On the Currents Carried by Electrons of Uniform Initial Velocity

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IN a Letter to the Editor, Mr. J. R. Pierce has compared my recent paper under the above title[1] with an earlier treatment of the same subject by C. E. Fay, A. L. Samuel, and W. Shockley.[2] Though Mr. Pierce's representation of my point of view is entirely correct, I should like to add a few remarks since it seems to me that a point of principle is involved.

The paper of the three authors was not known to me until Mr. Pierce drew my attention to it. However, the treatment therein is based on the treatment of Langmuir and Compton[3] to which I have referred.

The original derivation of the Child-Langmuir formula was subject to the following two criticisms:

(a) It does not contain the slightest indication of the phenomenon of saturation.

(b) It makes the solution singular in a mathematical sense by admitting an infinite value of the space charge density (at the cathode).

Admittedly the treatment of L. and C. and of the later authors, in the case of constant initial velocity, is open to objection (b) as regards the plane where the potential minimum occurs. It was the only aim in my investigation to do away with the mentioned objections. By introducing a modified form of boundary condition, I obtained solutions which admit of no indeterminacy, which are regular everywhere (also in the case of space-charge limited currents), and which automatically change into the saturated solution when a critical potential is reached.

If Mr. Pierce and others are willing to admit infinite values of the space-charge density, there is nothing to be said against the treatment of L. and C. except that a very puzzling ambiguity arises. Which solution is to be accepted as the valid one, the regular or the singular one? From general physical and mathematical principles there can hardly be a doubt as to the answer. However, there is a further argument in favor of the regular solution.

When the space-charge limitation sets in, the minimum of the potential is not yet as low as its lowest possible value. (Thus in the case $\xi_0=10$ represented by Fig. 3 of my paper, the limitation sets in for $\eta_{\min}=-0.45$, and for real cases of thermionic emission the minimum would be still less low.) If now the limitation sets in (e.g., by an increase in the potential) according to the solution of L. and C., the minimum would have to jump suddenly to $\eta_{\min}=-1$. It seems to me that such a behavior is quite inadmissible and that, therefore, the regular solution has to be continued into the domain of space-charge limitation. This is exactly what my modified boundary condition achieves.

How nearly the singular solution approximates the regular one remains to be discussed. It is easy to show that the treatment of L. and C. approximates the regular solution in two limiting cases, namely, either (in the notation of my paper) when $E_0/E\rightarrow 0$, or when $\xi_0\rightarrow 5/3$, which means when $16\pi i/(v_0E_0)^{\frac{1}{2}}x\rightarrow 5/3$. In the former case the minimum will be slight ($\eta_{\min}\rightarrow 0$) and near the cathode; in the latter case it will be near the lowest possible value ($\eta_{\min}\rightarrow -1$) and near the anode.

In all other cases deviations must be anticipated which concern the potential distribution as well as the characteristic. It will be sufficient to show the discrepancy in the latter case. Particularly large deviations are likely to occur for relatively large initial velocities, i.e., in the limiting case $E_0/E\gg 1$. In this limit my formula (30) yields the value $j=8E_0^{\frac{3}{2}}$. On the other hand, it can be deduced from L. and C.'s formulae (293) to (295) that $j=E_0^{\frac{3}{2}}((1+\gamma)/\gamma)^2$, where $\gamma=(2i_s/i-1)^{\frac{1}{2}}$. Since the saturation value i_s is always larger than i, γ will be a number larger than unity, and the factor multiplying $E_0^{\frac{3}{2}}$ will be between $\frac{1}{2}$ and $\frac{1}{8}$ of the value in my formula.

I should like to stress once more that my treatment regards exclusively the idealization that all electrons have the same velocity normal to the cathode. It is only in this case that the electrons cannot build up a minimum sufficiently low to make them revert (unless a singularity is admitted). If there is a velocity distribution, it has been shown long ago, and I have stressed the fact in my paper that the slower electrons will revert in the field created by the swifter ones.

As to the question of stability, I agree with Mr. Pierce that it cannot be answered completely without a special investigation. Therefore, I should have called the solution with the lower minimum not the "unstable solution" but the "less stable solution." As a matter of fact, I did admit a certain stability for the "unstable solution"—otherwise I would not have drawn diagrams giving the potential distribution in such cases. For the main point under discussion this is quite irrelevant. I pointed out that the solutions which make the minimum as low as $\eta_{\min}=-1$ are on the unstable branch, but I excluded them as physically inadmissible because they make the space-charge density infinite.

[1] George Jaffé, Phys. Rev. **65**, 91 (1944).
[2] C. E. Fay, A. L. Samuel, and W. Shockley, Bell Sys. Tech. J. **17**, 49 (1938).
[3] I. Langmuir and K. T. Compton, Rev. Mod. Phys. **3**, 191 (1931), see p. 239.

Retraction of Stressed Rubber

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THE process of the retraction of stressed rubber has been studied by us for some time, using several experimental methods. The most promising method employed was the use of a smoked revolving drum driven by a synchronous motor. A light stylus was fastened to the free end (or any position behind the tip) of a rubber sample; the other end of the sample was clamped. The elongation-time curves were recorded accurately and directly on the drum by the stylus. From these curves the velocity of the tip, as a function of elongation (or time), may be obtained by differentiation. Figure 1 shows elongation-time curves.