rium energy; (2) gives the instantaneous energy in terms of the equilibrium value and the phase variation, and (3) is the "equation of motion" for the phase. Equation (4) determines the radius of the orbit.

$$E_0=(300cH)/(2\pi f), \tag{1}$$

$$E=E_0[1-(d\phi)/(d\theta)], \tag{2}$$

$$2\pi\frac{d}{d\theta}\left(E_0\frac{d\phi}{d\theta}\right)+V\sin\phi =\left[\frac{1}{f}\frac{dE_0}{dt}-\frac{300}{c}\frac{dF_0}{dt}+L\right]+\left[\frac{E_0}{f^2}\frac{df}{dt}\right]\frac{d\phi}{d\theta}, \tag{3}$$

$$R=(E^2-E_r^2)^{\frac{1}{2}}/300H. \tag{4}$$

The symbols are:

E = total energy of particle (kinetic plus rest energy),
E_0 = equilibrium value of E,
E_r = rest energy,
V = energy gain per turn from electric field, at most favorable phase for acceleration,
L = loss of energy per turn from ionization and radiation,
H = magnetic field at orbit,
F_0 = magnetic flux through equilibrium orbit,
ϕ = phase of particle (angular position with respect to gap when electric field = 0),
θ = angular displacement of particle,
f = frequency of electric field,
c = light velocity,
R = radius of orbit.

(Energies are in electron volts, magnetic quantities in e.m.u., angles in radians, other quantities in c.g.s. units.)

Equation (3) is seen to be identical with the equation of motion of a pendulum of unrestricted amplitude, the terms on the right representing a constant torque and a damping force. The phase variation is, therefore, oscillatory so long as the amplitude is not too great, the allowable amplitude being $\pm\pi$ when the first bracket on the right is zero, and vanishing when that bracket is equal to V. According to the adiabatic theorem, the amplitude will diminish as the inverse fourth root of E_0, since E_0 occupies the role of a slowly varying mass in the first term of the equation; if the frequency is diminished, the last term on the right furnishes additional damping.

The application of the method will depend on the type of particles to be accelerated, since the initial energy will in any case be near the rest energy. In the case of electrons, E_0 will vary during the acceleration by a large factor. It is not practical at present to vary the frequency by such a large factor, so one would choose to vary H, which has the additional advantage that the orbit approaches a constant radius. In the case of heavy particles E_0 will vary much less; for example, in the acceleration of protons to 300 Mev it changes by 30 percent. Thus it may be practical to vary the frequency for heavy particle acceleration.

A possible design for a 300 Mev electron accelerator is outlined below:

peak H = 10,000 gauss,
final radius of orbit = 100 cm,
frequency = 48 megacycles/sec.,
injection energy = 300 kv,
initial radius of orbit = 78 cm.

Since the radius expands 22 cm during the acceleration, the magnetic field needs to cover only a ring of this width, with of course some additional width to shape the field properly. The field should decrease with radius slightly in order to give radial and axial stability to the orbits. The total magnetic flux is about $\frac{1}{5}$ of what would be needed to satisfy the betatron flux condition for the same final energy.

The voltage needed on the accelerating electrodes depends on the rate of change of the magnetic field. If the magnet is excited at 60 cycles, the peak value of $(1/f)(dE_0/dt)$ is 2300 volts. (The betatron term containing dF_0/dt is about $\frac{1}{5}$ of this and will be neglected.) If we let $V=10{,}000$ volts, the greatest phase shift will be 13°. The number of turns per phase oscillation will vary from 22 to 440 during the acceleration. The relative variation of E_0 during one period of the phase oscillation will be 6.3 percent at the time of injection, and will then diminish. Therefore, the assumptions of slow variation during a period used in deriving the equations are valid. The energy loss by radiation is discussed in the letter following this, and is shown not to be serious in the above case.

The application to heavy particles will not be discussed in detail, but it seems probable that the best method will be the variation of frequency. Since this variation does not have to be extremely rapid, it could be accomplished by means of motor-driven mechanical turning devices.

The synchrotron offers the possibility of reaching energies in the billion-volt range with either electrons or heavy particles; in the former case, it will accomplish this end at a smaller cost in materials and power than the betatron; in the latter, it lacks the relativistic energy limit of the cyclotron.

Construction of a 300-Mev electron accelerator using the above principle at the Radiation Laboratory of the University of California at Berkeley is now being planned.

Radiation from a Group of Electrons Moving in a Circular Orbit

EDWIN M. MCMILLAN
University of California, Berkeley, California
September 9, 1945

A SINGLE electron of total energy E (rest energy $=E_r$) moving in a circle of radius R, radiates energy at the rate L (electron volts per turn), given by:

$$L=400\pi(e/R)(E/E_r)^4, \tag{1}$$

where e is the electronic charge in e.s.u., and $E>>E_r$. In the synchrotron one has the case of a rather concentrated group of electrons moving in the orbit, and the total amount of radiation depends on the coherence between the waves emitted by the individual electrons. For example, if there were complete coherence, the radiation per electron would be N times that given by (1), where N is the number of electrons in the group.

It is apparent from the above that an answer to the coherence problem is very important for any device in which groups of electrons are made to move in a circle with high velocity. This answer is given by a formula due to J. Schwinger (communicated to the author by I. I. Rabi). Schwinger's formula gives the radiation in each harmonic of the period of revolution, in a form that allows easy computation for any distribution of electrons around the orbit. It leads to the following conclusions:

(a) Most of the energy in (1) lies in very high harmonics.
(b) The coherence between the high harmonics from different electrons tends to become very small if the group has an appreciable angular spread.
(c) The low harmonics are partially coherent, and give an energy loss per electron per turn (L') depending on N, but not on E if $E>>E_r$.
(d) Because of fluctuations from a uniform distribution, each electron also radiates the same amount L that it would if alone in the orbit. The total radiation per electron is thus $L+L'$.

Values of L' have been computed numerically from Schwinger's formula for the case of N electrons covering uniformly an arc with an angular extent which is $1/m$ of a circle. This was done for $m=2$, 4, and 6; also the asymptotic form for large m was obtained. These values can all be fitted within a few percent by the formula:

$$L' \sim 400\pi(e/R)\times 2.4(m^{4/3}-1)N. \tag{2}$$

Applying (1) and (2) to the case where $R=100$ cm, $E/E_r=600$, $N=10^{11}$ (1/60 microcoulomb, giving 1 microampere at a 60 cycle repetition rate), and $m=6$, we get:

$$L=780 \text{ volts}, \quad L'=1400 \text{ volts}.$$

Thus the radiation loss will not seriously affect the operation of the synchrotron. Furthermore, L. I. Schiff has shown that the coherent part L', which is mostly in the very low harmonics, can be strongly reduced by shielding.

A Note on the Separation of Gases by Diffusion into a Fast-Streaming Vapor

F. A. SCHWERTZ
*Mellon Institute of Industrial Research, Pittsburgh, Pennsylvania**
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WHILE it is well known that gas mixtures may be separated by diffusion into a fast-streaming vapor, the potential speed of the process is not generally appreciated. The simple calculation given below may serve to correct this situation. The calculation is meant to be illustrative and not quantitative in the sense that it would agree with experimental data.

In Fig. 1, A and B are two channels connected by a cross-duct, C, of very small cross-sectional area. A pure gas is assumed to be traveling in A at a high velocity in the downward direction. Similarly, a vapor is flowing in B. The pressure differential across C is assumed to be such that the fluid velocity in the x direction is constant. To express the rate of transfer of the gas the equation:

$$\mathbf{i}=\rho\mathbf{v}-D \text{ grad } \rho \tag{1}$$

may be used.[1] In this equation $\mathbf{i}$ is the rate of gas transfer per unit area; ρ, the density of the gas; $\mathbf{v}$, the fluid velocity; and D, the mutual coefficient of diffusion of gas and vapor. For the steady state $\text{div } \mathbf{i}=0$, so that

$$\text{div } (\rho\mathbf{v})-D \text{ div grad } \rho=0. \tag{2}$$

TABLE I. Rate of diffusion, i, of H_2 into water vapor. (i is expressed in $\text{cm}^3/\text{cm}^2\times\text{sec.}$ at 150°C and 1 atmos.)

v (cm/sec.)	$b=0.01$ cm	$b=0.1$ cm	$b=1.0$ cm
−1000	2.34	000	000
−100	120	0.234	000
−10	156	12.0	0.023
−1	164	15.6	1.20
0	165	16.5	1.65
+1	166	17.6	2.20
+10	177	22.0	10.0
+100	220	100	100
+1000	1002	1000	1000

TABLE II. Separation currents and compositions. (i is expressed in $\text{cm}^3/\text{cm}^2\times\text{sec.}$ at 150°C and 1 atmos.)

v	$b=0.01$ cm		$b=0.1$ cm		$b=1.0$ cm	
cm/sec.	i	f_{H_2}	i	f_{H_2}	i	f_{H_2}
−1000	1.17	1.00	000	—	000	—
−100	62.3	0.964	0.117	1.00	000	—
−10	91.5	0.852	6.23	0.964	0.012	1.00
−1	97.8	0.839	9.15	0.852	0.623	0.963
0	98.5	0.837	9.85	0.837	0.985	0.837
+1	99.3	0.836	107	0.826	1.63	0.678
+10	107	0.827	16.3	0.678	10.0	0.500
+100	162	0.678	100	0.500	100	0.500
+1000	1001	0.500	1000	0.500	1000	0.500

If ρ is considered a function of x only, and $v_x=v$ is considered constant, then

$$\frac{\partial^2\rho}{\partial x^2}-\frac{v}{D}\frac{\partial\rho}{\partial x}=0. \tag{3}$$

If it is further assumed that the velocities in A and B are so large that the concentration of gas in B and of the vapor in A are substantially zero, (3) must be solved subject to the boundary conditions:

$$\rho(x)=\rho_0 \quad \text{at} \quad x=0,$$

and

$$\rho(x)=0 \quad \text{at} \quad x=b.$$

The result is

$$\rho=\rho_0[e^{vx/D}-e^{vb/D}]/[1-e^{vb/D}]. \tag{4}$$

For the current (1) gives

$$i=\rho_0 v/[1-e^{-vb/D}]. \tag{5}$$

The limit of i as v approaches zero is

$$i_0=\rho_0 D/b. \tag{6}$$

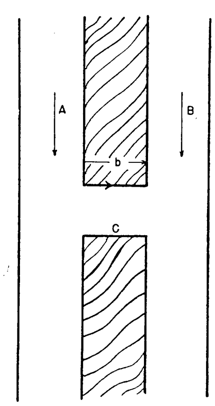

FIG. 1. Gas and vapor streaming by a connecting duct.