THE
PHYSICAL REVIEW
A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 67, NOS. 1 AND 2 JANUARY 1 AND 15, 1945

Threshold Measurements on the Nuclear Photo-Effect

GEORGE C. BALDWIN* AND H. W. KOCH
Department of Physics, University of Illinois, Urbana, Illinois
(Received March 27, 1944)

The high voltage x-radiation from the betatron has been employed to produce (γ, n) reactions in several elements of atomic number up to 47, the reaction observed by detection of induced radioactivity. The peak x-ray energy was controlled by integrating the voltage on the main coils with an RC circuit, which actuated the orbit expander at a predetermined electron energy. Upon irradiating samples at sequences of energies, measuring their activity with beta-counters, and plotting activities against peak energy, smooth curves were obtained from which thresholds were estimated. The accuracy of the method is limited by low activity in almost all cases and by uncertainties in energy output. The following thresholds in Mev have been measured: C^{11}, 18.7 to 19.4; N^{13}, 11.1±0.5; O^{15}, 16.3±0.4; Fe^{53}, 14.2±0.4; Cu^{62}, 10.9±0.3; Zn^{63}, 11.6±0.4; $Se^{79 \text{ or } 81}$, 9.8±0.5 for the lower, short period isomer; $Mo^{91 \text{ or } 93}$, 13.5±0.4; Ag^{108}, 9.3±0.5; Ag^{106}, 9.5. The first three are in fair agreement with other data, but because of exceptionally weak activities are not sufficiently reliable to afford a test of the method. Rough measurements on the uranium photo-fission threshold are also reported. Since the radiation from the betatron is continuous and its spectral distribution is unknown, the excitation curves are not susceptible to interpretation, and cross sections for the above reactions cannot be estimated at present.

INTRODUCTION

WITH the high energy x-rays now available from the betatron, it is energetically possible to photo-disintegrate nearly all nuclei. In all cases so far observed, photo-disintegration of a target nucleus of charge Z and mass number A leads to an isotope of the target with mass number $A-1$, upon ejection of a neutron from the target nucleus. The process may be detected by direct observation of the ejected neutron; however, frequently the product nucleus is radioactive and the reaction is then most conveniently detected by observation of the induced activity in an irradiated sample—this method having the added advantage of definitely identifying the particular isotopes of the target element involved in the reaction.

To produce the photo-effect it is necessary that the energy of the incident quantum exceed the binding energy of the particle to be ejected. The following reaction is typical:

$$N^{14}+h\nu \rightarrow N^{13}+n^1,$$
$$N^{13} \rightarrow C^{13}+e^+ +e^- +\eta +Q.$$

Conservation of energy requires that

$$h\nu \geq M(_6C^{13})+M(_0n^1)+2mc^2+Q-M(_7N^{14}),$$

where M denotes the atomic mass. Substitution of numerical values[1,2] for the masses of N^{14} and

*Now at General Electric Research Laboratory, Schenectady, New York.

[1] R. T. Birge, Rev. Mod. Phys. **13**, 233 (1941).
[2] J. J. Livingood and G. T. Seaborg, Rev. Mod. Phys. **12**, 30 (1940).

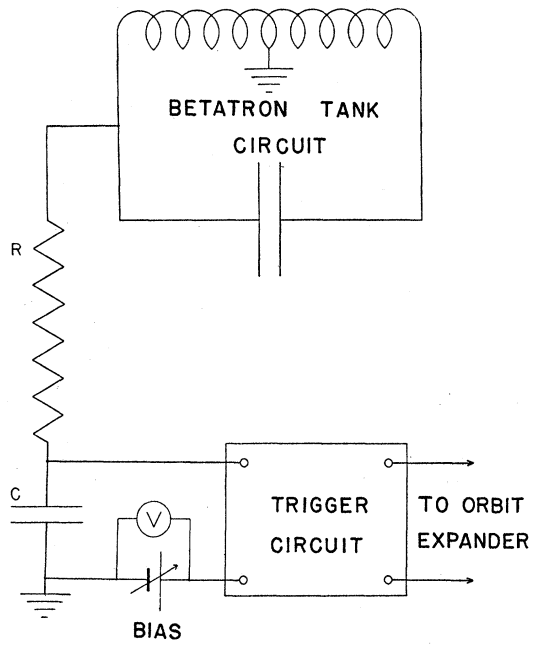

FIG. 1. Circuit scheme for control of peak x-ray energy.

C^{13} and for Q leads to

$$h\nu = 10.65 \pm 0.15 \text{ Mev}$$

for the threshold of the photo-effect in N^{14}. Similar calculations give thresholds of 15.6 ± 0.6 for O^{16} and 18.7 ± 0.3 for C^{12}. Calculation of the threshold is possible only in several cases, however. Since a knowledge of the binding energies of particles in nuclei is of considerable theoretical interest, we have undertaken measurements of photo-effect thresholds in a sequence of nuclei from $Z=6$ to $Z=43$. This work has been reported elsewhere.[3]

Previous work on the photo-effect, except in the cases of beryllium and deuterium, has been limited by the unavailability of x-rays of sufficiently high energy. For heavier nuclei at least 8 Mev are required; in all cases reported herein, considerably higher energies are necessary. Gamma-rays from proton bombardment of lithium and boron have been the chief photo-disintegrative agents employed in past work. Bothe and Gentner[4] have measured the cross section for the reaction $Cu^{63}(\gamma, n)Cu^{62}$ with lithium and boron gamma-rays, and have compared the yields induced by both radiations in other elements. Inhomogeneity of the boron radiation, together with low yields of activity, makes interpretation of their results uncertain. Thresholds could not be estimated in this sort of measurement.

METHOD

The threshold measurements reported in this paper were made by irradiating samples at a sequence of energies, the energy being raised in small steps until the process was observed to occur, as evidenced by radioactivity induced in the sample. The yield of activity was plotted as a function of peak x-ray energy, giving a curve from which the threshold was obtained by inspection. This method involved the problems of (1) accurate control of peak energy, (2) monitoring of x-ray intensity, and (3) detection of the reaction.

For energy control, use was made of the fact that the energy acquired by electrons during acceleration in the betatron is proportional to the magnetic flux linking their orbit, which is in turn proportional to the flux linking the main coils.[5] A resistor and capacitor were placed in series across one of the main coils, the time constant large compared with the period of the alternating flux. The condenser voltage, proportional to the integral of the applied voltage, was proportional to the flux linkage and hence to the energy. This voltage was applied in series with an adjustable bias to a trigger circuit to actuate the orbit expander at a chosen critical input voltage read on the bias voltmeter. The block diagram of Fig. 1 illustrates the circuit scheme. With the constants employed ($R=10^6$ ohms, $C=10^{-7}$ farad) a voltage of 88.5 v on the integrating condenser corresponded to an electron energy of 20 Mev. The trigger point, reproducible to within 0.2 v, showed negligible drift after adequate warm-up. All voltages were electronically stabilized.

This circuit was calibrated by a search coil in the orbit plane. It was known that the magnet flux is an accurate sine wave, the third harmonic amounting to only 0.07 percent at the highest amplitude. The flux within the search coil could thus be readily determined by a voltage measure-

[3] G. C. Baldwin and H. W. Koch, Phys. Rev. **63**, 59A and 426A (1943).

[4] W. Bothe and W. Gentner, Zeits. f. Physik **112**, 45 (1939).

[5] D. W. Kerst, Phys. Rev. **60**, 47 (1941); Rev. Sci. Inst. **13**, 387 (1942).

ment and simultaneously compared with the voltage integrator response and with the magnet voltage read on a 2-turn coil about the magnet yoke. The latter reading was made a permanent substandard to which each setting was referred by determining the magnet voltage at which the integrator circuit would cause orbit expansion at peak flux. From these calibration data the Hr value at which the electrons struck the target could be computed. The probable error of this calibration was 1.5 percent; in actual energy settings further error arose. The total error in most cases did not exceed 2 percent.

Samples were irradiated at a standard position in the x-ray beam, 46 cm from the target, ordinarily to half-saturation. Irregularities in the resulting activity could be largely accounted for by variations in x-ray intensity from run to run. The intensity, monitored by an ionization chamber, was recorded in roentgens per minute at 1 meter from the target. The method of correction to standard intensity will be evident from a consideration of the process of radioactive growth under decay. During an infinitesimal time interval dt, the number of active nuclei N will change by

$$dN=kIdt-Ndt/\tau \qquad (1)$$

where k is a constant, I the x-ray intensity, and τ the mean life of the expected activity. This gives for the number of active nuclei produced between

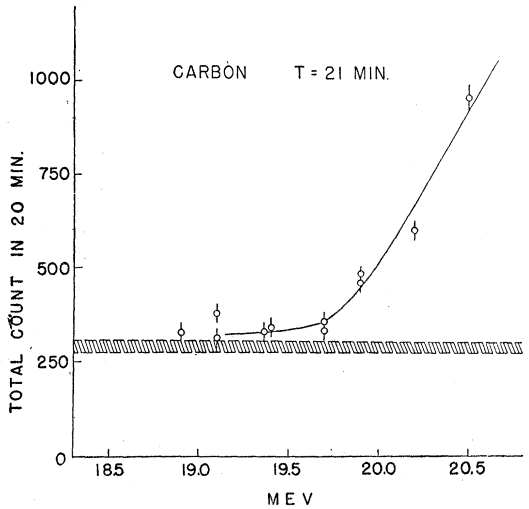

FIG. 2. Excitation curve for production of C^{11} by x-rays. The cross-hatched area represents the statistical spread in natural background of the Geiger-Müller counter.

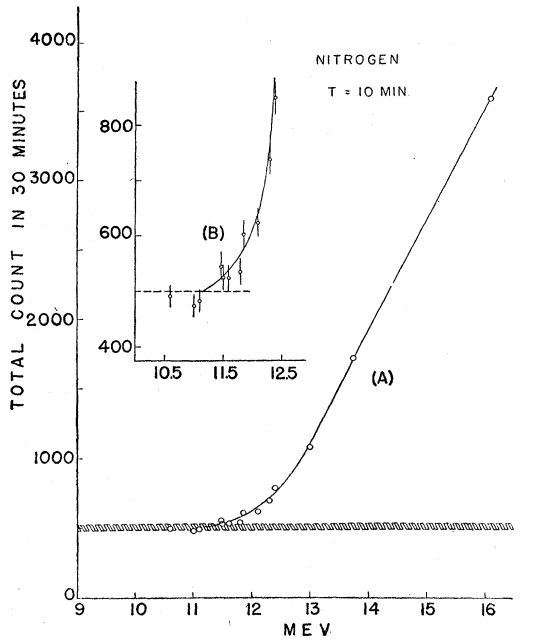

FIG. 3. Excitation curve for N^{13}: (A) Corrected for intensity variations; (B) Uncorrected points near threshold.

$t=0$ and $t=T$

$$N=k\exp(-T/\tau)\int_0^T I(t)\exp(t/\tau)dt=kR. \qquad (2)$$

In practice the integral was replaced by a weighted sum; the monitor was read at regular intervals during irradiation, each reading multiplied by the weighting factor, $\exp(t/\tau)\cdot\exp(-T/\tau)$, and the results added. The resulting sum R is called the "irradiation."

To reduce as far as possible the effect of intensity fluctuations all points were then corrected to the irradiation which the sample ordinarily received near threshold. This did not change points near threshold, where the effects were small or doubtful, but smoothed the curves somewhat and thereby facilitated threshold estimation.

The resulting smoothed curves are not susceptible to ready interpretation. It is not yet possible to determine from them the trend of the cross section for the process. That portion of the x-ray spectrum which is responsible for the reaction is responsible for only a small portion of the monitor response. If the cross section of the process be given by $\sigma(E)$ and the unknown spectral distribution by $N(E)$, the yield at some peak

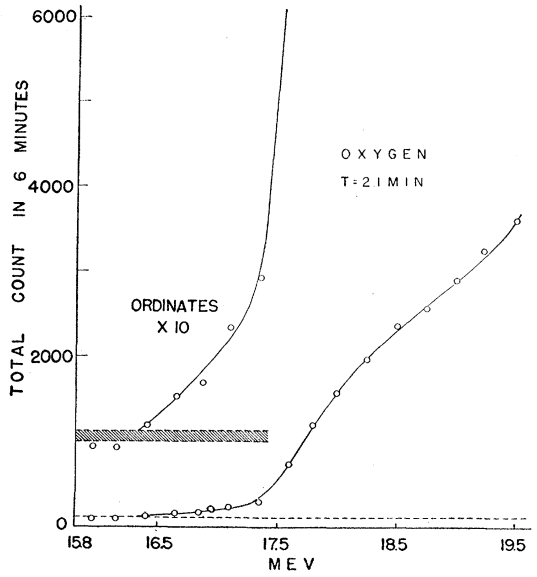

FIG. 4. Excitation curve for O^{15}. Points near threshold are replotted to larger scale in the inset. The first two points are below background because of the shielding effect of the water sample in which the counter was immersed.

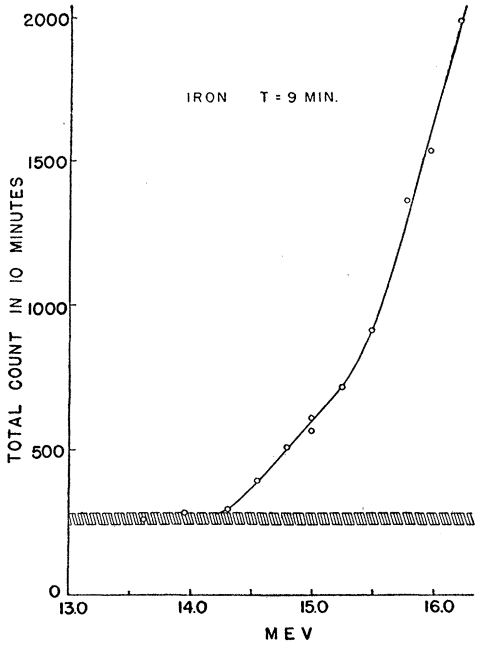

FIG. 5. Excitation curve for Fe^{53}.

energy E_{max} is proportional to

$$\int_{E_0}^{E_{max}} N(E)\sigma(E)dE,$$

where E_0 is the threshold for the reaction. The monitor response is given by a similar expression, but the lower limit and the energy-dependence of the integrand are entirely different. There is thus no simple correlation between activities observed at two different values of E_{max}, even though both were produced by the same total irradiation.

Activity, usually rather weak, was detected with β-ray counters. The counter walls were silvered glass 7 cm long, 16 mm in diameter, and about 0.2 mm thick. They were filled with 9:1 argon-alcohol to 9-cm pressure. The counter was coupled to a Neher-Harper circuit followed by a scale-of-16 counter circuit. Background of about 35 min.$^{-1}$ was reduced to 15 min.$^{-1}$ by enclosure in a lead shield 10 cm thick. Still further reduction of background is desirable. Samples were usually thick cylinders which fitted snugly over the counters.

RESULTS

For the first three of the following elements it was possible to predict the threshold from mass and beta-decay data. These reactions would provide a check on the accuracy of the method, except that the resulting activities are the weakest of all those studied. All other elements give higher yields and sharper thresholds, but reliable mass and beta-decay data are not available for comparison.

CARBON

This reaction should appear above 18.75 ± 0.3 Mev,[6] hence, it has not been produced with natural sources of gamma-radiation.

Cylinders of graphite 7.5 cm long, 2.5 cm in diameter, and 2 mm thick were irradiated at 21 Mev for 20 minutes and gave an initial activity of 200 counts per minute with a half-life of 20 ± 3 minutes. At lower energies the yields became so small that it was uncertain whether the activity was actually 21-minute C^{11}—almost any other substance, such as adsorbed air, becomes active at considerably lower energy. The total counts obtained at various energies, reduced to irradiations of 50R, are shown in Fig. 2. The cross-hatched area is the statistical spread of counter

[6] F. W. Aston, *Mass Spectra and Isotopes* (Longmans, Green and Company, New York, 1942); Delsasso, White, Barkas, and Creutz, Phys. Rev. **58**, 586 (1940); S. K. Allison, Phys. Rev. **55**, 624 (1939); L. P. Smith, Phys. Rev. **56**, 548 (1939).

background. The threshold of such a curve is taken as the energy below which points consistently fall within the background spread. The points below 19.4 Mev may or may not be genuine carbon activity. The threshold is probably below 19.4 and above 18.7 Mev, in fair agreement with the predicted value—a high result naturally being observed with such a weak activity. More accurate work with carbon must await the production of higher intensity x-radiation than has yet been obtained with the betatron.

NITROGEN

As with carbon, yields from nitrogen[7,8] are extremely weak, but there is less danger of contamination of samples, as the threshold is low. At 16-Mev peak energy the observed activity was only 600 counts per minute. Samples of NaN_3, irradiated for 10-minute periods, were held for counting between concentric aluminum cylinders. The inner cylinder, 0.8 mm thick, was perforated with 600 evenly-spaced 2-mm holes, which were covered by 0.001″ aluminum foil. This was placed over the counter. The annular section of NaN_3 thus employed was 4.5 mm thick.

Figure 3, curve *A*, shows 30-minute counting totals corrected to an irradiation of 25R. Uncorrected counts in the vicinity of threshold are plotted to larger scale in the inset, curve *B*. The difficulty of estimating a threshold is apparent, but no activity is observable below 11.1 Mev, while the sample is definitely active at 11.5 Mev. We have therefore placed the threshold for the reaction $N^{14}(\gamma, n)N^{13}$ at 11.1 ± 0.5 Mev. The stated uncertainty includes estimated uncertainties both in energy settings and in determination of the intercept of the excitation curve. This result is to be compared with the value predicted in the introduction, above.

OXYGEN

The 2.1-minute positron activity of O^{15} should have a threshold of 15.6 ± 0.6 Mev.[1,9] It has been weakly produced with lithium gamma-rays.[4,8,10]

[7] W. Bothe and W. Gentner, Zeits. f. Physik **106**, 236 (1937).
[8] O. Huber, P. Scherrer, O. Lienhard, and H. Waffler, Helv. Phys. Acta **XV**, 312 (1942).
[9] W. A. Fowler, L. A. Delsasso, and C. C. Lauritsen, Phys. Rev. **49**, 561 (1936).
[10] W. Chang, M. Goldhaber, and R. Sagane, Nature **139**, 962 (1937).

Preliminary measurements[3] indicated a higher threshold, between 16.3 and 16.7 Mev. These measurements were repeated with improved technique, using water samples and an immersion counter of the same dimensions as the other counters used in this work. The volume of each water sample was 130 cm³, which surrounded the counter to a thickness of 1 cm.

Samples were irradiated for 4 minutes to a total irradiation of 50R; counting was begun 30 seconds after irradiation and continued for three half-lives to give optimum measure of activity.

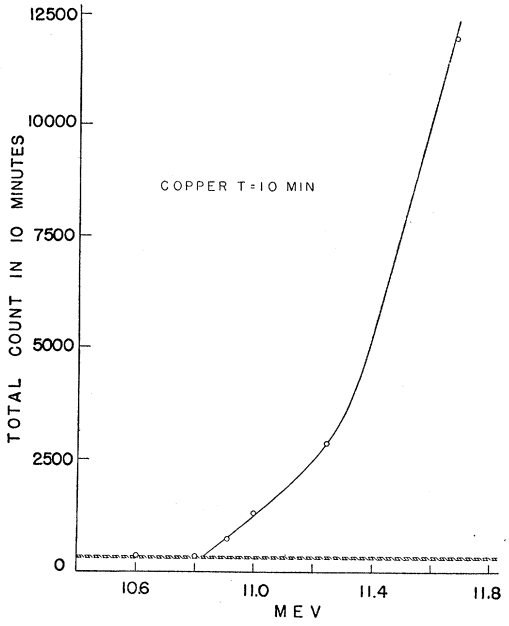

FIG. 6. Excitation curve for Cu^{62}. The energy values should be increased by about 0.1 Mev, as explained in the text.

Figure 4 shows the resulting excitation curve up to 19.5 Mev, with points near threshold replotted to larger scale. The threshold is 16.3 ± 0.4 Mev.

IRON

Iron gave a moderately strong 9-minute activity, which has been assigned to Fe^{53}.[2,8] Bothe and Gentner[4,7] did not report this activity; Carlson and Henderson[11] failed to obtain it with higher intensity lithium gamma-rays. Huber *et al.*[8] recently obtained it with 17-Mev gamma-

[11] P. R. Carlson and J. E. Henderson, Phys. Rev. **58**, 193A (1940).

rays and reported a yield of 38 percent that observed with copper, based on the original isotope Fe^{54}. It is surprising that it was completely missed in previous work—the yields obtained with the betatron were quite strong. Flat iron plates, $10\times8\times1$ cm, were irradiated for 10 minutes to 29R. For detection they were placed 5 mm from an unshielded counter. The initial counting rate at 1 Mev above threshold was three times the background of 26 min.$^{-1}$ From Fig. 5 the threshold is 14.2 ± 0.4 Mev.

COPPER

The 10-minute Cu^{62} is the strongest activity observed in a (γ, n) reaction,[7,8] and its threshold is quite sharp. Bothe and Gentner[4] measured the cross section for lithium gamma-rays as 5×10^{-26} cm^2, and with boron gamma-rays to be 2.3 times smaller.

Copper tubes 7.5 cm long, 1 mm thick, and 2.5 cm in diameter were used as samples. Irradiation was for 10-minute periods. The data are shown in

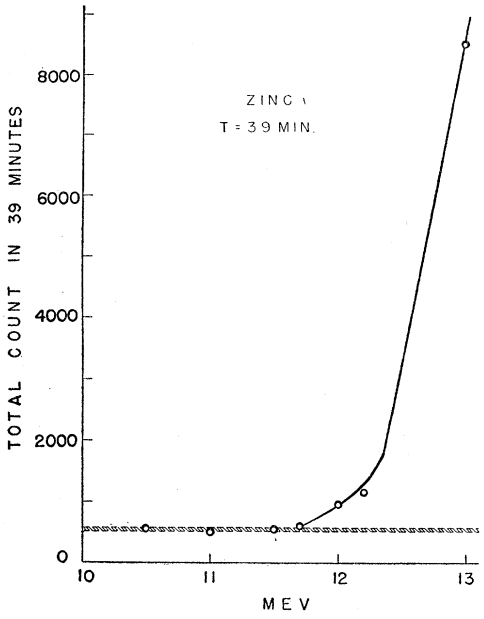

FIG. 7. Excitation curve for Zn^{63}.

Fig. 6, reduced to an irradiation of 15R. There is a slight systematic error in energy-values here because of phase-shift in the integrating unit, which is appreciable only when expansion occurs well below peak flux. The correction to be applied in this case was found to be not more than 0.1 Mev. The threshold has accordingly been taken as 10.9 ± 0.3 Mev. This is considerably lower than measurements of the $(n, 2n)$ threshold have indicated.[12]

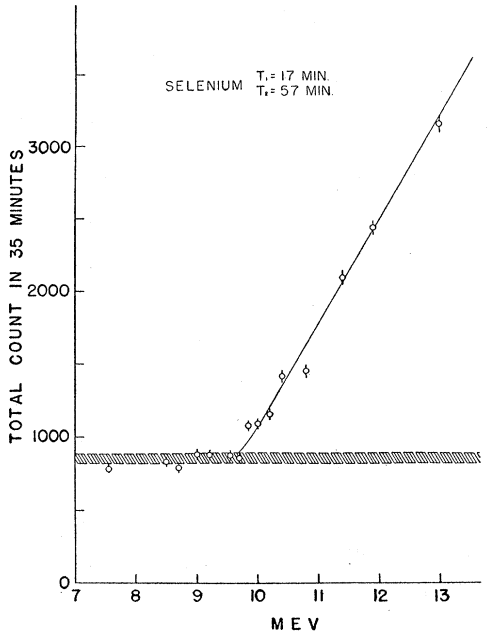

FIG. 8. Excitation curve for total activity in selenium. The observed count is owing to two isomeric activities.

ZINC

Cylinders of zinc 6 cm long, 2.5 cm in diameter, of 3-mm wall-thickness were irradiated for 25 minutes. Strong yields of Zn^{63} were obtained.[2,8] The counts for irradiations of 50R are shown in Fig. 7. The threshold is 11.6 ± 0.4 Mev.

SELENIUM

X-rays induce two activities in this element, one of about 17-minute half-life (also reported as 19-minute),[2] the other about an hour. Bothe and Gentner[4] obtained both by 1-hour irradiation of selenium with 17-Mev gamma-rays and also produced them with slow neutrons. Langsdorf and Segrè[13] showed that they are isomers of either Se^{79} or Se^{81}. These isomers were separated by chemical methods, and it was established that the 57-minute selenium decays into the 17-minute

[12] R. Sagane, Phys. Rev. **53**, 492 (1938).
[13] A. Langsdorf and E. Segrè, Phys. Rev. **57**, 105 (1939).

isomer, emitting a 98-kev gamma-ray; the lower isomer decays by emission of a 1.5-Mev beta-particle. There is no evidence that the upper state decays directly by beta-emission.

Plates of pure selenium, $7.5\times5\times1$ cm, were irradiated for 15-minute periods, then placed 1 mm from the counter surface. Total activity in 35-minute counting periods, reduced to irradiation of 15R (computed on the basis of a 17-minute half-life), is plotted in Fig. 8. No activity results below 9.8 ± 0.5 Mev (there was an uncertainty of as high as 3 percent in the energy settings in this experiment). If the short period

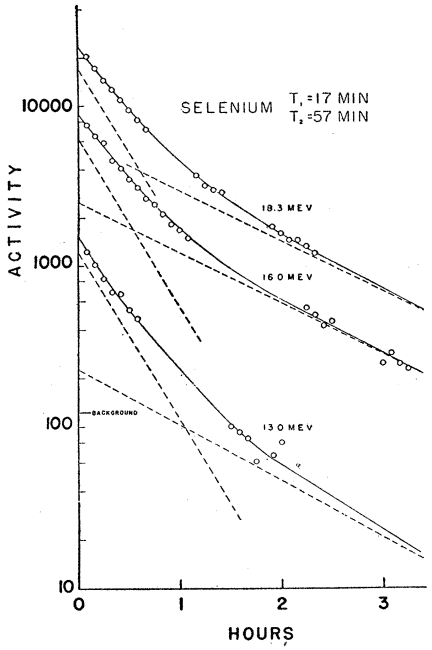

FIG. 9. Decay of selenium activity, $Se^{79\text{ or }81}$.

is the lower isomer, this threshold should be that of the 17-minute period. Analysis of decay curves indicates that this is probably true.

Designating the upper state by the subscript γ and the lower by β, the differential equations for production are

$$-\dot{N}_\gamma=\frac{N_\gamma}{\tau_\gamma}-k_\gamma I, \tag{3}$$

$$-\dot{N}_\beta=\frac{N_\beta}{\tau_\beta}-\frac{N_\gamma}{\tau_\gamma}-k_\beta I, \tag{4}$$

the lower isomer being produced by decay of the upper state as well as by the (γ, n) process. These equations could be solved for the initial populations of the two states and for the ratio of initial activities which should be observed if the constants k_β and k_γ were given. The beta-decay exhibits both half-lives. Since the beta-ray counter employed in the experiment had very low efficiency for gamma-rays, we need not consider the gamma-activity. With the simplifying assumption that the x-ray intensity is constant, it can be shown that

$$\frac{k_\beta}{k_\gamma}=r\left[\frac{1-\exp(-T/\tau_\gamma)}{1-\exp(-T/\tau_\beta)}\right]\left[\frac{\tau_\gamma}{\tau_\gamma-\tau_\beta}\right]+\frac{\tau_\beta}{\tau_\gamma-\tau_\beta}, \tag{5}$$

where

$$r=\frac{\text{initial activity decaying with lifetime } \tau_\beta}{\text{initial activity decaying with lifetime } \tau_\gamma}.$$

Note that in the absence of direct photoelectric production of the lower isomer (i.e., for $k_\beta=0$), r would be negative, indicating that growth, rather than decay, should be initially observed for the activity with lifetime τ_β.

From decay curves (Fig. 9) it is evident that r is always positive and tends to increase at lower energies. Near threshold the curves become difficult to analyze because of large relative statistical fluctuations, but it is still possible to estimate upper and lower limits for the 57-minute activity. These estimates indicate that k_γ/k_β decreases with decreasing energy, hence that the 57-minute threshold is probably higher than the 17-minute. This trend is shown in Fig. 10; as the energy is increased, k_γ/k_β appears to attain a limiting value of about 0.6. In making any deductions from this trend, it should be borne in mind that the activity observed at a given peak energy is the integrated effect of all x-ray quanta above threshold energy. This treatment must be regarded as essentially qualitative as it was not possible to make a reliable analysis of decay curves near threshold, but it clearly indicates the trend. A chemical separation would be out of the question with the weak activities obtained with this reaction.

MOLYBDENUM

The 17-minute isotope of this element is the only one of convenient lifetime for the present

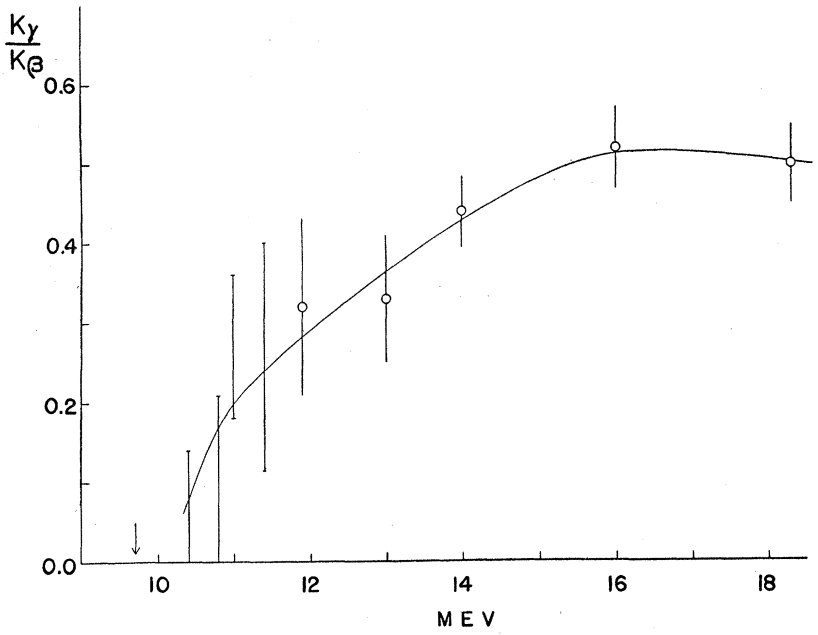

FIG. 10. Ratio of rate of production for upper isomer to that for lower isomer of $Se^{79 \text{ or } 81}$.

experiments.[2] Activities were moderately strong. A shorter period of about 1-minute half-life was observed above 14 Mev; it was allowed to decay for 5 minutes before counting was begun on the 17-minute activity, and did not interfere with determination of the latter threshold. Analysis of the molybdenum samples revealed no impurity other than a slight trace of tungsten. No attempt has yet been made to identify this period.

Samples of rolled sheet molybdenum, approximately 0.8 mm thick, measuring 6 cm ×7 cm, were irradiated for 10-minute periods. The counts plotted in Fig. 11 have been corrected to an irradiation of 30R. The threshold is 13.5±0.4 Mev.

SILVER

Two activities are induced in silver by photodisintegration.[2,7] Ag^{106} is positron-active with a 24.5-minute half-life (the isomeric 8.2-day period is too long to be observed in the present study); Ag^{108} emits electrons with a half-life of 2.3 minutes. To determine a threshold for each, it is necessary to separate their effects. This was done by analyzing the decay curves by somewhat the same procedure as that used for selenium.

A silver plate 7×6.5 cm, 1 mm thick, was rolled into a cylinder which fit snugly over the counter. This was allowed to decay completely between successive runs. A series of 25-minute runs gave the decay curves shown in Fig. 12. From this and from Fig. 13*A* it is evident that the short period predominates at lower energies. Below 10 Mev it is difficult to estimate the longer activity, but an

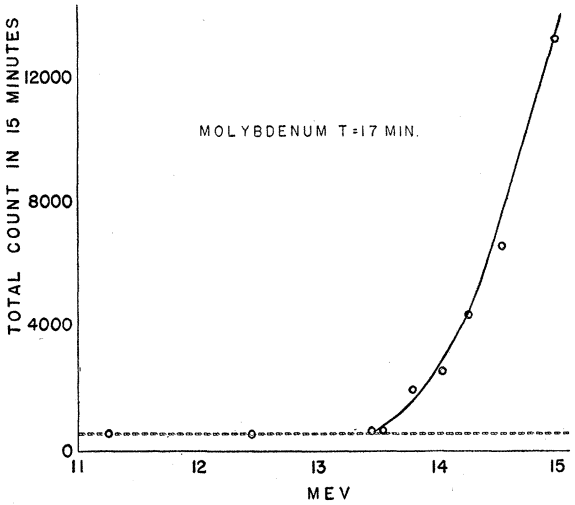

FIG. 11. Excitation curve for 17-min. Mo activity.

upper limit for it can be set as in the case of selenium. Part *B* of Fig. 13 gives totals in a series of 3-minute runs to irradiations of 3.2R, indicating a threshold of 9.3±0.5 Mev for the 2.3-minute period.

Since each isotope is produced independently of the other, growth under irradiation follows Eq. (1). The ratio of the constants k_{106} and k_{108} is

$$\frac{k_{106}}{k_{108}}=r\left[\frac{24.5}{2.3}\right]\left[\frac{R_{108}}{R_{106}}\right],$$

where the R's are the weighted irradiations for each isotope and

$$r=\frac{\text{initial 24.5-minute activity}}{\text{initial 2.3-minute activity}}.$$

Were each isotope irradiated to saturation at the same x-ray intensity, k_{106}/k_{108} would be the ratio of activities observed. Values of this ratio computed from decay-curve data are plotted in Fig. 14. At and below 9.5 Mev the 24.5-minute activity is not produced. It is therefore concluded that the 2.3-minute threshold lies below the 24.5-minute threshold by at least 0.25 Mev.

TABLE I. Threshold energy values for nuclear photo-effects.

Z	Target element	Active isotope	Half-life (minutes)	Threshold (Mev)
6	Carbon	C^{11}	21	18.7 to 19.4
7	Nitrogen	N^{13}	10	11.1±0.5
8	Oxygen	O^{15}	2.1	16.3±0.4
26	Iron	Fe^{53}	9	14.2±0.4
29	Copper	Cu^{62}	10.3	10.9±0.3
30	Zinc	Zn^{63}	39	11.6±0.4
34	Selenium	$Se^{79 \text{ or } 81}$	17	9.8±0.5
		$Se^{79 \text{ or } 81}$	57	higher
42	Molybdenum	$Mo^{91 \text{ or } 93}$	17	13.5±0.4
47	Silver	Ag^{108}	2.3	9.3±0.5
		Ag^{106}	24.5	above 9.5

CONCLUSIONS

Thresholds which have been measured so far are tabulated in Table I. Probable errors listed include uncertainties both in energy settings and in determination of threshold from inspection of the excitation curve. The chief difficulty in the latter arises from statistical fluctuation in counter background. A threshold determination is limited by the sensitivity of the means of detecting the effect. Since the effects are comparable with

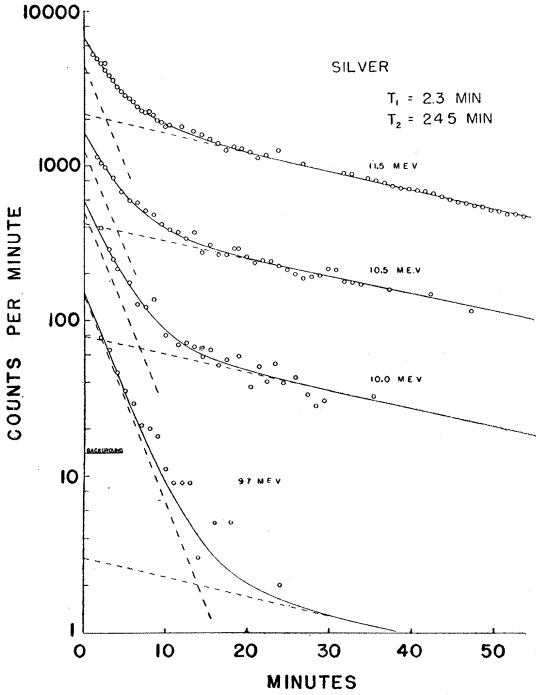

FIG. 12. Decay of silver activity at various irradiation energies.

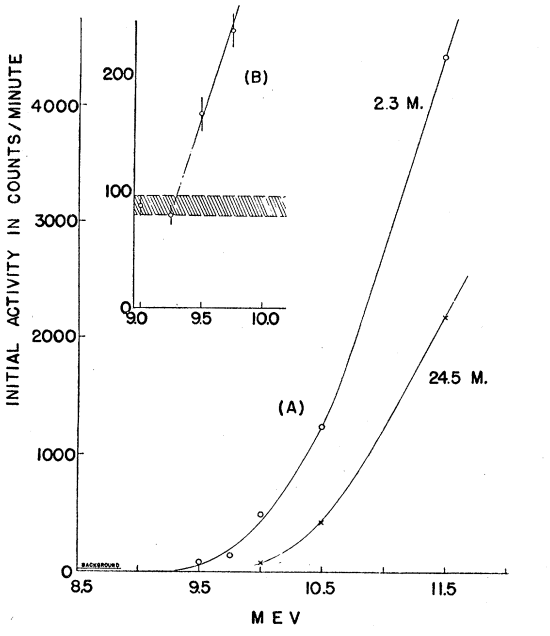

FIG. 13. (*A*) Initial activities of Ag^{106} and Ag^{108} *vs.* irradiation energy. (*B*) Excitation curve for silver with short irradiation period.

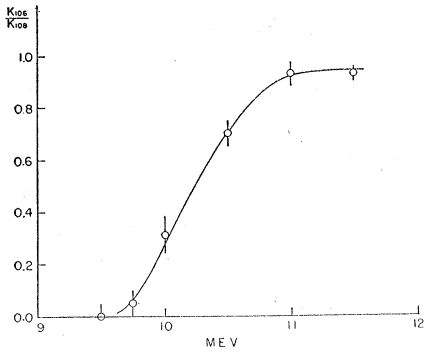

FIG. 14. Ratio of production rates for Ag^{106} and Ag^{108}.

background fluctuations in several cases, one cannot be completely satisfied that the true threshold may not lie considerably below the point where the effect first becomes detectable. The first three elements, yielding weak activities, gave thresholds consistently higher than predicted values. This may indicate that the true thresholds are below those read from the curves. It should be pointed out, however, that emission of gamma-rays after the beta-decay would make the calculated threshold seem too low. This might be the case for oxygen.

In other elements, the relatively better data warrant confidence that the true thresholds do not differ from those listed by more than the errors given.

No interpretation of the shapes of the excitation curves has been attempted. The spectral distribution of the radiation employed, which arises from a target of intermediate, variable thickness, must be known before actual cross sections and true excitation curves can be found. There is a general similarity of shape in all the curves.

All the measured thresholds are appreciably above the mean binding energy per particle, which is under 8 Mev in all cases studied. The trend is toward lower values in the heavier elements, but the list is not yet large enough to warrant generalizations from the data. An extension of the list to heavier elements is contemplated, together with a thorough rechecking of the present list. Present conditions make it unlikely that this work can be completed in the near future, however.

PHOTO-FISSION

Photo-fission of uranium was also investigated by collecting fission recoils on a paper catcher and measuring the collected activity. Uranium oxide was coated on a cylindrical form. A cylinder of paper was slipped over this sample and held by

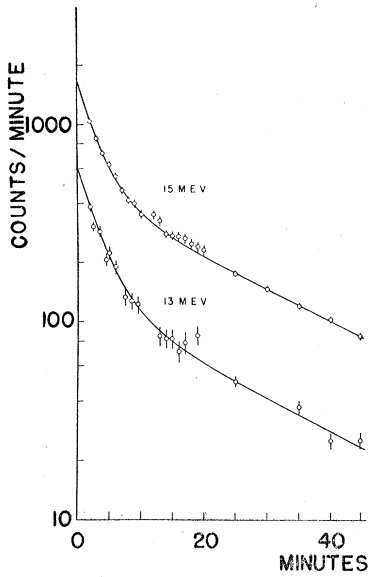

FIG. 15. Decay curves for photo-fission of uranium, observed by collection of fission recoils.

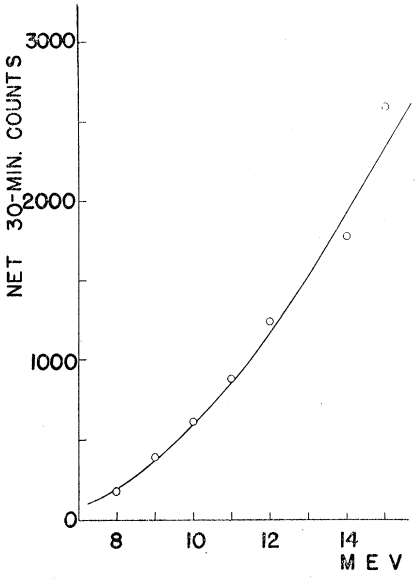

FIG. 16. Excitation curve for photo-fission of uranium from measured activity of collected recoils.

spacers 1 mm from the uranium. This was irradiated for 10-minute intervals at various energies, and the activity was measured with a counter. Blank tests on ordinary bond paper showed no activity below the oxygen threshold at 16.3 Mev.

Decay of the collected fission fragments was followed for 45 minutes after runs at 13 and 15 Mev. The points shown in Fig. 15 were fitted by a double exponential of half-lives 2.2 and 18 minutes, though the full decay scheme is much more complex. The activity is not strong enough to permit resolving the various activities present.

Net 30-minute counting totals (background of 500 subtracted out), corrected to an x-ray intensity of 1R per minute at one meter (or an irradiation of 10R, if no allowance be made for decay), are plotted in Fig. 16. Below 8 Mev the collected activity becomes too weak to follow. The threshold is apparently below 7 Mev, however.

Attempts to observe photo-fission in lead by this method were unsuccessful at energies up to 16 Mev, where the collector became active.

ACKNOWLEDGMENT

The writers wish to express their appreciation to Professor Donald W. Kerst for his direction of this research, and to Dr. Philip Morrison for many helpful discussions and suggestions.

PHYSICAL REVIEW VOLUME 67, NUMBERS 1 AND 2 JANUARY 1 AND 15, 1945

Expansion of Positive Energy Coulomb Wave Functions in Powers of the Energy*

J. G. BECKERLEY
Pupin Physics Laboratories, Columbia University, New York, New York

(Received September 16, 1944)

The non-relativistic wave function (in spherical polar coordinates) for a charged particle in a Coulomb field is expressed in a form suitable for problems in which the particle has a small positive energy. This formulation amounts to expanding the radial part of the wave function in powers of the energy E and is achieved by simple algebraic manipulation of power series and recurrence formulas. The coefficients of the expansion are functions of the radial coordinate and are identified with Bessel functions of integral order.

FUNDAMENTAL EQUATIONS

A PARTICLE of mass m and charge[1] $Z'e$ is moving in the Coulomb field of a nucleus with charge Ze. If the energy of the particle is $E(\geq 0)$ and if it is described in terms of spherical polar coordinates, r, θ, ϕ (origin at Ze), then a suitable wave function for the particle is:[2]

$$\psi_{klm}(r,\theta,\phi)=Ne^{im\phi}P_l^{|m|}(\cos\theta)L_{k,l}(r),$$

$$L_{k,l}(r)=e^{-\pi\beta/(2k)}\left[\left|\Gamma\left(l+1+i\frac{\beta}{k}\right)\right|/\Gamma(2l+2)\right] \times(2kr)^l e^{ikr}\,{}_1F_1\left(i\frac{\beta}{k}+l+1,\,2l+2,\,-2ikr\right), \quad (1)$$

$$\beta=ZZ'me^2/\hbar^2;\quad k^2=2mE/\hbar^2$$

in which the constant factors in $L_{k,l}(r)$ have been chosen so that asymptotically

$$L_{k,l}\sim(kr)^{-1}\sin\left[kr-l\frac{\pi}{2}+\arg\Gamma\left(l+1+i\frac{\beta}{k}\right)-\frac{\beta}{k}\ln 2kr\right].$$

The confluent hypergeometric function ${}_1F_1$ is defined[3] as

$${}_1F_1(a,b,z)=1+\frac{a}{b}z+\frac{a(a+1)}{b(b+1)}\frac{z^2}{2!}+\cdots. \quad (2)$$

To simplify the analysis consider, instead of $L_{k,l}(r)$, the function [essentially $r^{-l}L_{k,l}(r)$]

$$u_{k,l}(r)=e^{ikr}\,{}_1F_1\left(i\frac{\beta}{k}+l+1,\,2l+2,\,-2ikr\right), \quad (3)$$

* Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University.

[1] e is positive throughout, so that, e.g., $Z'=-1$ for an electron or $+2$ for an alpha-particle.

[2] N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Oxford University Press, 1933), p. 39.

[3] E. T. Whittaker and G. N. Watson, *Modern Analysis* (Cambridge University Press, 1927), p. 338.