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Researches on the Magnetic Deflection of the Hard Component of Cosmic Rays*

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The energy spectrum and positive excess of the hard component of cosmic rays have been investigated both in Rome (50 m above sea level) and at Pian Rosà (3500 m above sea level) by means of a counter system with deflecting magnetized cores. The results of several measurements are discussed, and the following interpretation is suggested: (1) A positive excess of the order of 20 percent is found in the hard component, in agreement with the results of other investigators. (2) The hypothesis of the existence of several types of mesons is not confirmed in the lower atmosphere (between 0 and 5000 m). (3) Assuming for the ratio $\tau/\mu c^2$ between the meson proper lifetime τ and the rest energy μc^2 the value $\tau/\mu c^2 = 3\times 10^{-8}$ sec./Mev, one has to assume that the energy spectrum of mesons at the top of the atmosphere does not follow a power law $E^{-2.9}$ for low values of the energy ($E < 4\times 10^9$ ev).

1. INTRODUCTION

IN the years 1941–43 we have performed in the Physical Institute of the University of Rome a number of experiments on the mesonic component of cosmic rays by means of magnetized iron cores. We present here an outline of this work: some details that will be omitted may be found elsewhere.[1]

The first attempts to detect the deflection of cosmic particles by the magnetic field in an iron core were made in 1930 by Rossi[2] and Mott-Smith[3] independently. Their apparatus were rather different, but similar in two respects: both used a counter telescope to detect the particles, and in both cases the effect investigated was essentially a differential one, depending on the charged particles in the radiation being of one sign only or, at least, prevalently so.

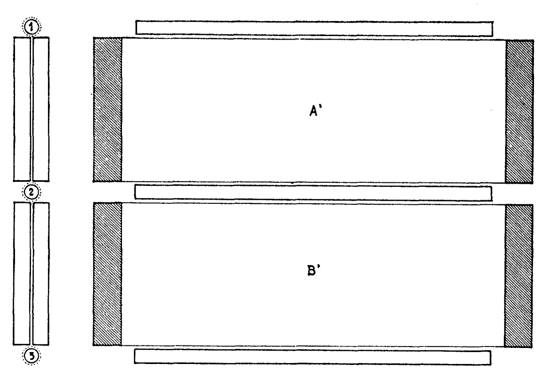

FIG. 1. Arrangement of counters and magnetic cores.

* A fund for researches on cosmic rays was granted by the Consiglio Nazionale delle Ricerche.

[1] G. Bernardini and M. Conversi, La Ric. Scient. **11**, 858 (1940). G. Bernardini, M. Conversi, E. Pancini, and G. C. Wick, La Ric. Scient. **12**, 1227 (1941). M. Conversi and E. Scrocco, N. Cim. **I**, 372 (1943); Bernardini, Conversi, Pancini, and Wick, Phys. Rev. **60**, 535 (1941).

[2] B. Rossi, Nature **128**, 300 (1931).

[3] L. M. Mott-Smith, Phys. Rev. **35**, 1125 (1930); **37**, 1001 (1931); **39**, 403 (1932).

FIG. 2. Coils of pair of cores ($A'A''$ or $B'B''$).

Mott-Smith obtained a negative result.[4] Rossi on the other hand, by means of a telescope consisting of only two counters found an indication of a small effect, corresponding to an excess of the positive over the negative particles; the evidence, however, was partly contradictory and was not regarded as definite by the author.

Looking at these results in the light of later knowledge we can see that: (a) the hypothesis that the effective field inside iron is not given by the induction vector B and is considerably smaller than B (this, of course, would easily explain the negative result of the deflection experiments of Mott-Smith and Rossi) has been dismissed by a theoretical discussion by v. Weizsäcker;[5] (b) an alternative explanation is offered by Blackett's result,[6] namely that the radiation consists of positive and negative particles in approximately equal numbers; (c) while this is probably the correct explanation, we must not forget that there is a certain discrepancy between Blackett's statement and the subsequent investigations of Leprince-Ringuet and Crussard,[7] Jones,[8] and Hughes.[9] Especially the last two authors have given quite definite evidence of the existence of a small but real excess in the number of the positive over the negative mesons; the excess is of the order of 20 percent, and according to this, one should find in an experiment like that of Rossi a small but definite effect.

It would seem, therefore, that the subject deserves further investigation; but there are more cogent reasons too. The positive excess in the meson spectrum is probably connected with the positive nature of the primary radiation that is revealed by the azimuthal effect;[10] a study of the variation of the positive excess with height would probably be interesting and might throw some light on the process of creation of the mesons. Now the Wilson chamber method

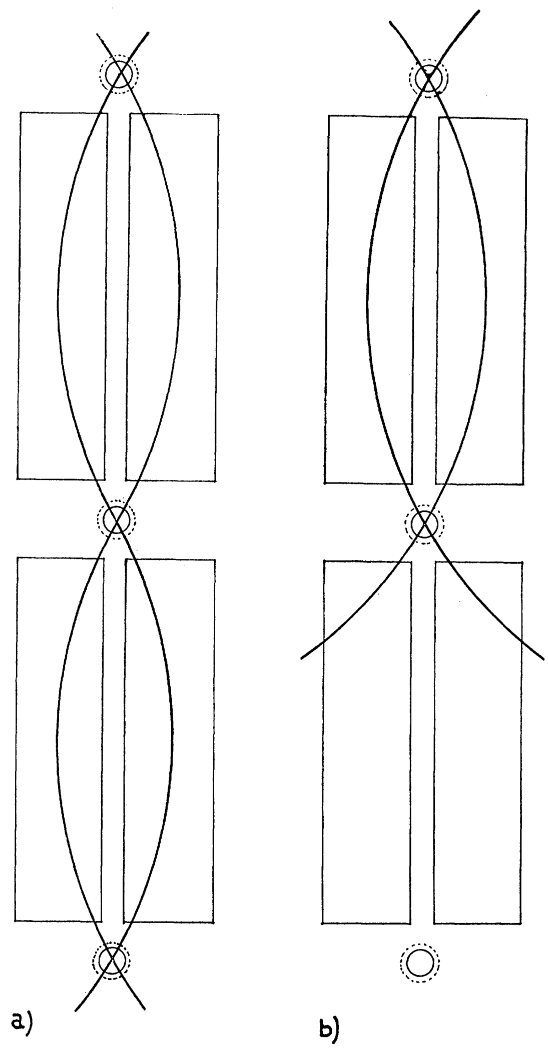

FIG. 3. Qualitative shape of trajectories: (a) "cc" arrangement; (b) "cd" arrangement.

[4] In a subsequent investigation, however, Danforth and Swann, Phys. Rev. **49**, 582 (1936) have found a small but real effect by means of an apparatus not very different from that of Mott-Smith.

[5] C. F. Weizsäcker, Ann. d. Physik **17**, 869 (1933).

[6] P. S. M. Blackett, Proc. Roy. Soc. **159**, 1 (1937).

[7] L. Leprince Ringuet and J. Crussard, J. de. Phys. **8**, 207 (1937).

[8] H. Jones, Rev. Mod. Phys. **11**, 235 (1939).

[9] D. J. Hughes, Phys. Rev. **57**, 592 (1940).

[10] T. H. Johnson, Rev. Mod. Phys. **11**, 208 (1939).

employed by all the authors mentioned above is obviously unsuitable for this purpose owing to the rather heavy equipment it requires.

Bearing this in mind, we have tried to improve Rossi's experiment, and we have planned an apparatus consisting of a triple coincidence system with four iron cores; we have found that it gives a quite measurable effect, depending upon the positive excess. We hope to use it for an investigation of the positive excess in the upper atmosphere.

This apparatus also allows an entirely different use since the magnetization of the cores can be arranged in such a way as to cause a deflection effect which is not differential. That is, the effect depends on the energy of the particles, but not on their sign; accordingly, we have found in this case a substantially larger effect, which can be used for investigation of the energy spectrum of the particles.[11]

2. APPARATUS

Our apparatus is shown schematically in Fig. 1; it consists primarily of three coincident counters having a radius $r=1$ cm and useful length of about 50 cm. The distance d between the axis of the intermediate counter "2" and that of either counter "1" or "3" is 22 cm. The two pairs of iron cores A', A'' and B', B'' are magnetized by coils as is shown by Fig. 2. The field in the cores is parallel to the axis of the counters and has opposite directions in A', A'' (similarly in B' B''). The magnetic circuit is closed by iron bars applied at both ends of A', A'' (or B', B''). Thus each pair of iron cores, for instance A', A'', acts, roughly speaking, like a cylindrical magnetic lens which concentrates the particles of one sign. We shall hence-forward speak of this "lens" as convergent (or briefly "c") or divergent (briefly "d") for positive ($c+$ or $d+$) or negative ($c-$ or $d-$) particles.

[11] The main results of the present paper have already been indicated in a preliminary communication (Phys. Rev. reference 1). After the present paper was completed we have learned from a private communication that F. Rasetti has recently succeeded in obtaining a deflection effect by an arrangement of the type used by Mott-Smith. His results are in accord with the theoretical prediction that the effective field inside iron is B and with the existence of a positive excess. All this seems to be in the best agreement with our own previous conclusions and with these of the present paper.

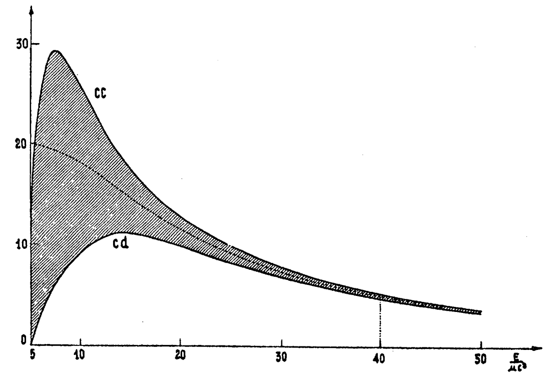

FIG. 4. Modification impressed upon the total energy spectrum (positive+negative mesons) by the action of the instrument with either arrangement "cc" or "cd."

Now the upper (A', A'') and lower (B', B'') pairs can be magnetized in the same way or not, so that on the whole we have the following *four* possibilities:

$$cc+\ (=dd-);\quad dd+\ (=cc-);$$
$$cd+\ (=dc-);\quad dc+\ (=cd-).$$

The two alternatives cc and cd are schematically shown in Fig. 3.

With the "cc" arrangement, the number of positive particles detected by the counter telescope is increased, the number of the negative particles is decreased, and the reverse takes place with the "dd" arrangement. The difference $N_{cc+}-N_{cc-}$ between the number of triple coincidences, N_{cc+} or N_{cc-} registered in a given time with the two arrangements depends on the asymmetry of the meson spectrum; *it may be used to measure the positive excess*, as a more quantitative analysis will show.

We shall later develop a theory of the action of the instrument. Figure 4 gives an idea of the modification impressed upon the total energy spectrum (positives+negatives) by the action of the magnetic lenses with either arrangement cc or cd. In this case the positive excess has been neglected; the broken line in Fig. 4 shows the energy spectrum at sea level in the vertical direction and in open air. This spectrum has been evaluated by assuming, as usual, a power spectrum for the mesons at the creation level in the upper atmosphere, and by taking into account the disintegration of the mesons, and by assuming for the mean lifetime of τ of a

TABLE I. Measurements of the positive excess.

Meters above sea level	$10^{-3}B$ (gauss)	Threefold coinc./hour N_{cc+}	N_{cc-}	Double coinc./hour N_{c+}	N_{c-}	δ	δ'
50 (Roma)	2.4	81.1 ± 1.2	77.7 ± 1.2	235.4 ± 2.4	234.4 ± 2.3	4.3 ± 2.1	0.4 ± 1.4
	5.0	83.1 ± 1.2	77.2 ± 1.1	243.6 ± 2.4	235.7 ± 2.3	7.4 ± 2.0	3.3 ± 1.4
	10.4	88.5 ± 0.9	80.5 ± 0.9	245.2 ± 1.9	236.2 ± 1.8	9.5 ± 1.5	3.7 ± 1.1
	15.5	97.5 ± 1.0	84.7 ± 0.9	253.7 ± 2.0	242.3 ± 1.9	14.0 ± 1.5	4.6 ± 1.1
	17.0	103.3 ± 1.5	91.2 ± 1.3	260.2 ± 2.8	250.5 ± 2.5	12.4 ± 2.0	3.8 ± 1.5
3500 (Pian Rosà)	2.4	170.0 ± 2.1	175.0 ± 2.2	615 ± 8.0	601 ± 8.0	1.1 ± 1.7	2.3 ± 1.9
	5.0	180.5 ± 2.5	172.0 ± 2.5	597 ± 9.5	589 ± 9.5	4.8 ± 2.0	1.3 ± 2.3
	10.4	190.5 ± 2.1	174.5 ± 2.0	609 ± 4.5	583 ± 4.5	8.8 ± 1.6	4.4 ± 1.1
	15.5	221.0 ± 1.8	191.0 ± 2.0	664 ± 4.0	624 ± 4.0	14.6 ± 1.3	6.2 ± 0.9
	17.0	223.0 ± 2.7	200.0 ± 2.9	678 ± 7.5	631 ± 7.5	15.2 ± 1.8	7.2 ± 1.6

meson a value:[12] $\tau/\mu c^2=2\times10^{-8}$ sec./Mev (μ is the meson mass) with $\mu c^2=100$ Mev.

The full lines give the spectrum as modified by the field (and the absorption in the iron cores) when the field B in the two pairs of iron coils is $B=17{,}000$ gauss. It can be seen that the spectrum is noticeably modified in the energy interval, between 5×10^8 ev—the minimum energy for a meson to get through the iron bars—and 4×10^9 ev. The spectrum becomes richer in low energy particles with the *cc* arrangement; the reverse takes place with the *cd*, or *dc*, arrangement.

One can see, also that the energy range of field sensitive particles is about the same as for the Wilson chamber technique.[13]

[12] This value of $\tau/\mu c^2$ is, in our opinion, rather too low; but we have chosen it in our calculation because it is intermediate between the several values found by different experiments.

[13] In comparing the sensitivity of our apparatus with that attained in the Wilson chamber, it is well to remember that in the latter case the limit to the measurement of large energies is not set by the difficulty of measuring the small curvatures involved, but rather by the spurious curvatures produced by irregular motions of the chamber gas. In our case this cause of error does not exist; unfortunately, it is superseded to a certain extent by a similar effect due to scattering of the mesons in the iron cores. This causes a spurious curvature of the trajectories, and thus tends to obscure the effect of the magnetic field. The following evaluation shows, however, that the change in the action of the magnetic "lens" by the scattering, must not be very large (and, at any rate, the spurious curvatures due to scattering vary as the reciprocal of the particle energy; that is, they become smaller at the same rate as the true curvatures, which is not the case in the Wilson chamber!).

The order of magnitude of the spurious curvatures introduced by scattering can be evaluated by means of Williams' formulae, Proc. Roy. Soc. **169**, 531 (1939), which give the mean deflection $\bar{\vartheta}$ suffered by a particle in going through a plate of thickness t, of atomic number Z, density ρ, atomic weight A. Consider, for instance, a particle moving in a plane normal to the axis of the counters and call θ the component of the deflection parallel to this plane—this is in fact the component in which we are mainly interested, as it is indistinguishable from a deflection due to the magnetic field—we find, according to Williams:

$$\bar{\vartheta}=\sim\frac{600Ze(Nt)^{\frac{1}{2}}}{\beta E}\left[3.7+0.28\log\frac{Z^{4/3}\rho t}{A\beta^2}\right],$$

where E is the energy of the particle in e-volts, βc its velocity ($\beta\approx1$), and N is the number of atoms per cm^3. In iron we find: $\bar{\vartheta}=\sim((\sqrt{t})/E)10^7$, when $t\approx20$ up to 40 cm.

The deflection caused by the magnetic field is: $\vartheta'=300Bt/E$, which becomes, when $B=15000$ gauss: $\vartheta'=(t/E)10^6$. Putting $t=40$ cm, one can see that ϑ' is about three times as large as $\bar{\vartheta}$, independently of energy.

The Wilson chamber, of course, allows a much more detailed analysis of the energy spectrum to be made, but this advantage is attained only by the much greater labor which the method requires.

On the whole we think that the two methods are not mutually exclusive, but that they can usefully integrate each other.

3. RESULTS

(a) The Positive Excess

These measurements were made partly in the Physical Institute of the University of Rome and partly at Pian Rosà (Cervinia) 3500 m above sea level in a wooden hut.[14]

During the measurements the two arrangements $cc+$ and $cc-$ were, of course, systematically alternated; also, several values of the magnetic field were tested. The electric pulses of counters were sent to a coincidence system which registered besides the triple coincidences also the double coincidences between the two upper counters.

[14] A series of measurements in a plane flying at 9000 m above sea level was also begun, but unfortunately a flying accident destroyed the apparatus before a sufficient set of data had been secured.

These could be compared with Rossi's results, although the distance between the counters and the counter diameter are not quite the same. Our results are shown in Table I.

The effect δ in the sixth column is defined by:

$$\delta = 200\frac{N_{cc+} - N_{cc-}}{N_{cc+} + N_{cc-}}. \quad (1)$$

The effect δ' in the seventh column is defined in a similar way for the double coincidences.

The value of δ has been evaluated without taking into account the effect of *showers*, which had been shown in the usual way (that is by shifting the upper counter horizontally, out of the solid angle defined by the remaining two counters) to be negligible.

The correction for random coincidences is negligible in the case of the triple ones; the double coincidences due to chance were about 40/hr. at Pian Rosà as well as in Rome[15] and they are already taken account of in the N_{c+} and N_{c-} values given in Table I.

In Figs. 5 and 6, the values of δ and δ', found in Rome and at Pian Rosà, are plotted against the field B. The effect on double coincidences, δ', is seen to be quite a bit smaller than on δ; this is understandable since doubling the distance between the extreme counters not only increases the angular definition of the beam selected by the telescope, but also doubles the deflection caused by the field. In fact, the theory of the apparatus (Section 4) shows that the sensitivity increases, roughly, with the square of the distance between the counters.

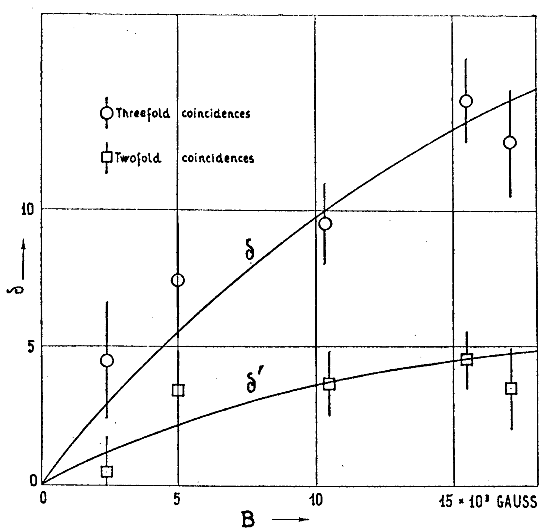

FIG. 5. Results of the measurements of the positive excess performed in Rome (50 m above sea level).

Whatever the reason may be, the effect δ' which we have found with the double coincidences is much less reliable than δ.

Considering the values of δ found at Rome and Pian Rosà, one can see that the results are quite similar, indicating that the positive excess in the meson spectrum does not materially change between sea level and 3500 meters.

With regard to the dependence of δ on B, one may remark that δ can never be larger than the positive excess ϵ, if ϵ is distributed uniformly over the energy spectrum. Indeed one can see that δ would finally become equal to ϵ if the action of the "lenses" would become strong enough to exclude the particles of one sign completely. The $\delta(B)$ curve should therefore resemble a saturation curve. But with the field values we have reached, there is no indication of a bend. Our results, however, are not inconsistent with the hypothesis that for higher values of B, δ might tend to a saturation value.

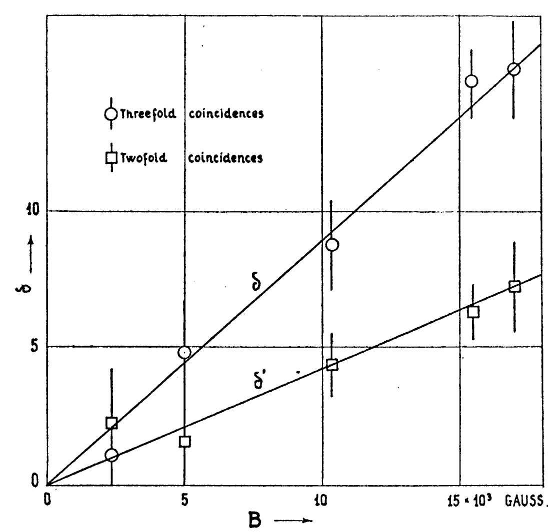

FIG. 6. Results of the measurements of the positive excess performed at Pian Rosà (3500 m above sea level).

[15] The building material of the Institute is weakly radioactive and increases the natural pulses of counters.

TABLE II. Measurements on the meson energy spectrum ($B = 17{,}000$ gauss).

Series	Meters above sea level	Absorbing layer (g/cm²)	Arrangement	Threefold coincidences per hour ($N_{cc\pm}$; $N_{cd\pm}$)	$\alpha\pm$	α	$\Delta = \alpha_+ - \alpha_-$
1st	50 (Roma)	0	$cc+$	104.0 ± 1.4	50.8 ± 3.0	42.3 ± 2.0	16.1 ± 3.9
			$cd+$	69.6 ± 1.0			
			$cc-$	94.6 ± 1.2	34.7 ± 2.5		
			$cd-$	70.7 ± 0.9			
2nd	50 (Roma)	250	$cc+$	91.9 ± 1.4	48.0 ± 3.3	40.0 ± 2.3	15.8 ± 4.6
			$cd+$	62.7 ± 1.0			
			$cc-$	80.0 ± 1.4	32.2 ± 2.2		
			$cd-$	61.0 ± 1.0			
3rd	3500 (Pian Rosà)	0	$cc+$	233.7 ± 3.8	67.2 ± 4.5		
			$cd+$	145.6 ± 2.7			

(b) Measurements of the Meson Energy Spectrum

The results of these measurements, Table II, are grouped in three separate series, each series comprising measurements made at the same height above sea level and with the same absorber thickness above the apparatus.

In the first two series of measurements, made in Rome, the direction of the current was systematically changed in the coils around $A'A''$ and independently in the coils around $B'B''$, so that all four arrangements $cc+$, $cc-$, $cd+$, $cd-$ were alternated.

With the mean values

$$N_{cc} = \tfrac{1}{2}[N_{cc+} + N_{cc-}]; \quad N_{cd} = \tfrac{1}{2}[N_{cd+} + N_{cd-}], \quad (2)$$

the effect we were looking for was defined by

$$\alpha = 100(N_{cc} - N_{cd})/N_{cd}. \quad (3)$$

This is given in the next to the last column of Table II. In the third series (at Pian Rosà 3500 m above sea level) the current in the $A'A''$ coils was never changed so that the upper "lens" was always convergent for positive particles. The effect was, therefore, defined by

$$\alpha_+ = 100(N_{cc+} - N_{cd+})/N_{cd+}. \quad (4)$$

The second series of measurements was planned after Nielsen, Ryerson, Nordheim, and Morgan[16] had announced that a new careful investigation of the proper lifetime of a meson had given a value $\tau/\mu c^2 = 1.25 \times 10^{-8}$ sec./Mev, considerably lower than all values previously found.

With this value one can see that the maximum in the energy spectrum of the mesons arriving at sea level in the vertical direction should lie above the minimum energy of the mesons which are not stopped in the 40 cm Fe of our apparatus. Now if a thick absorber is placed above the counter telescope, it will cut off the lower part of the energy spectrum; for a properly chosen thickness of the absorber one can see that the resulting spectrum—that is the spectrum impinging on the apparatus as modified by the absorber—is appreciably *softer* than the unmodified spectrum.

We might then expect the effect α to be increased; this is also confirmed by the more detailed considerations of Section 5. Accordingly we built a brick wall, under which our apparatus could be placed, covering the solid angle defined by the counter telescope.

The vertical thickness of the wall was 250 g/cm², and the results are grouped in the second series.

The effect of showers has been deducted from

[16] W. M. Nielsen, C. M. Ryerson, L. W. Nordheim, and K. Z. Morgan, Phys. Rev. 59, 547 (1941).

the results included in the first two columns of Table II; the correction was, however, very small. The data given in the last column can be used for a further evaluation of the positive excess.

4. THEORY OF THE INSTRUMENT

We shall now try to draw from the experimental data a quantitative evaluation of the positive excess as well as some more detailed information about the meson spectrum.

The direction of motion of an incoming particle shall be specified by the polar angles ϑ (colatitude) and φ (longitude). As polar axis we chose the axis of a counter, or the direction of the magnetic field inside the cores. Let further $\varphi=0$ indicate the vertical plane containing the three counter axes. Put

$$dn=J(E,\vartheta,\varphi)\sin\vartheta d\vartheta d\varphi dE,$$

for the number of incoming particles per second and per cm^2, of energy E within the interval dE, and of direction ϑ, φ within the solid angle $\sin\vartheta d\vartheta d\varphi$.

For particles of a given sign, there must be a "cross section" of the instrument $\Sigma(E,\vartheta,\varphi)$ such that the number dN of incoming particles within the specified intervals of energy and direction, that cross the useful volume of the three counters, is given by

$$dN=dn\sum(E,\vartheta,\varphi).$$

Naturally Σ depends also on the field B, on the geometrical constants of the instrument, and on the field "arrangement" (whether *cc*, or *cd*, etc.).

Since the instrument is sensitive (or $\Sigma\neq0$) only to particles with $|\varphi|\ll1$ we may neglect the dependence of J on φ and use for the total number of mesons crossing the counter system

$$N=\int\int J(E,\vartheta)dE\sin\vartheta d\vartheta\int\sum(E,\vartheta,\varphi)d\varphi. \quad (5)$$

Now a more detailed investigation[17] shows that the second integral can be put in the form

$$\int\sum(E,\vartheta,\varphi)d\varphi=\psi_1(\vartheta)f(\sigma,\gamma), \quad (6)$$

[17] Bernardini, Conversi, Pancini, and Wick, reference 1.

where σ and γ are dimensionless measures for the energy E and the stopping power L of iron, respectively,

$$\sigma=\frac{r\cdot\sin\vartheta}{d^2Be}E;\quad \gamma=\frac{r}{dBe}L. \quad (7)$$

Here d is the distance between the axis of the central counter and either the lower or the upper counter, r is the radius of a counter, e is the charge of the particle. In Eq. (6) ψ_1 is a simple function which needs not to be known exactly; the function $f(\sigma,\gamma)$ shall be indicated as the "characteristic function" of the instrument and expresses the dependence of the cross section on all parameters involved through the relation (7).

We shall now suppose that the dependence of the spectrum $J(E,\vartheta)$ on energy and ϑ can be separated so that we can put

$$J(E,\vartheta)=\psi_2(\vartheta)\Phi(\sigma). \quad (8)$$

This hypothesis can be justified approximately; remember that the intensity of mesons of energy E, incident under a zenith angle ζ, is well represented[18] by the formula

$$\left[E+\frac{m}{\cos\zeta}\right]^{-n}dE, \quad (9)$$

with a proper choice of the constants m and n. Now since in our measurements the counter telescope was vertical, and remembering that $|\varphi|\ll1$, we have: $\cos\zeta\approx\sin\vartheta$ and putting

$$a=\frac{d^2Be}{r}=\frac{Ld}{\gamma};\quad \psi_2(\vartheta)=\sin^n\vartheta;$$
$$\Phi(\sigma)=(a\sigma+m)^{-n}, \quad (10)$$

we really find formula (8).

Substituting (6) and (8) into (5) and integrating over ϑ, we find

$$N=\int\Phi(\sigma)f(\sigma,\gamma)d\sigma, \quad (11)$$

neglecting a constant factor which is unessential in the following development.

We have termed f the characteristic function, but in fact, there are four such functions, corre-

[18] Conversi and Scrocco, reference 1.

TABLE III. Results of the α and α_+ calculated effects.

	1st series (α)		2nd series (α)		3rd series (α_+)	
$\tau/\mu c^2$	$k=0$	$k=\frac{1}{8}$	$k=0$	$k=\frac{1}{8}$	$k=0$	$k=\frac{1}{8}$
1	38.7	17.9	44.8	24.4	56.0	36.2
2	79.5	41.8	78.8	38.6	119.0	66.8
3	100.0	51.1	93.8	47.5	152.1	81.5
4	111.7	55.6	102.6	51.3	171.1	92.5

sponding to the four possible arrangements, cc, dd, cd, dc ("convergent" "divergent" referring of course to particles of the specified sign). We may indicate these functions by f_{cc}, f_{dd}, $\cdots$ respectively; if they are known, we may proceed to a theoretical evaluation of (1), (3), and (4).

How these functions can be determined has been discussed elsewhere.[17,18] We shall only recall some results.

We shall put

$$\bar{\sigma}=\sigma-\gamma, \tag{12}$$

which is the value of the "energy" σ possessed by a particle when it reaches the central counter, that is after suffering one-half of the total energy loss in crossing the iron cores. The theory then shows that one has, with sufficient approximation

$$f_{cc}(\sigma,\gamma)=f_{cc}(\bar{\sigma},0);\quad f_{dd}(\sigma,\gamma)=f_{dd}(\bar{\sigma},0);$$
$$\tfrac{1}{2}[f_{cd}(\sigma,\gamma)+f_{dc}(\sigma,\gamma)]=f_{cd}(\bar{\sigma},0)=f_{dc}(\bar{\sigma},0). \tag{13}$$

These formulae have a rather simple meaning; the first two mean, for instance, that the characteristic function can be evaluated by neglecting the energy loss (that is putting $\gamma=0$ in $f(\sigma,\gamma)$) provided the energy $\bar{\sigma}$ instead of the initial energy σ is taken. Then $f(\sigma,\gamma)$ may be indicated more briefly with $f(\bar{\sigma})$. We have found

$$f_{cc}(\bar{\sigma})=1+(1/\bar{\sigma})-(k/\bar{\sigma}^2);$$

$$f_{dd}(\bar{\sigma})=\begin{cases}1-(1/\bar{\sigma})+(1/4\bar{\sigma}^2); & (\bar{\sigma}>\frac{1}{2})\\ 0; & [\bar{\sigma}<\frac{1}{2};\ B>\sim 4.10^3 \text{ gauss}]\end{cases};$$

$$f_{cd}(\bar{\sigma})=f_{dc}(\bar{\sigma})=1+\frac{1+\bar{\sigma}}{2\bar{\sigma}^2}\xi_m{}^2-\frac{1}{4\bar{\sigma}^2}(3\xi_m+1),$$

where ξ_m is amongst the three real roots of

$$\xi_m{}^3-2(1+\sigma)\xi_m-1=0,$$

the one which satisfies $-1<\xi_m<0$.

The value of the coefficient k must still be considered. The term containing k has been introduced to take into account in a rough way the effect of scattering within the iron cores, which could not be included in the theory. It is natural to assume that this effect will be important especially at the low energies and especially in the "cc" case when the cross section of the counter system tends to increase to very large values with decreasing energy. In our simplified theory this increase continues until the cross section is suddenly brought down to zero, because the particles are stopped in the iron before reaching the lowest counter. Scattering, however, must set a limit to this increase before the stopping limit is reached, and this effect may be represented by a term of the type considered. The choice of the value of k has no considerable influence on the evaluation of the positive excess (see Section 5(a)); unfortunately this is not so in the case of the meson spectrum. We have avoided the difficulty by the assumption that the proper value of k can be determined by comparing our experimental results in air and at sea level with calculations based on formulae (11) and assuming as energy spectrum one that can be derived from the absorption curve in lead measured by Clay and van Gemert. For further details see one of our previous papers;[18] the result is: $k=0.13\pm0.01$.

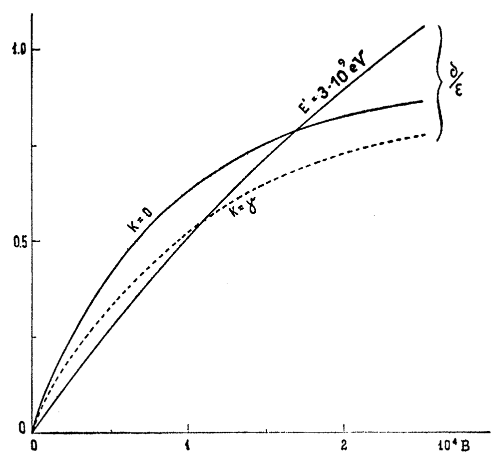

FIG. 7. Theoretical $\delta(B)$ curves (see formula (1)) under the assumptions of a uniformly distributed positive excess (curves $k=0$ and $k=\gamma$) and of an excess localized about $E'=3\times10^9$ ev.

5. DISCUSSION

(a) The Positive Excess

Remembering the definition of N_{cc}, N_{dd}, by means of Eq. (11) we have

$$N_{cc+}=\int_{\gamma}^{\infty}\{\Phi^{+}(\sigma)f_{cc}(\sigma)+\Phi^{-}(\sigma)f_{dd}(\sigma)\}d\sigma,$$
$$N_{dd+}=\int_{\gamma}^{\infty}\{\Phi^{-}(\sigma)f_{cc}(\sigma)+\Phi^{+}(\sigma)f_{dd}(\sigma)\}d\sigma, \quad (14)$$

the meaning of the symbols being self-evident. Assume the positive excess ϵ to be distributed uniformly over the spectrum

$$\Phi^{+}(\sigma)=\tfrac{1}{2}\left(1+\frac{\epsilon}{2}\right)\Phi(\sigma);$$
$$\Phi^{-}(\sigma)=\tfrac{1}{2}\left(1-\frac{\epsilon}{2}\right)\Phi(\sigma). \quad (15)$$

Using the third of Eqs. (10), with $n=3$, we have calculated N_{cc}, N_{dd} as functions of B. From these we derive the $\delta(B)$ curve represented in Fig. 7,[19] which is in good agreement with the approximately linear increase of the experimental δ (Figs. 5, 6). We have found experimentally $\delta=0.15$ for $B=17{,}000$ gauss; comparing this with the two curves of Fig. 7, we find $\epsilon=0.19$ or 0.23 according as we make for k the two extreme assumptions $k=0$, and $k=\gamma$. The exact value of k is therefore rather unimportant, and we may conclude that $\epsilon=20$ percent approximately in very good agreement with Hughes and Jones.

The agreement between the result of Hughes and Jones and our own gives a plausible argument in favor of B as the effective field inside iron.

Subsequently we have also examined different hypotheses about the distribution of the positive excess over the spectrum, but we have found that the shape of the δ *vs.* B curve is rather insensitive to such changes, so that closer investigation of the shape of this curve does not open the possibility of investigating the distribution of the positive excess over the spectrum. The only thing we can say is, that if the excess were located in the low energy part of the spectrum, say around $E<\sim10^{9}$ ev the value of δ to be expected (for a positive excess of 20 percent) would be considerably larger than observed.

[19] We have not included in the figure the N_{cc} *vs.* B and N_{dd} *vs.* B curves, as derived theoretically; but they also are in good accord with the data of Table I.

TABLE IV. Results of the comparison of theoretical curves for $\alpha(\tau/\mu c^2)$ and experimental results.

Series	$10^{8}\tau/\mu c^{2}$
1st	$2.15\begin{cases}+0.40\\-0.28\end{cases}$
2nd	$2.25\begin{cases}+0.55\\-0.38\end{cases}$
3rd	$2.10\begin{cases}+0.52\\-0.32\end{cases}$

(b) Energy Spectrum

In this case, we assume, for greater accuracy, the meson spectrum to be given by the product of (9) by the probability $W(E, h)$ for a meson of energy E to reach the depth h without disintegrating. In this way the effects α and α_{+}, defined by (3) and (4) are seen to depend on the $\tau/\mu c^2$ ratio, which appears in the expression for $W(E, h)$. If j is the stopping power of air, we shall have $m=hj$. Further we have chosen $n=2.87$ as the most accurate value known to us.[20] For $W(E, h)$ the usual expression was used, and by means of (7) the spectrum was expressed as a function of the dimensionless energy σ.

Assume the positive excess to be uniformly distributed over the spectrum. Writing the explicit expressions of N_{cd} and N_{dc} (analogous to (20)) and putting for shortness sake:

$$F(\sigma)=\tfrac{1}{2}[f_{cc}(\sigma)+f_{dd}(\sigma)];$$
$$g(\sigma)=\tfrac{1}{2}[f_{cc}-f_{dd}+f_{cd}-f_{dc}],$$

we shall have (see Eq. (2)),

$$N_{cc}=\int_{\gamma}^{\infty}\Phi(\sigma)F(\sigma)d\sigma;$$
$$N_{cd}=\int_{\gamma}^{\infty}\Phi(\sigma)f_{cd}(\sigma)d\sigma \quad (16)$$

while (4) can be expressed approximately[21] in

[20] A. Emhert, Zeits. f. Physik **106**, 751 (1937).

[21] That is neglecting a term $\frac{\epsilon}{4}\int_{\gamma}^{\infty}[f_{dc}(\sigma,\gamma)-f_{cd}(\sigma,\gamma)]d\sigma$ compared to N_{cd}.

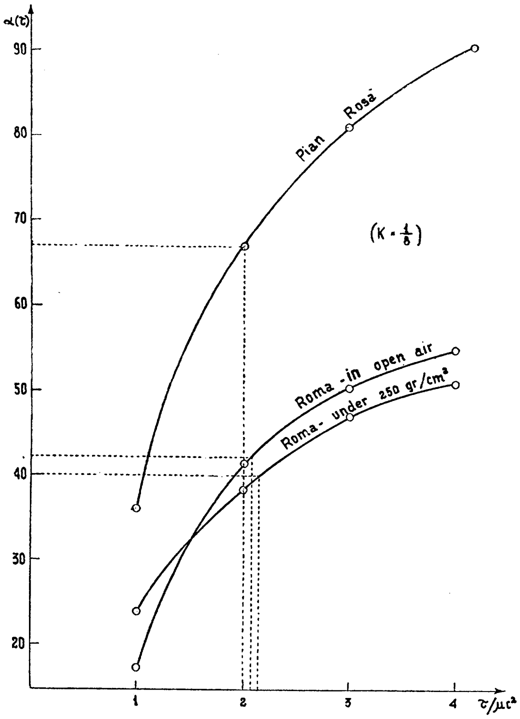

FIG. 8. Comparison of theoretical $\alpha(\tau/\mu c^2)$ curves (see formulas (2)—(4)) and experimental results.

the form

$$\alpha_+ = \alpha + \frac{\epsilon}{2N_{cd}} \int_\gamma^\infty \Phi(\sigma) g(\sigma) d\sigma. \qquad (17)$$

The integrals in (16) and (17) have been evaluated numerically, for each of the three series of measurements, for two values of k ($k=0$ and $k=1/8$) and for four values of the ratio $\tau/\mu c^2$ ($=1$; 2; 3; $4 \cdot 10^{-8}$ sec./Mev). The results are given in Table III. Comparing the calculated $\alpha(\tau/\mu c^2)$ curves (these can be drawn from the results of Table III also for the k values considered in Section 4, remembering the linear dependence of α on k) with the experimental results of Table II, we obtain for $\tau/\mu c^2$ the values contained in Table IV. The essential hypothesis made is that the meson spectrum at the top of the atmosphere can be represented by the law $E^{-2.87}$.

The calculated curves of Fig. 8 confirm the statements already made in Section 3(b) about the influence on α of an absorber 250 g/cm² thick, if one assumes for $\tau/\mu c^2$ a value as low as that found by Nielsen *et al*;[16] the effect observed experimentally has not the sign to be expected on the assumption of a value as low as that.

The values for $\tau/\mu c^2$ derived from the three series of measurements are in surprising agreement with each other, especially considering the fact that the agreement is not sensitive to a change in the initial spectrum. This seems to be one further reason to exclude a complex nature of the meson component,[22] at least between sea level and 4000 m.

Let us now briefly discuss our results. The evaluation of $\tau/\mu c^2$ was based on formula (11); hence it is subject to errors due to (a) the approximations adopted in the theoretical treatment of the characteristic functions and (b) deviations of the actual energy distribution at the top of the atmosphere from the distribution assumed, formula (9).

As regards the first cause of uncertainty a discussion of the hypotheses involved has been given elsewhere[18] but it is rather hard to estimate accurately the possible error. We feel, however, that to assume that this is the source of the difference between our present results and the value, $\tau/\mu c^2 \approx 3\times 10^{-8}$ sec./Mev, which is indicated by the most recent and reliable measurements by the anomalous absorption method,[23] would be to stretch the limits of error of the calculations more than is probable.

We shall, therefore, examine the second possible cause of discrepancy. Obviously one can interpret the discrepancy by the assumption, that the meson energy spectrum on the low energy side is less rich in particles than corresponds to (9). According to this formula the spectrum is given by the product of (9), with $n=2.87$, by the probability of survival $W(E, h)$. This probability was evaluated on the usual assumption that the creation of mesons takes place at a depth of 100 g/cm² below the top. It is well to remark that the value of $W(E, h)$ at sea level is not very sensitive to changes in the depth of the creation layer. Turning now to formula (9), we notice that it is supported by the work of Blackett, Jones, Hughes, and others, for $n=2.9$ and for $E=$values comprised between

[22] P. Weisz, Phys. Rev. **59**, 554 (1941).
[23] G. Bernardini, Zeits. f. Physik **120**, 413 (1943).

some 10^9 ev and the maximum energies measurable by the Wilson chamber method. The intensity or depth curves of Ehmert and others indicate that the validity of the formula extends to much higher energies. Since moreover the exact shape of the spectrum above 10^{10} ev is immaterial for the present purpose, we may assume the formula $E^{-2.87}$ to represent the *initial* spectrum correctly for all E-values larger than a certain E^*. Considering that in crossing the atmosphere a meson loses about 2.5×10^9 ev, we shall put $E^*\approx 4\times 10^9$ ev.

For $E<E^*$ the $E^{-2.87}$ law is not sufficiently supported by any experimental data.

There is even some indication to the contrary, for the energy spectra of Hughes and Jones seem to be less rich on the low energy side than one would expect assuming a creation spectrum according to (9), this conclusion being practically unaffected by a variation of the assumed height of the creation layer between 50 and 200 g/cm^2.

Such a conclusion would be in agreement with our observations and would not lead to any difficulty in connection with present knowledge about the meson radiation in the upper atmosphere. As a matter of fact, the experiments of Schein, Wollan, and Groetzinger,[24] and of Herzog and Bostick,[25] have revealed that a large number of rather slow mesons are created in the upper atmosphere; but these particles are in general much too slow to reach our apparatus and be recorded by it.

One further remark is that the meson spectrum must probably be affected in some way by the cutting off in the spectrum of the primary protons (?) by the magnetic fields of the earth and sun. This would cause a falling off on the low energy side. On the other hand one may compare these suggestions with the results of the measurements of $\tau/\mu c^2$ by the anomalous absorption method based on integral effects. These require hypotheses on the creation spectrum of mesons which are quite analogous to those which are necessary to discuss our experimental results.

As a matter of fact, in the anomalous absorption method, one finds that the agreement between the $\tau/\mu c^2$ values based on differential effects, which are independent of any hypothesis concerning the spectrum, and those based on integral effects, which are not, is in general rather poor. This points to some hypothesis such as we have indicated. Two notable exceptions are given by the measurements discussed by Bernardini and Festa[26] and by the ionization chamber measurements of Neher and Stever.[27] As regards the first exception, however, one may remark that the agreement found may be owing to the evaluation method employed by those authors, a method whose precise aim was to eliminate the uncertainty of the low-energy side of the meson spectrum from the final result.

As regards the experiment of Neher and Stever, one has to remark that it refers to the meson radiation incident from the whole hemisphere; consequently the mean path in air of the particles was much larger than with a vertical counter set. This naturally increases the importance of the high energy side of the spectrum, where no large discrepancies from the $E^{-2.87}$ law need be assumed.

Let us finally turn our attention to the data of the last column in Table II. These allow (see also Section 3(b)) a further evaluation of the positive excess ϵ, on the assumption that it is uniformly distributed over the spectrum. In fact, putting

$$\mu=\frac{1}{N_{cd}}\int_{\gamma}^{\infty}\Phi(\sigma)g(\sigma)d\sigma,$$

we get from (17) and from the analogous expression for α_-

$$\Delta=\mu\epsilon.$$

The evaluation gives:

$$\epsilon=\begin{cases}(18.3\pm 4.4)\text{ percent in open air;}\\ (18.6\pm 5.4)\text{ percent under a layer 250 g/cm}^2\\ \quad\text{thick,}\end{cases}$$

in good agreement with the ϵ-value derived from Table I. There is as yet no evidence that the 250 g/cm^2 layer causes an appreciable change in the positive excess.

We finally wish to emphasize the possibilities

[24] M. Shein, E. O. Wollan, and G. Groetzinger, Phys. Rev. **58**, 1027 (1940).

[25] G. Herzog, and W. H. Bostick, Phys. Rev. **58**, 278 (1940); **59**, 122 (1941).

[26] G. Bernardini and C. Festa, Atti d. R. Acc. D'Italia **7**, 4 (1943).

[27] H. V. Neher and H. G. Stever, Phys. Rev. **58**, 766 (1940).

which the present method may have in the investigation of the meson spectrum at great heights (notice in this connection how the slope of the α *vs.* $\tau/\mu c^2$ curves increases with increasing height). An experiment of this kind by Bernardini and Scrocco was in progress in 1941, but was interrupted by a serious flying accident to the airplane which carried the apparatus. As a consequence the apparatus was gravely damaged, and for this and other reasons connected with Italian events, the experiment was given up for the time being.

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Disorder Scattering of X-Rays by Local Distortions

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The disorder scattering (background) owing to local distortions is caused mainly by the elastic strain field surrounding the distorted zone rather than by the misfit atoms in the zone of distortion. As an example, the scattering caused by a distortion of spherical symmetry is calculated. The result is extended for more general types of distortions. The background intensity increases strongly in the neighborhood of a line, and it tends toward a constant value for small scattering angles. This explains previous observations on rolled copper.

1

To explain the plastic properties of solid bodies, it is necessary to assume local lattice distortions or irregularities. Most of the evidence for the assumed models of dislocation is indirect. Several papers have attempted to connect the observed line intensities and widths with the internal distortions.[1–6] Little attention has been given to the continuous background scattering arising from the distortions. The experimental conditions for this type of research are not easy, because other effects such as fluorescent radiation, Compton radiation, diffuse scattering by air, coherent temperature scattering and background owing to the continuous x-ray spectrum, all contribute to the observed background. Yet experiments by Guinier[7] have shown the effect of plastic deformation of copper upon the background to be appreciable. If the scattering caused by local distortions can be isolated, it will provide more useful information than the study of the lines; first, because this scattering is entirely absent in an ideal crystal, while the line intensity decreases only by a few percent when distortion is introduced, and secondly, because slowly variable internal stresses affect the line, but not the background.

The following theoretical discussion gives a correlation between local distortions and disorder scattering, which explains some of Guinier's results, and may be helpful for further experimental work.

For a simple Bravais lattice, the scattering intensity is given by:

$$I=I_e|F|^2|\textstyle\sum_n \exp(2\pi i\mathbf{R}\cdot\mathbf{r}_n)|^2, \qquad (1)$$

where I_e is the scattering intensity from one electron, F the atomic form factor,

$$\mathbf{R}=\mathbf{k}-\mathbf{k}_0, \qquad (2)$$

$\mathbf{k}$ and $\mathbf{k}_0$ the wave vectors of incident and scattered wave, and $\mathbf{r}_n$ the radius vector of the nth atom. For $\mathbf{r}_n$, we write

$$\mathbf{r}_n=\mathbf{a}_n+\mathbf{u}_n, \qquad (3)$$

[1] J. Hengstenberg and H. Mark, Zeits. f. Physik **61**, 435 (1930).
[2] G. W. Brindley and F. W. Spiers, Phil. Mag. **20**, 882 (1935).
[3] U. Dehlinger, Zeits. f. Krist. **65**, 615 (1927).
[4] W. Boas, Zeits. f. Krist. **97**, 354 (1937).
[5] U. Dehlinger and A. Kochendörfer, Zeits. f. Metallkunde **31**, 231 (1939).
[6] A. Kochendörfer, Zeits. f. Krist. **101**, 149 (1939).
[7] A. Guinier, Comptes rendus **208**, 894 (1939).