the fission fragments was determined in this way. Later on, small improvements of the apparatus have made it possible to measure the Hρ-distribution for each of the two groups of fission fragments separately.

Figure 1 shows the energy distribution as obtained when

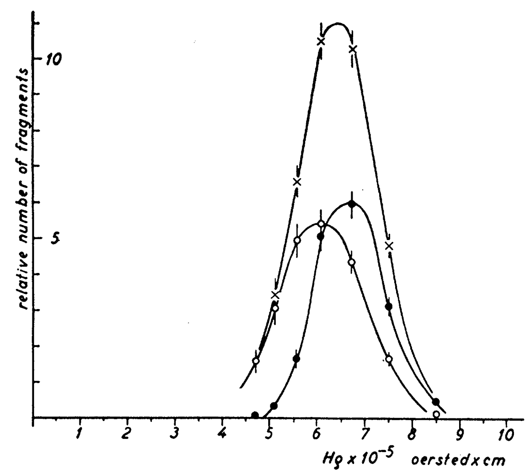

FIG. 2. Hρ-distribution of fission fragments. Full circles: light group. Open circles: heavy group. Crosses: Both groups.

the slit S_1 was made so wide (7 mm) that the fission fragments entering through S_2 could have all possible values of Hρ. The figure clearly shows that, apart from a very few exceptions, the pulses of energies higher than 34 Mev are caused by fragments of the light, most energetic group, while the fragments of the heavy group correspond to pulses between 9 and 34 Mev; pulses of energies lower than 10 Mev are caused by coincidences between recoiling argon atoms. In the Hρ-measurements the slit S_1 had a width of 1.0 mm and it was moved in steps of 1.0 mm; for the different positions of S_1 the number of fragments were counted, of course for equal neutron doses, and by means of the photographic record divided into two groups corresponding to the sizes of the pulses. In this way we get the number of fragments of each group as a function of the displacement of S_1 and we then calculate by means of the known magnetic field (which is not homogeneous) the Hρ-distribution curves shown on Fig. 2.

As seen, the two groups have nearly the same values of Hρ, but the light group has evidently slightly higher Hρ-values than the heavy one. Putting the most frequent value of mE to 8.5×10^3 mass units$\times$Mev[1] we get for the total charge of the light and the heavy group 20ϵ and 22ϵ, respectively, when ϵ denotes the electronic charge. This is in agreement with calculations by Professor Bohr[2] which predict an effective charge of fission fragments of about 20ϵ; yet the agreement is not perfect, as the theory involves a higher effective charge of the light group than of the heavy one in contradiction to the present experiments. Nevertheless, it is no real disagreement; as pointed out by Professor Bohr in the paper, the total fragment charge need not be identical with the charge effective in electronic interactions.

The author wishes to express his heartiest thanks to the Director of the Institute of Theoretical Physics, Professor Niels Bohr, and to Professor J. C. Jacobsen for their interest in the work and their great encouragement.

[1] N. O. Lassen, Kgl. Danske Vid. Sels. Math.-Fys. Medd. **22**, No. 2 (1945). In press.
[2] Niels Bohr, Phys. Rev. **59**, 270 (1941).

The Synchrotron—A Proposed High Energy Particle Accelerator

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ONE of the most successful methods for accelerating charged particles to very high energies involves the repeated application of an oscillating electric field, as in the cyclotron. If a very large number of individual accelerations is required, there may be difficulty in keeping the particles in step with the electric field. In the case of the cyclotron this difficulty appears when the relativistic mass change causes an appreciable variation in the angular velocity of the particles.

The device proposed here makes use of a "phase stability" possessed by certain orbits in a cyclotron. Consider, for example, a particle whose energy is such that its angular velocity is just right to match the frequency of the electric field. This will be called the equilibrium energy. Suppose further that the particle crosses the accelerating gaps just as the electric field passes through zero, changing in such a sense that an earlier arrival of the particle would result in an acceleration. This orbit is obviously stationary. To show that it is stable, suppose that a displacement in phase is made such that the particle arrives at the gaps too early. It is then accelerated; the increase in energy causes a decrease in angular velocity, which makes the time of arrival tend to become later. A similar argument shows that a change of energy from the equilibrium value tends to correct itself. These displaced orbits will continue to oscillate, with both phase and energy varying about their equilibrium values.

In order to accelerate the particles it is now necessary to change the value of the equilibrium energy, which can be done by varying either the magnetic field or the frequency. While the equilibrium energy is changing, the phase of the motion will shift ahead just enough to provide the necessary accelerating force; the similarity of this behavior to that of a synchronous motor suggested the name of the device.

The equations describing the phase and energy variations have been derived by taking into account time variation of both magnetic field and frequency, acceleration by the "betatron effect" (rate of change of flux), variation of the latter with orbit radius during the oscillations, and energy losses by ionization or radiation. It was assumed that the period of the phase oscillations is long compared to the period of orbital motion. The charge was taken to be one electronic charge. Equation (1) defines the equilibrium energy; (2) gives the instantaneous energy in terms of the equilibrium value and the phase variation, and (3) is the "equation of motion" for the phase. Equation (4) determines the radius of the orbit.

$$E_0=(300cH)/(2\pi f), \quad (1)$$

$$E=E_0[1-(d\phi)/(d\theta)], \quad (2)$$

$$2\pi\frac{d}{d\theta}\left(E_0\frac{d\phi}{d\theta}\right)+V\sin\phi =\left[\frac{1}{f}\frac{dE_0}{dt}-\frac{300}{c}\frac{dF_0}{dt}+L\right]+\left[\frac{E_0}{f^2}\frac{df}{dt}\right]\frac{d\phi}{d\theta}, \quad (3)$$

$$R=(E^2-E_r^2)^{\frac{1}{2}}/300H. \quad (4)$$

The symbols are:

E =total energy of particle (kinetic plus rest energy),
E_0 =equilibrium value of E,
E_r =rest energy,
V =energy gain per turn from electric field, at most favorable phase for acceleration,
L =loss of energy per turn from ionization and radiation,
H =magnetic field at orbit,
F_0 =magnetic flux through equilibrium orbit,
ϕ =phase of particle (angular position with respect to gap when electric field =0),
θ =angular displacement of particle,
f =frequency of electric field,
c =light velocity,
R =radius of orbit.

(Energies are in electron volts, magnetic quantities in e.m.u., angles in radians, other quantities in c.g.s. units.)

Equation (3) is seen to be identical with the equation of motion of a pendulum of unrestricted amplitude, the terms on the right representing a constant torque and a damping force. The phase variation is, therefore, oscillatory so long as the amplitude is not too great, the allowable amplitude being $\pm\pi$ when the first bracket on the right is zero, and vanishing when that bracket is equal to V. According to the adiabatic theorem, the amplitude will diminish as the inverse fourth root of E_0, since E_0 occupies the role of a slowly varying mass in the first term of the equation; if the frequency is diminished, the last term on the right furnishes additional damping.

The application of the method will depend on the type of particles to be accelerated, since the initial energy will in any case be near the rest energy. In the case of electrons, E_0 will vary during the acceleration by a large factor. It is not practical at present to vary the frequency by such a large factor, so one would choose to vary H, which has the additional advantage that the orbit approaches a constant radius. In the case of heavy particles E_0 will vary much less; for example, in the acceleration of protons to 300 Mev it changes by 30 percent. Thus it may be practical to vary the frequency for heavy particle acceleration.

A possible design for a 300 Mev electron accelerator is outlined below:

peak H =10,000 gauss,
final radius of orbit =100 cm,
frequency =48 megacycles/sec.,
injection energy =300 kv,
initial radius of orbit =78 cm.

Since the radius expands 22 cm during the acceleration, the magnetic field needs to cover only a ring of this width, with of course some additional width to shape the field properly. The field should decrease with radius slightly in order to give radial and axial stability to the orbits. The total magnetic flux is about $\frac{1}{5}$ of what would be needed to satisfy the betatron flux condition for the same final energy.

The voltage needed on the accelerating electrodes depends on the rate of change of the magnetic field. If the magnet is excited at 60 cycles, the peak value of $(1/f)(dE_0/dt)$ is 2300 volts. (The betatron term containing dF_0/dt is about $\frac{1}{5}$ of this and will be neglected.) If we let V=10,000 volts, the greatest phase shift will be 13°. The number of turns per phase oscillation will vary from 22 to 440 during the acceleration. The relative variation of E_0 during one period of the phase oscillation will be 6.3 percent at the time of injection, and will then diminish. Therefore, the assumptions of slow variation during a period used in deriving the equations are valid. The energy loss by radiation is discussed in the letter following this, and is shown not to be serious in the above case.

The application to heavy particles will not be discussed in detail, but it seems probable that the best method will be the variation of frequency. Since this variation does not have to be extremely rapid, it could be accomplished by means of motor-driven mechanical turning devices.

The synchrotron offers the possibility of reaching energies in the billion-volt range with either electrons or heavy particles; in the former case, it will accomplish this end at a smaller cost in materials and power than the betatron; in the latter, it lacks the relativistic energy limit of the cyclotron.

Construction of a 300-Mev electron accelerator using the above principle at the Radiation Laboratory of the University of California at Berkeley is now being planned.

Radiation from a Group of Electrons Moving in a Circular Orbit

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A SINGLE electron of total energy E (rest energy $=E_r$) moving in a circle of radius R, radiates energy at the rate L (electron volts per turn), given by:

$$L=400\pi(e/R)(E/E_r)^4, \quad (1)$$

where e is the electronic charge in e.s.u., and $E>>E_r$. In the synchrotron one has the case of a rather concentrated group of electrons moving in the orbit, and the total amount of radiation depends on the coherence between the waves emitted by the individual electrons. For example, if there were complete coherence, the radiation per electron would be N times that given by (1), where N is the number of electrons in the group.

It is apparent from the above that an answer to the coherence problem is very important for any device in which groups of electrons are made to move in a circle with high velocity. This answer is given by a formula due to J. Schwinger (communicated to the author by I. I. Rabi). Schwinger's formula gives the radiation in each harmonic of the period of revolution, in a form that allows easy computation for any distribution of electrons around the orbit. It leads to the following conclusions: