been calculated in the same way as Shutt. According to Shutt a correction of 30 percent is to be added to this for using the projections of the angles in the plain of the cloud chamber instead of the actual angles in three dimensions. Making this correction, the single scattering in one centimeter of lead comes to

$$K=12.65\pm 3.2 \text{ percent}.$$

The average cross section σ per nucleon is given by

$$\sigma=K/N,$$

where N is the number of nucleons per cubic centimeter of the material, since K is the percentage scattered in 1 cm of the material. Inserting values we get

$$\sigma=\frac{12.65\times 10^{-2}}{6.024\times 10^{23}\times 11.4}=1.84\times 10^{-26}$$

± 25 percent per nucleon.

This is about twenty-five times the average value obtained by Shutt for the whole energy distribution. Bhabha[9] has calculated the nuclear scattering cross section for tranversely polarized mesons taking radiation reaction into account according to the classical theory. Taking the constant $g_2{}^2\kappa^2/\hbar=1/13.3$, as is required by the theory of nuclear forces, he finds that the cross section attains a maximum of 3×10^{-26} for $E=3.5\mu c^2$ or 3.5×10^8 ev. The mean value of our small energy spectrum is 2×10^8 ev, and the value of the cross section found in this experiment agrees very well with Bhabha's theoretical value 1.6×10^{-26} for this energy. Recently Ma and Hsueh[11] have calculated the nuclear scattering cross section for transverse mesons according to the quantum theory, and their value is 2×10^{-26} cm^2 per nucleon for $E=2\times 10^8$ ev. Weinberg's[10] theoretical value of the cross section for mesons of

TABLE II. Values of $f(u)$.

$u=\theta/\sqrt{t}$ degrees x cm$^{\frac{1}{2}}$	$I(\theta, t)\sqrt{t}=f(u)$ $t=2$ cm, % x cm$^{\frac{1}{2}}$	$t=4$ cm, % x cm$^{\frac{1}{2}}$
2– 4	27.1	70.8
4– 6	13.5	34.0
6– 8	6.8	6.0
8–10	5.6	3.0
10–12	3.4	3.0
2–12	56.4	116.8

[13] k is a function of θ but not of t. The K that occurs in the formula for δ further down is the integral of this k over all θ.

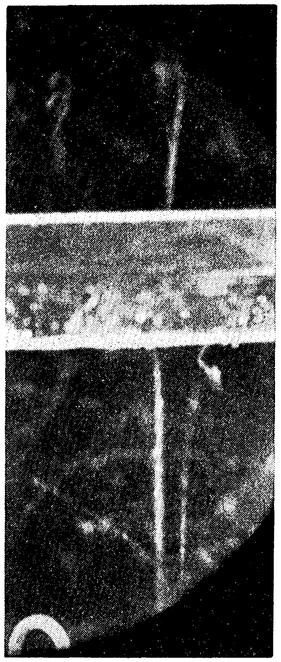


FIG. 4. Track of ionizing particle scattered in a 4-cm lead plate.

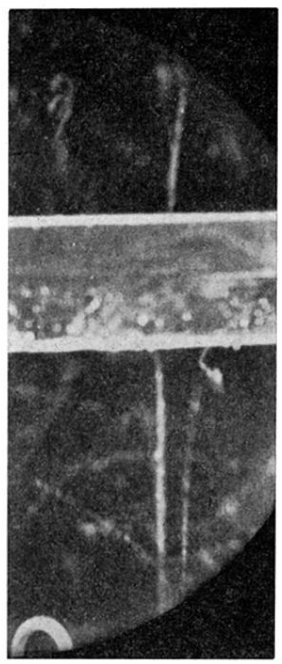
spin $\frac{1}{2}$ is of the same order of magnitude as ours, and agrees better with our value than with Shutt's. The experimental value of the cross section obtained by Shutt should be necessarily lower than ours since his value is an average for the whole energy spectrum. The presence of high energy mesons in large numbers completely masks the higher value of the cross section for low energy mesons. This is the reason why Wilson[1] or Code[2] (as calculated by Shutt[7]), who have deliberately excluded mesons of energy below 2×10^8 ev, get values for the nuclear scattering cross section which are too low. Here on the contrary, we have excluded the high energy mesons, and hence the value that we obtain agrees better with theoretical values for low energy mesons.

SCATTERING OF A PROTON

Figure 4 shows the track of a heavily ionizing particle being scattered through an angle of 5.6°, in a 4-cm lead plate. The track cannot be that of a meson, for if it were a meson slow enough to produce such large ionization it would not have penetrated the 4-cm plate of lead.

The range of the particle lies between 4 and 10 cm of lead and hence the minimum kinetic energy possible for the particle, taking it to be a proton is 10^8 ev, or the total energy is 10^9 ev. The mean angle of multiple scattering for this energy, according to Eq. (3) is 2.4°. The probability that a proton of this energy be scattered through 5.6° (the observed angle) is 7.3 percent. It will be still lower if the range is nearer to 10 cm. So the scattering seems to be more probably a nuclear one. There is another alternative. The charge of the particle may be more than one. If the incident particle is of charge Ze, the right-hand side of Eq. (3) is to be multiplied by Z. If we take the particle to be doubly-charged, the mean angle would come to 4.8° which is not very different from the observed angle. This alternative cannot be excluded as the ionization produced by the particle is almost that of an α-particle and this is what would be expected for a doubly-charged proton.

The author wishes to express his grateful thanks to Professor H. J. Bhabha for suggesting the experiment and his inspiring guidance throughout the work. He also wishes to thank Professor Sir C. V. Raman for lending the apparatus used for measuring the scattering angles.



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