Letters to the Editor

PROMPT publication of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is the third of the month. Because of the late closing date for the section no proof can be shown to authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not in general exceed 600 words in length.

Comments on Bethe's Theory of Diffraction of Electromagnetic Waves by Small Holes*

C. L. PEKERIS**
Columbia University, New York, New York
November 3, 1944

IN this interesting paper Professor Bethe develops a vectorial theory of diffraction of plane electromagnetic waves through a circular aperture in an infinite plane conducting screen, for the limiting case when the aperture is small in comparison with the wave-length. In the introduction Professor Bethe points out that if the problem is attacked by Kirchhoff's method one obtains a solution which fails to satisfy the boundary conditions, and this result he designates as a "failure of the Kirchhoff theory." The method adopted by Bethe is to replace the aperture by an equivalent distribution of magnetic currents, for which he obtains an explicit solution satisfying the boundary conditions.

It should be pointed out that Kirchhoff's method consists in making a certain approximation when solving Helmholtz's equation which, as Professor Bethe states, is justified in the limit of short wave-lengths. Kirchhoff was careful to stress the limitation of his method to wave-lengths which are small in comparison with the size of the aperture. When he took up the case of diffraction by a grating he made the following statement: "The equation derived above makes the essential assumption that the dimensions of the aperture are very large in comparison with the wave-length, and its application to diffraction spectra produced by gratings whose lines are only a few wave-lengths wide, is not justified." From this statement it is clear that Kirchhoff considered his approximation to be unjustified in the case where the wave-length is a moderate fraction of the aperture, let alone in the other extreme case, considered by Bethe, where the aperture is very small compared with the wave-length. For this case there is no "Kirchhoff theory," and therefore one cannot speak of a failure of the Kirchhoff theory.

A method of solving Helmholtz's equation in the case of long wave-lengths, when Kirchhoff's approximation cannot be made, was given some fifty years ago by Lord Rayleigh.[1] The problem solved by Rayleigh is scalar, but his method is essentially the one used by Bethe in the more complicated vectorial problem. Rayleigh's results, especially for the pertinent case where the wave function is made to vanish on the screen,[2] agree with Bethe's as far as order of magnitude is concerned. In a later paper[3] Rayleigh extends his method (in the two-dimensional case) to apertures which are of the dimension of a wave-length.

A systematic exposition of the application of the Helmholtz-Kirchhoff theory, including the case of "long" wave-lengths, to the scalar problem will be found in Lamb's *Hydrodynamics*[4] where will also be found a discussion of Rayleigh's paper.[5] The question of the retention of the various terms in Helmholtz's equation which Professor Bethe raises in Section 2 of his paper is taken up on page 500.

* H. A. Bethe, Phys. Rev. **66**, 163 (1944).
** On leave of absence from the Massachusetts Institute of Technology.
[1] Lord Rayleigh, *Sci. Papers*, Vol. IV, p. 283.
[2] See reference 1, pp. 286, 287.
[3] Lord Rayleigh, *Sci. Papers*, Vol. VI, p. 11.
[4] Lamb, *Hydrodynamics*, pp. 492–520.
[5] See reference 4, pp. 517, 518.

Double Bragg Reflections of X-Rays in a Single Crystal

C. J. DAVISSON AND F. E. HAWORTH
Bell Telephone Laboratories, Inc., New York, New York
October 6, 1944

WHEN a beam of monochromatic x-rays traverses a crystal in a direction to satisfy the condition for ***hkl*** reflection, the resultant Bragg beam traverses the same crystal in a direction to satisfy the condition for $\langle hkl\rangle_{Av}$ reflection. The two beams constitute a symmetrical pair. Radiation is reflected out of the primary beam into the Bragg beam, and out of the Bragg beam into the primary beam. There is a continuous interchange of radiation between the two beams with a net flow to the less intense.

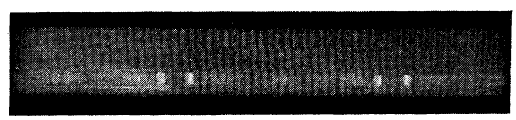

FIG. 1. Double x-ray reflection in quartz.

In the case in which two Bragg beams are produced simultaneously the two beams interchange radiation, not only with the primary beam, but also, through associated reflections, with one another. If the two reflections are $h_1k_1l_1$ and $h_2k_2l_2$, radiation flows from beam 1 to beam 2 via the reflection $(h_2-h_1)(k_2-k_1)(l_2-l_1)$ and from 2 to 1 via $(h_1-h_2)(k_1-k_2)(l_1-l_2)$. These are the associated reflections.

In the still more special case in which the structure factor of one of the reflections is zero, no radiation flows to this beam from the primary beam directly, but radiation does flow to it indirectly via the other reflection and one of the associated reflections. A doubly-reflected beam appears in the place of the non-existent once-reflected beam. The condition for this occurrence is, to repeat, that the primary beam shall satisfy the Bragg condition for the non-existent reflection, and that it shall, at the same time, satisfy the Bragg condition for some allowed reflection. There is the further requirement that the associated reflection also is allowed.