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The Apparent Breakdown of Meek's Streamer Criterion in Divergent Gaps due to the Failure of Townsend's Ionization Function

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Meek's criterion postulates that a streamer will propagate if the positive ion space charge field close to the anode is equal to K times the impressed field, where K should be a constant independent of gap configuration and pressure. Measurements in confocal paraboloid gaps, designed to substantiate this, revealed that K was not constant, but decreased with decreasing pressures. Morton showed that on a negatively charged smaller electrode Townsend's ionization functions do not represent the ionization produced in divergent fields. Consequently it was believed that the calculations of the space charge field of the positive ions produced in such gaps must be inaccurate in view of this erroneous assumption. A series of measurements on the dark current i in the same gap showed that it was not represented by $i=i_0 \exp[\int \alpha dx]$ at lower pressures. The trend and the apparent quantitative variation of K with pressure is explained if the apparent i_0 *versus* voltage curves are extrapolated to streamer onset for each pressure. Thus for practical purposes under the conditions studied K is sensibly constant in conformity with Meek's theory. *Per contra,* the applications of Meek's theory to gaps with very divergent fields where the currents can be expected to deviate markedly from those computed by the relation $i=i_0 \exp[\int \alpha dx]$ are not justified, thus placing a practical limitation on the use of Meek's theory to streamer studies. Roughly this phenomenon appears when the change of the field exceeds two percent change over an average electron free path.

INTRODUCTION

FOR some time it has been clear that the classical Townsend theory of sparking cannot account for the formation of sparks in air when the product of pressure and plate distance exceeds 200 mm of Hg×cm. Summaries of the inadequacy of the Townsend theory have been given by Meek[1] and by Loeb and Meek.[2]

The discovery of the streamer in the positive point-to-plane corona and its explanation by Loeb, Kip, and Trichel[3] supplied a new mechanism for spark breakdown. Almost simultaneously the same mechanism was proposed by Raether[4] on the basis of cloud-track pictures. The streamer theory of sparking was developed qualitatively by Loeb, but it was not until Meek proposed his criterion for streamer formation that the theory was placed on a quantitative basis.

Meek's criterion states that a streamer will form when the positive ion space charge field close to the anode is equal to K times the impressed external field (providing the photo-ionization in the gas is adequate). The space charge field is computed on the assumption that most of the positive ions created by the first electron avalanches are contained in a hemi-

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[1] J. M. Meek, Phys. Rev. **57**, 722 (1940).
[2] L. B. Loeb and J. M. Meek, *The Mechanism of the Electric Spark* (Stanford University Press, Stanford, 1941).
[3] A. F. Kip, Phys. Rev. **54**, 139 (1938); G. W. Trichel, *ibid.* **55**, 382 (1939); L. B. Loeb and A. F. Kip, J. App. Phys. **10**, 142 (1939).
[4] H. Raether, Zeits. f. Physik **107**, 91 (1937); **110**, 611 (1938); **112**, 464 (1939); H. Costa, *ibid.* **113**, 531 (1939).

spherical boss which has a radius equal to the radius of electron diffusion at the head of the avalanche. The radius of electron diffusion depends upon the quantity $\lambda_0/\sqrt{f}$, where λ_0 is the electron free path at 760 mm of Hg, and f is the fraction of energy lost by an electron per impact. Both λ_0 and f are functions of the electron energy. Since the electron energy distribution function is not well known, especially at high pressures and in strong electric fields, there is some uncertainty in the value of $\lambda_0/\sqrt{f}$ which should be used. It has been shown, however, that the error introduced by this uncertainty is not serious.[5]

Originally, Meek chose the empirical condition that $K=1.0$, and his calculations gave a fair agreement with published values of sparking potentials, but a more careful scrutiny of the data indicated that a value of $K=0.1$ was more nearly correct.

If the streamer theory is to have physical significance, K should be a constant, independent of pressure and gap configuration. In order to study the behavior of K, one can use values of measured sparking potentials in plane parallel gaps. Analysis of reliable sparking potentials in the literature yields a factor of K which varies by a factor of 60 at atmospheric pressure. More recent measurements[6] of sparking potentials yielded data for which K varies only by a factor of two. At lower pressures, however, there are no adequate sparking potential values and, as recent studies[6] have indicated, there are not likely to be any reliable values of sparking potentials at lower pressures for some time.

Another method can be used to evaluate K. Fitzsimmons[7] determined the onset potentials for streamers in air in confocal paraboloid gaps at atmospheric pressure. By knowing the field distribution over the axis of the gap at the measured potential, it is possible to evaluate the positive ion space charge field, and values of K may be calculated. The results of Fitzsimmons showed that K has the value 0.1 for air, and this conclusion seemed consistent with the value of K selected by Loeb and Meek from sparking potential data. But there was still no information about the behavior of K with pressure. It was therefore decided to extend Fitzsimmons' observations to lower pressures, and for the sake of generality to other gases. The results of these measurements, based on the assumptions given above, revealed that K is not a constant but decreased with decreasing pressure. A careful analysis of the physical processes involved in streamer formation indicated that no *known* factor could be responsible for the results. It was therefore concluded that either some unknown factor was being neglected, or that Meek's criterion was in principle fundamentally wrong. In view of the fact that the criterion, rough as it was, appeared to be reasonable, and was based on physical processes believed to operate in the streamer, it was hoped that a reasonable explanation could be found. At this time, Morton[8] was investigating the validity of applying Townsend's ionization coefficients to ionization produced in divergent fields and discovered that they were not applicable where the field-change per electron mean free path exceeded 2 percent. It occurred to one of the writers (L. H. F.) to inquire whether the space charge field of the positive ions in applying Meek's criterion to divergent gaps was thus not being calculated upon erroneous assumptions. Experimental data bearing out this hypothesis were obtained in the very gap in which the variation of K was observed. The two sets of experiments will be presented separately, while a combined treatment of both will be given in the discussion.

THE DEPENDENCE OF K ON PRESSURE

The experimental arrangements were the same as those used by Fitzsimmons[7] except for the gap itself. A hollow paraboloid electrode of focal length four centimeters was used as a plate in conjunction with a paraboloid point of focal length either 0.02 or 0.005 cm. Both point paraboloids were considerably sharper than those used by Fitzsimmons. The surfaces of the electrodes were nickel. These gaps were enclosed in a large Pyrex bulb and were connected to a mercury diffusion pump and gas purification system. Mercury, however, was carefully excluded from the chamber. Provisions were made

[5] See reference 2, page 52, last paragraph.
[6] L. Fisher, Phys. Rev. **65**, 153 (1944).
[7] K. E. Fitzsimmons, Phys. Rev. **61**, 175 (1942).
[8] P. L. Morton, Thesis, University of California Library, Berkeley, California.

to circulate air through the chamber, but this procedure was found to be unnecessary if the currents passing through the gap were smaller than one microampere. If the currents exceeded this value by increasing the potential above that for streamer onset, oxides of nitrogen were formed which changed the streamer onset potential. The chamber was baked out under vacuum at 400°C for several hours and filled with either air or hydrogen. However, after hydrogen had been admitted to the chamber, baking alone was not adequate to remove the hydrogen. It was found necessary to subject the gap to a prolonged glow discharge with nitrogen at low

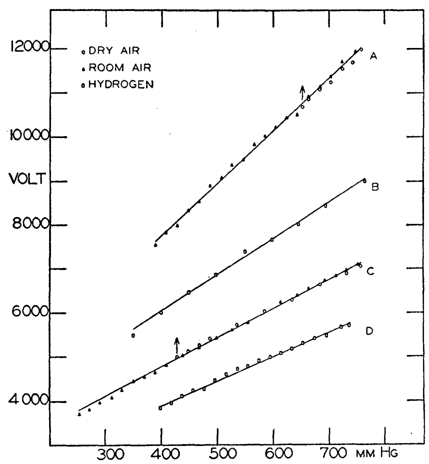

FIG. 1. Streamer onset potentials *versus* pressure. Curves A and B were obtained with a point paraboloid of 0.02-cm focal length, curves C and D with a point paraboloid of 0.005-cm focal length. The arrows indicate the lowest pressure where streamers were observed in dry air.

pressure. Only in this way could the hydrogen dissolved in the nickel be eliminated. The air was dried by passing it over three traps surrounded by a mixture of alcohol and dry ice, while the hydrogen was cleaned over hot copper filings in order to remove oxygen.

In order to facilitate the pulse formation preceding corona onset, a sample of thorianite was placed about eight inches from the gap, thus increasing the dark current, and making the observations more reproducible.

The chamber was initially filled with the gas to approximately atmospheric pressure and the potential difference across the gap was increased until pre-corona onset streamers appeared. The pressure was then decreased in steps of 20 mm of Hg, and corresponding streamer onset potentials were observed at each pressure, until a pressure was reached at which streamers were no longer formed. The pressure at which streamers no longer appear is in no case clearly defined. On decreasing the pressure of a given gas, and maintaining a suitable potential difference across the gap, fewer and fewer streamers occur, while the number of burst pulses increases. This observation indicates that at lower pressures, the discharge tends to spread laterally over the point forming a burst pulse rather than to grow out of the high field region in the form of a streamer. This preferential formation of burst pulses at low pressures is probably due to a decrease in the *density* of photo-ionization in the gas. The curves for pre-corona streamer onset potentials *versus* pressure showed a linear relationship in all cases (Fig. 1). Within the common pressure range for streamer formation, the curves for dry air coincided with those for room air within one percent. No attempt was made to determine the relative humidity of the room air used. Hydrogen would not show any pre-corona onset streamers if the gas had been purified by negative corona from an auxiliary point in the chamber[9] in addition to the hot copper treatment. However, since all measurements of the first Townsend coefficient α in hydrogen were taken without electrical purification of the gas, this step was omitted in the present experiments. Consequently, streamers were observed in the potential region preceding corona onset.

Since the constant K is the ratio of the field X_1 of the positive ion space charge to the impressed field X at the tip of the point parabola, it was necessary to calculate both X_1 and X. The surface field corresponding to a measured streamer onset potential was calculated from the relation

$$X=\frac{V}{\log_e (f/F)}\frac{1}{x+f}\text{ volt/cm.} \qquad (1)$$

This expression is the solution of Laplace's equation for the field on the axis between two confocal paraboloids with focal lengths, f for the point parabola, and F for the plate parabola,

[9] G. L. Weissler, Phys. Rev. **63**, 96 (1943).

TABLE I. Values of K and associated factors.

Gas	Pressure mm Hg	Streamer onset Point potential	Streamer onset Surface field X v/cm	Focal length (cm)	x_s	$(x_s/p)^{\frac{1}{2}}$	Space charge field X_1 v/cm	$K=X_1/X$	α_s	$_\epsilon\int \alpha dx$	$\alpha_{s\epsilon}\int \alpha dx$	$\int \alpha dx$
Dry air	748	11950v	1.13×10^5	0.02	0.034	6.73×10^{-3}	7.5×10^4	0.67	1210	7.9×10^5	9.6×10^8	13.6
Dry air	654	10600	1.00×10^5	0.02	0.034	7.2×10^{-3}	3.2×10^4	0.32	1100	4.0×10^5	4.4×10^8	12.9
Room air	469	8575	8.10×10^4	0.02	0.034	8.5×10^{-3}	1.50×10^4	0.18	990	2.4×10^5	2.4×10^8	12.4
Room air	370	7260	6.85×10^4	0.02	0.034	9.6×10^{-3}	3.53×10^3	0.05	880	7.3×10^4	6.4×10^7	11.2
H_2	766	9000	8.5×10^4	0.02	0.09	10.8×10^{-3}	3.9×10^4	0.45	613	1.2×10^6	7.35×10^8	14.05
H_2	550	7400	7.0×10^4	0.02	0.09	12.8×10^{-3}	7.4×10^3	0.11	533	3.2×10^5	1.7×10^8	12.7
H_2	450	6470	6.11×10^4	0.02	0.09	14.1×10^{-3}	2.1×10^3	0.04	481	1.1×10^5	5.3×10^7	11.6
H_2	350	5500	5.2×10^4	0.02	0.09	16.1×10^{-3}	6.8×10^2	0.013	427	4.6×10^4	2.0×10^7	10.7
Dry air	746	7070	2.12×10^5	0.005	0.017	4.85×10^{-3}	2.33×10^5	1.1	3000	7.16×10^5	2.15×10^9	13.5
Dry air	547	5810	1.74×10^5	0.005	0.018	5.74×10^{-3}	1.2×10^5	0.69	2460	5.3×10^5	1.31×10^9	13.2
Dry air	429	5040	1.50×10^5	0.005	0.018	6.47×10^{-3}	7.2×10^4	0.48	2250	3.9×10^5	8.8×10^8	12.9
Room air	399	4780	1.43×10^5	0.005	0.018	6.7×10^{-3}	3.8×10^4	0.27	2120	2.24×10^5	4.75×10^8	12.3
Room air	252	3700	1.11×10^5	0.005	0.019	8.7×10^{-3}	7.2×10^3	0.07	1620	7.3×10^4	1.18×10^8	11.2
H_2	738	5700	1.71×10^5	0.005	0.0475	8×10^{-3}	2.2×10^4	0.13	1796	1.78×10^5	3.2×10^8	12.1
H_2	478	4370	1.31×10^5	0.005	0.044	9.6×10^{-3}	4.0×10^3	0.03	1400	4.9×10^4	6.86×10^7	10.8
H_2	399	3850	1.16×10^5	0.005	0.040	10×10^{-3}	8.33×10^2	0.007	1250	1.2×10^4	1.5×10^7	9.4

respectively. The quantity x is the distance from the point parabola to the point in question.

For the calculation of the field X_1 the equation developed by Loeb and Meek[10] was employed.

$$X_1=5.27\times10^{-7}\frac{\alpha_s \exp\left[\int_0^{x_s}\alpha dx\right]}{(x_s/p)^{\frac{1}{2}}}\ \text{volt/cm}, \quad (2)$$

where the factor 5.27×10^{-7} contains the numerical value of $\lambda_0/\sqrt{f}$, α_s is that value of the first Townsend coefficient which contributes the largest number of positive ions to the formation of the space charge boss (and is consequently found at or near the surface of the point), p is the gas pressure in mmm of Hg, and x_s is that distance from the point surface in cm where, on the average, one ion pair is formed by an electron advancing toward the point, that is, $\int_0^{x_s}\alpha dx=1$. Since the field varies rapidly over the gap, the calculation of X_1 involves the graphical integration of αdx over the axis of the gap. For this purpose, α must be known as a function of distance from the point. Such values are obtained from the measurements of α/p as a function of X/p *in uniform fields* from the measurements of Sanders[11] for air and from those of Hale[12] for hydrogen. Since the integral occurs as an exponent in Eq. (2), its value will be the dominant factor. Even if the value of α_s is replaced by the value of α corresponding to a distance of one ionizing free path away from the surface, and even if one allows x_s to take on different values, the final value of X_1 will be changed only slightly.

The numerical constant in (2) refers to air only. In order to calculate the constant for hydrogen, it is assumed that the electrons have an average energy of about two volts. For this case Ramien[13] gives $f=0.03$, and Brose and Saayman[14] give $\lambda_0=3\times10^{-5}$ cm, and thus $\lambda_0/\sqrt{f}=1.74\times10^{-4}$ cm. (If we assume an average electron energy of seven ev we find $\lambda_0/\sqrt{f}=4.8\times10^{-4}$ cm.) Thus for hydrogen, Eq. (2) becomes

$$X_1=\frac{5.55\times10^{-7}\,\alpha_s \exp\left[\int_0^{x_s}\alpha dx\right]}{(x_s/p)^{\frac{1}{2}}}\ \text{volt/cm}. \quad (2')$$

The calculated results of the pre-onset streamer studies at lower pressures are compiled in Table I. In the case of room air, K varies by a factor of 12 in either gap, while for hydrogen the factor is 20 for the more divergent gap and even larger for the other gap. In dry air, and in both gaps, K varies only by a factor of two. Not too much

[10] See reference 2, page 45, Eq. (25).
[11] F. H. Sanders, Phys. Rev. **41**, 667 (1932); **44**, 1020 (1933).
[12] D. H. Hale, Phys. Rev. **54**, 241 (1938); **56**, 815 (1939).
[13] H. Ramien, Zeits. f. Physik **70**, 351 (1931).
[14] H. L. Brose and Saayman, Ann. d. Physik **5**, 797 (1930).

weight can be attributed to the absolute value of K since both x_s and α_s are not uniquely defined. Nevertheless, the decrease in K with decreasing pressure appears to be real.

The range of pressure in which onset streamers occur is much larger in room air than in dry air. It has been pointed out[9] that the efficiency of photo-ionization is very sensitive to extremely slight changes in the composition of a gas. This dependency of photo-ionization on purity, together with the knowledge that photo-ionization in the *gas* is the primary mechanism involved in streamer formation, provides a probable explanation for the difference in pressure ranges for streamer formation in dry and in room air.

ION CURRENTS IN DIVERGENCE GAPS

In order to determine K from the onset potentials of streamers, it has been seen that the space charge field of the positive ions must be calculated, and that this field is proportional to $\exp\left[\int\alpha dx\right]$. It has been recently pointed out by Morton[8] that in the glow discharge, the ionization does not take place where the values of the field strength are high, and that one cannot use the Townsend ionization functions when the field strength varies appreciably over a mean free path. Such considerations induced Morton to study Townsend currents in concentric cylinders in hydrogen and have led to the most interesting results. In much less divergent fields than were used in the present study, and at pressures of the order of a millimeter, he finds that the ionization is not given properly by integrating the ionization functions for uniform fields.

The question arose as to whether the calculations of the space charge field X_1 given in the earlier part of the paper could be in error due to the same phenomenon, thus explaining the variations in K. True, the pressures employed in the streamer studies were 300 to 700 times as large as the pressures used by Morton, but then the fields of the confocal paraboloids were very much more divergent than the fields of Morton's cylinders.

In order to decide whether the space charge calculations in the confocal paraboloid gap were falsified, the more divergent gap was provided with an external source of radiation, and the currents *well below* streamer onset were measured as a function of potential and pressure. From these measurements it is possible to determine the importance of the divergent field in calculating the space charge field X_1.

The sharpest point parabola (focal length 0.005 cm) was used and the gap length was again four centimeters. Ultraviolet light from a quartz mercury arc was focused by means of a quartz lens through a graded quartz seal on the center of the plate paraboloid. The radius of the circular image was estimated as two millimeters. The chamber was thoroughly baked out and was filled with hydrogen purified over hot copper. The current below onset was measured by means of a Dolezalek quadrant electrometer (sensitivity 1000 mm/volt) which was used to determine the potential drop across a high ohmage resistor. By keeping the ultraviolet arc on continuously during the run, the intensity of illumination was kept constant. The entire set of readings was taken during one continuous run, and the current through the arc changed by less than one percent.

Table II gives the different pressures and

TABLE II. Dark currents and related quantities as a function of voltage and pressure.

p_{22°	Voltage	$\int\alpha dx$	$e^{\int\alpha dx}$	Current (ampere)	i_0 (apparent) $\times10^{13}$ (ampere)
676	3200	4.74	114	1.68×10^{-11}	1.47
676	3550	5.67	290	4.32×10^{-11}	1.49
676	3770	6.31	550	9.16×10^{-11}	1.67
676	3850	6.75	854	1.32×10^{-10}	1.55
676	3910	6.95	1043	1.82×10^{-10}	1.74
676	4150	7.63	2059	4.96×10^{-10}	2.41
676	4200	7.81	2463	6.68×10^{-10}	2.71
676	4270	8.10	3294	9.60×10^{-10}	2.91
676	4390	8.67	5825	1.71×10^{-9}	2.93
616	3350	5.58	265	4.32×10^{-11}	1.63
616	3560	6.25	493	1.02×10^{-10}	2.07
616	3660	6.33	561	1.68×10^{-10}	3.00
616	3950	7.24	1394	6.20×10^{-10}	4.43
616	4060	7.87	2617	1.13×10^{-9}	4.33
616	4150	8.22	3714	1.70×10^{-9}	4.57
548	3110	5.47	237	4.12×10^{-11}	1.74
548	3310	5.88	358	1.01×10^{-10}	2.83
548	3410	6.36	578	1.64×10^{-10}	2.84
548	3750	7.48	1772	6.96×10^{-10}	3.93
548	3840	7.74	2298	1.25×10^{-9}	5.43
477	2910	4.89	133	5.24×10^{-11}	3.94
477	3070	5.38	217	1.03×10^{-10}	4.74
477	3200	5.90	365	2.07×10^{-10}	5.67
477	3480	6.65	773	7.48×10^{-10}	9.68
408	2660	5.09	162	5.52×10^{-11}	3.41
408	2860	5.75	314	1.30×10^{-10}	4.14
408	2970	6.17	478	2.20×10^{-10}	4.60
408	3200	6.95	1043	7.80×10^{-10}	7.48
408	3350	7.56	1920	1.92×10^{-9}	10.0

potentials at which currents were measured. Also given are values of $\int \alpha dx$ and of $\exp[\int \alpha dx]$ calculated from the field distribution by graphical integration over the axis of the gap. The current *versus* potential curves are given by Fig. 2 for various pressures.

If the current is correctly determined by the use of Townsend's coefficient alpha, the measured current i is given by $i_0 \exp[\int \alpha dx]$, where i_0 is the initial photoelectric current. By dividing the measured current by $\exp[\int \alpha dx]$, one obtains an *apparent* i_0 which should be constant. The departure of i_0 from constancy is a direct measure of the discrepancy of the true ionization from the Townsend value predicted from $i_0 \exp[\int \alpha dx]$. A plot of the apparent i_0 against potential is given in Fig. 3 for the various pressures.

Experiments of the nature of those reported above were also performed in air, but the currents below streamer onset were too small to be measured conveniently. The currents in air were certainly less than one-hundredth of the currents measured in hydrogen. Comparison of values of Townsend's alpha for air and hydrogen leads one to expect currents of the same order of magnitude in the two gases. The large currents in hydrogen must be ascribed to the great transparency of the gas to electrons and photons, and to the small energy losses of electrons in hydrogen as shown by the smaller threshold potentials for streamers in hydrogen as compared to air. Kip[15] has reported measurable currents in air with positive points below streamer onset only under strong radioactive irradiation.

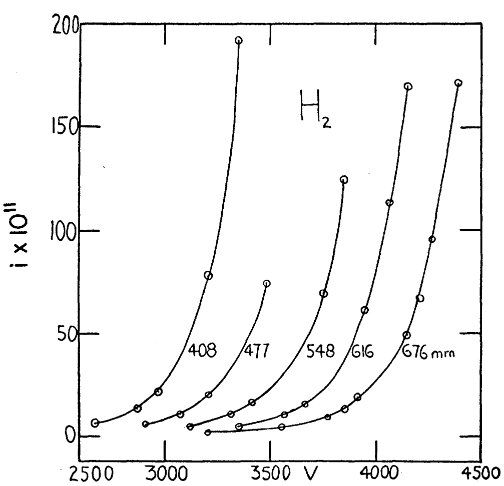

FIG. 2. Dark current *versus* potential curves for various pressures.

CONCLUSIONS CONCERNING THE TWO EXPERIMENTS

It is clear from Fig. 3 that the ionization predicted from Townsend's function is far from correct, and that the apparent value of i_0 is not constant even at a constant pressure at varying potentials. At the highest pressure the value of the apparent i_0 is sensibly constant for a considerable voltage range and Townsend's equation is not far from correct. This pressure region corresponds to the pressure studied by Fitzsimmons. But as the pressure decreases, the deviation of the apparent i_0 from constancy becomes more and more marked. At high pressures the mean free path of the electrons are short; at low voltages all of the apparent i_0 *versus* voltage curves seem to be approaching roughly the same limiting value of i_0. From these considerations it seems reasonable to assume that the ionization at high pressure and low voltage may be taken as being accuractely represented by $i = i_0 \exp[\int \alpha dx]$, but that there is more ionization at lower pressures than is accounted for by Townsend's coefficient as applied to this equation. Such increased ionization at low pressures effectively introduces an *apparent* decrease in the value of K, not because of the failure of Meek's criterion, but because the theoretical calculation of the space charge field X_1 with Townsend's alpha does not correspond to reality.

The experiments on the onset of streamers actually determined the factor by which the actual space charge field in the gap differed from the space charge field calculated by means of $\exp[\int \alpha dx]$. Therefore the apparent variation of K is not due to a failure of Meek's criterion. The current measurements below onset are not only in the right direction to explain the trend in K, but the quantiative variation of K with pressure is explained in order of magnitude if the apparent i_0 *versus* voltage curves are extrapolated to streamer onset for each pressure. Thus the apparent failure of a constant value of K is to

[15] A. F. Kip, Phys. Rev. **54**, 139 (1938).

be ascribed to the use of the first Townsend coefficient in calculations which Morton showed cannot be applied when the change of field strength per electron free path is approximately two percent or larger, which applies to these investigations.

AN ATTEMPT TO EVALUATE THE FAILURE OF TOWNSEND'S IONIZATION FUNCTIONS

Between impacts, an electron in a gas in an electric field acquires the full energy of the work done by the field over the length of the component of the mean free path directed in the field direction. Suppose electrons exist in a gas in a divergent field such that the field strength varies appreciably over the distance of an electron free path, and such that the electrons travel from strong to weak fields. In such cases, where the field changes very rapidly over a few mean free paths, as they do about sharp points, the electrons gain a large share of the energy of the total potential drop (hundreds of electron volts) in a very few free paths. Under these conditions, the excitation functions and free paths are such that energy loss due to excitation and inelastic impacts is very small. The electrons gain a terminal energy nearly equal to the potential drop and thereafter can ionize more efficiently than they excite. Such ionization is not materially different from cathode-ray or slow β-ray ionization, where ionization reaches the order of an ion pair for 20 to 30 ev of energy expended.

In a uniform field of the same length as the gap, the energy gain of an electron free path near sparking is of the order of one electron volt. The electrons gain their terminal energy slowly, passing through regions in which excitation and energy losing impacts are quite frequent before and after gaining the ionizing energy. The electrons rarely gain much more than twice or three times the ionizing energy, with the result that the average energy expenditure per ion pair is much more than the 20 to 30 ev where electrons obtain an initial high energy at once and then proceed to ionize in relatively weak field regions.

Thus, under nearly uniform field conditions, where Townsend's equation holds, the ion production is small compared to the case where, as in glow discharges or about negative points, the energy gain is very rapid and is succeeded by a low field region where the energy is expended in ionization.

In the present experiments, however, the point is positive, and it would appear as if the ionization should not be greater than for the equilibrium case where Townsend's equation applies. In fact, it might even be expected to be *less*. In the equilibrium Townsend case the value of $\int \alpha dx$ is constant, irrespective of whether the electrons travel from high to low fields, or vice versa; that is, the function $\int \alpha dx$ is independent of the sign of the field. However, where the free paths are longer and the field changes appreciably over a free path it was seen that as in Morton's case, where the high field was negative, the ionization could be expected to be, and was, in fact, found to be greater than for the uniform Townsend case.

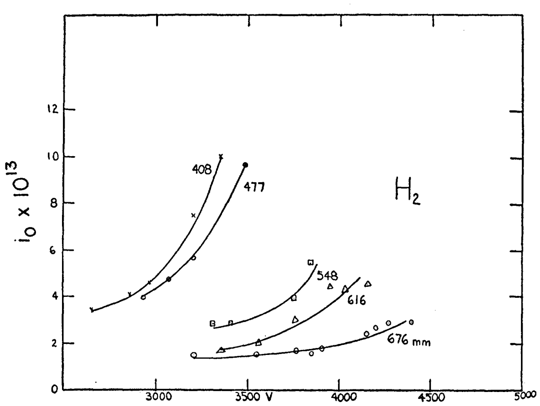

FIG. 3. The variation of the apparent i_0 with different potentials for various pressures.

If now we consider the case where the high field region is positive with large gradients over the free paths near the anode, the situation is different. In the cathode region, electrons gain energy and may fritter away even more energy in excitation and in elastic impacts than in the higher Townsend gradients. However, near the anode, the sharp gradients over a mean free path will again rapidly raise the electron energy to possibly hundreds of electron volts over the last free paths with very little energy loss to excitation and elastic impacts. Now if pressures are so low that the electrons make very few collisions before they strike the anode, then in fact their whole energy will be lost and the ionization will be *less* than in Townsend's case. If, on the other hand, the pressure is not too low, the high energy of the electrons and their random motions, despite the high anode field, will cause again a greater ionization than in the Townsend case. It is not inconceivable that owing to the attraction of the anode, these high energy electrons will not have as many collisions with gas molecules before capture by the anode as they did in moving through the low field region in Morton's case. However, both the electrons and their progeny generated in the high field region can be most effective before capture. This is indicated in Bradbury's[16] study of photoelectric currents in gases, where back diffusion of electrons is large even in the presence of fields, especially when the electronic energy is large. The magnitude of the excess ionization will obviously be critically pressure dependent, and will depend on Ramsauer free paths, excitation, and ionization functions. The effect will vary in different gases. Thus the excess ionization observed in the present experiment will be expected, as indicated above, provided the electrons, before being drawn in, spend more time than anticipated in the neighborhood of the point. Actually, high energy electrons of 100 ev near the anode will do just this at higher pressures, as has been recognized in Bradbury's study of photoelectric currents in gases. Thus it is not surprising that, under the conditions of the present experiments, the currents were found to be in excess of the value of $i_0 \exp [\int \alpha dx]$ calculated from Townsend's alpha, though one would expect the currents to be relatively less in excess than would be the case with the high field region at the cathode as in Morton's experiments.

In the present experiment, the field strength changes by 50 percent over a distance of 2.5×10^{-3} cm from the point, and the electrons just capable of ionization have approximately a mean free path of 10^{-4} cm at the lowest pressure studied. Hence, in the most extreme case, the field strength varies by approximately two percent over a mean free path. This is the amount of variation in the field strength over mean free path at which Morton finds abnormalities to begin in his currents.

The effect is undoubtedly a very complicated one, and its explanation must lie in the behavior of the energy distribution functions of the electrons as a function of voltage and pressure in a divergent field, as well as in the spatial distribution of collisions of higher energy electrons with atoms.

The present results in air may be explained by the attachment of electrons to oxygen molecules in the weak portion of the field. Loeb[17] has shown that such ions do not shed their electrons until they are in a field of $X/p=90$ volts/cm/mm. In these extremely divergent fields, perhaps, the electron is released even closer to the point than corresponds to $X/p=90$. If this explanation is correct, the calculation of Townsend currents in a divergent gap in air should be approached with even more caution than a similar calculation in hydrogen.

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[16] N. E. Bradbury, Phys. Rev. **40**, 980 (1932); L. B. Loeb, *Fundamental Processes of Electrical Discharges in Gases* (John Wiley & Sons, Inc., New York, 1939), Chapter 7.

[17] L. B. Loeb, Phys. Rev. **48**, 684 (1935).