Letters to the Editor

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The Magnetic Field Inside a Ferromagnet

G. H. WANNIER
University of Iowa, Iowa City, Iowa
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IT is generally agreed that for a fast charged particle (in the following, referred to as the "test charge") traversing a magnetized iron bar, the average magnetic field **b** is equal to the induction **B**.[1,2] However, doubts as to the correctness of this statement have been expressed on classical grounds[3] because the dominating contribution to this average is made by the enormous field which exists inside the spinning electron. This same state of affairs prevails in quantum theory. It expresses itself in the formula

$$\mathbf{b}=\mathbf{h}+4\pi p\mathbf{M}. \qquad (1)$$

Here **b** is the average field, **M** is the magnetization, and **h** is the field which would exist if the electrons were true magnetic "di-poles." p is a numerical factor which is equal to the relative probability of coincidence of test charge and electron, as compared to randomness.

The formula can be derived without specifying the structure of the wave function, except for the following assumptions:

(a) that the magnetic interaction is sufficiently small to be treated as a first-order perturbation,

(b) that the magnetic field is owing entirely to the electronic spin,

(c) that the ferromagnetic electrons move in orbits independent of each other,

(d) that it is sufficiently accurate to solve the Dirac equation in the Schroedinger type approximation of Darwin-Pauli, and

(e) that the test charge is much heavier than an electron.

Formula (1) becomes standard when the coincidence probability p equals 1, that is for plane waves. However, the term $4\pi\mathbf{M}$ is owing entirely to head-on collisions. This gives rise to two objections:[3]

(1) Head-on collisions may not be frequent enough to average out for each particle. This objection is actually unfounded as such a collision occurs in a crystal about once every hundred atoms.

(2) The coincidence probability p of the test charge and electron may be different from unity.

The latter proposition is certainly true to some extent, partly because of the crystalline field. But the primary reason is the direct attraction or repulsion between electron and test charge upon close approach. Barring the case of uncharged particles such as neutrons, or the existence of short range forces yet unknown, this direct interaction is primarily the Coulomb interaction between electric charges. For this case, the numerical value of p is known.[4] It equals

$$p=x/(1-e^{-x}), \qquad (2)$$

where

$$x=4\pi^2Ze^2/hv=\pm 1.39\times 10^9|Z|/v. \qquad (3)$$

Here v is the velocity of the test charge in cm/sec. and Ze is the charge of the test charge. The sign of x depends on the sign of this charge. A few values of p are listed in Table I.

TABLE I.

$v/\|Z\|$	p +charge	p −charge
$\frac{1}{4}\cdot 10^9$	5.58	0.021
$\frac{1}{2}\cdot 10^9$	2.97	0.184
$1\cdot 10^9$	1.85	0.461
$2\cdot 10^9$	1.39	0.692
$4\cdot 10^9$	1.18	0.837

Formula (1) suggests that if **b** is different from **B**, or **h** different from **H**, it will be primarily caused by short range Coulomb interaction between the test charge and the ferromagnetic electron. Such a calculation can be handled as a collision problem for two particles which are free otherwise. The necessary integrations can then be performed exactly and give

$$\mathbf{b}=\mathbf{H}+2\pi(p+1)\mathbf{M}, \qquad (4a)$$

or

$$\mathbf{b}=\mathbf{B}+2\pi(p-1)\mathbf{M}, \qquad (4b)$$

where p is again the parameter used in (1) and (2). This gives a field larger than **B** for positive charges and smaller than **B** for negative charges.

From an experimental point of view, the most hopeful feature for a check is the assymmetry of the equations for positive and negative charges. Positive particles should be deflected more under otherwise similar circumstances, provided their speed is not much greater than 5×10^9 cm/sec. It should be added in conclusion that because of assumption (d), all formulas are subject to relativistic modifications which are being investigated by the author.

[1] C. F. v. Weizsäcker, Ann. d. Physik **17**, 869 (1933).
[2] F. Rasetti, Phys. Rev. **66**, 1 (1944).
[3] W. F. G. Swann, Phys. Rev. **49**, 574 (1936).
[4] Mott and Massey, *Theory of Atomic Collisions* (Oxford University Press, New York), p. 36, formula (17).

Development of Electromagnetic Theory for Non-Homogeneous Spaces—A Correction

BENJAMIN LIEBOWITZ
New York, New York
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MY paper under the above title[1] purported to effect a separation of the components which define an electromagnetic field for the general case in which ϵ, the specific inductive capacity of the space, is an arbitrary twice-differentiable function of the coordinates. The