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Further Studies on the Origin of Cosmic Rays

Helium Annihilation Rays and the Cause of Their Variability with Time

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We find definite evidence that a new band of rays which we interpret as helium annihilation rays does come in vertically at about the predicted latitude. We present a discussion of the possible composite character of the so-called silicon annihilation band and of the so-called oxygen annihilation band. We then bring forward an explanation of the cause of our large and already reported variability in the cosmic-ray intensities found in high altitude electroscope flights at Bismarck, Omaha, and Oklahoma City. We also make a new and more accurate determination of the value of the field sensitive and the non-field sensitive components of the incoming cosmic rays.

I. HISTORICAL SUMMARY OF THE DISCOVERY OF THE SILICON COSMIC-RAY BAND

IN our expedition to India in late 1939 and early 1940 we had found unambiguous evidence that in going from the magnetic equator near Bangalore north to Peshawar (Mag. Lat. 25° N) there was no change at all in the intensity of the vertically incoming cosmic rays between Bangalore and Agra (Mag. Lat. 17.3), but in going the next 7.7°, from Agra to Peshawar, there was a marked increase in that intensity, amounting to more than 20 percent.

We had gone to India primarily to look for just this cosmic-ray effect, for we had predicted on the basis of the hypothesis that the birth of the cosmic rays in interstellar space consists merely in the complete transformation into a "charged particle pair" (which we treat as an "electron pair"[1]) of the total rest-mass energy of the most abundant of the atoms found in interstellar space, namely, the atoms of silicon, oxygen, nitrogen, carbon, and helium.

If this were so we had computed from the Einstein equation $E=mc^2$ that each electron of the "electron pair" arising from such a transformation of the silicon atom (we have called these "silicon annihilation rays") would have an energy of 13.2 billion electron volts (Bev), not enough to break vertically through the earth's magnetic field at the magnetic equator in India, where we had computed that the requisite incoming charged particle energy was 17 billion electron volts, but it would have just about enough incoming energy to break through at somewhere near 20° N Magnetic Latitude, i.e., somewhere between Agra and Peshawar.

Having then found *experimentally* by balloon flights to very high altitudes a plateau of constant vertically incoming cosmic-ray energy between Bangalore and Agra and a new band of rays coming in between Agra and Peshawar, we provisionally identified this new band as the cosmic-ray silicon annihilation band.

We found additional evidence for the correctness of the foregoing interpretation in the fact that when in our high balloon flights we again sent up to the very top of the atmosphere in place of the pairs of counters (one parallel to the other and some six inches above it) either a recording *electroscope* or a *single counter*, both of these two latter receivers being of the kind that respond to rays coming in *from all directions* rather than merely to rays coming in vertically, then, instead of finding this plateau of constant cosmic-ray intensity between Bangalore and Agra succeeded by a sudden rise in intensity between Agra and Peshawar, we found both the electroscope and the single-counter readings rising continuously both between Bangalore and Agra and also between Agra and Peshawar.

[1] A mesotron pair is barred out by its short lifetime. The only choice is between an electron pair and a proton pair, but the difference between the latitude of entrance of electrons and protons entering the earth's magnetic field from this mode of origin is in no case, not even in the case of the He annihilation rays, large enough to be detected with the resolving power of the experimental techniques we have so far used. However, we hope to obtain a resolution which will differentiate between these alternatives. See footnote 4, Phys. Rev. 61, 398 (1942). We miscalculated the difference in Phys. Rev. 63, 245 (1943), so that the conclusion there reached is not yet a valid one.

This last behavior is just what is to be expected when the electroscope or the single counter is the measuring instrument. The reason for this difference can be seen from a glance at our discussion of the Lemaitre-Vallarta curves reproduced in one of our preceding papers.[2]

II. THE MORE PRECISE MEANING OF THE SILICON BAND

It is desirable to call attention to one further characteristic of this band of incoming cosmic rays that we have above and, in previous papers, designated simply as the silicon-band, namely, its possible composite character. Although, according to Bowen's measurements[3] Si (at. wt. 28.3) is probably ten times more abundant in interstellar space than are any of the three closely adjacent elements Al (at. wt. 27.1), P (at. wt. 31), and S (at. wt. 32) these are all so close to Si in weight that their annihilation rays would be quite inseparable by the type of tests we have so far made.

Hence, for simplicity we designated, and shall continue to designate, this observed band of incoming rays the "*silicon-band*" without wishing to deny to it the possibility of a composite structure. Indeed, P. Swings, who, with his collaborators, has recently made elaborate studies on the abundance of both atoms and molecules in interstellar space,[4] says, "It may safely be assumed that the atomic populations in interstellar space of H, O, N, and C per cm^3 are between 1 and 10^{-3}." These estimates of the actual interstellar abundance of these atoms are not out of harmony with Bowen's measurements of the relative-interstellar abundance of atoms. From these he finds[3] the He atom 1/10 as abundant as the H atom, and the atoms of C, N, O, and Si each of about one-tenth the abundance of the atom of He.

Swings also writes us "that the interstellar abundance of the carbon compound molecules CH and CH^+ must be of the order 10^{-6} per cm^3." In other words, the most abundant interstellar *molecules* would have only 1/10,000 of the abundance of the foregoing abundant atoms C, O, N, and Si, so that so far as the origin of the "field sensitive cosmic rays"[5] is concerned, *no molecules and no atoms heavier than the atoms causing the Si band need to be considered at all.* This introduces a very great simplification into the whole problem of origin.

III. THE 1940 PREDICTIONS OF COSMIC-RAY BANDS AND OF PLATEAUS OF CONSTANT COSMIC-RAY INTENSITIES

If, then, our hypothesis as to origin is correct, there can be four, and only four, cosmic-ray bands in addition to the Si band to look for, namely, the O, N, C, and He bands, which should have the energies required to enable them to begin to break vertically through the resistance of the earth's magnetic field at a series of four different latitudes as the observer goes north from the latitude of first entrance of the silicon band.

However, on account of the dissymmetry of the earth's magnetic field we had computed as early as 1939 that the silicon band (energy 13.2 Bev), though it could not get vertically through at the equator in India, where the required energy was 17 Bev) could get through at the equator off the coast of Peru where we computed that the requisite energy was but 13 Bev.

As soon, then, as we had found the silicon band in India we were able to predict, and in fact published the prediction, that there should be a very long plateau of constant vertically incoming cosmic-ray intensity extending from Mollendo, near the magnetic equator in Peru, clear up to the middle of Mexico (Mag. Lat. 32° to 34°) at which latitude the oxygen annihilation band should begin to come in and of course remain in at full strength for all more northerly latitudes.

But the oxygen band like the silicon band might have a bit of complexity about it for, according to Bowen's table,[3] the Ne atom has one-tenth the interstellar abundance of O. Be-

[2] See Fig. 2, R. A. Millikan, H. V. Neher, and W. H. Pickering, Phys. Rev. 61, 401 (1942).

[3] See R. A. Millikan, H. V. Neher, and W. H. Pickering, Phys. Rev. 61, 399 (1942), footnote 5.

[4] P. Swings, Astrophys. J. 95, 270 (1942). See also *ibid.* 92, 292 (1942), and Astronom. Soc. Pac. 54, 3 (1942), and J. Roy. Astronom. Soc. Canada 35, 75 (1941).

[5] We define non-field sensitive cosmic rays as those found at the magnetic equator in India when the measuring instrument that is carried to the top of the atmosphere is an electroscope. Field sensitive rays are the *additional* rays that come in when the electroscope measurements are carried out at successive latitudes between the equator and the pole.

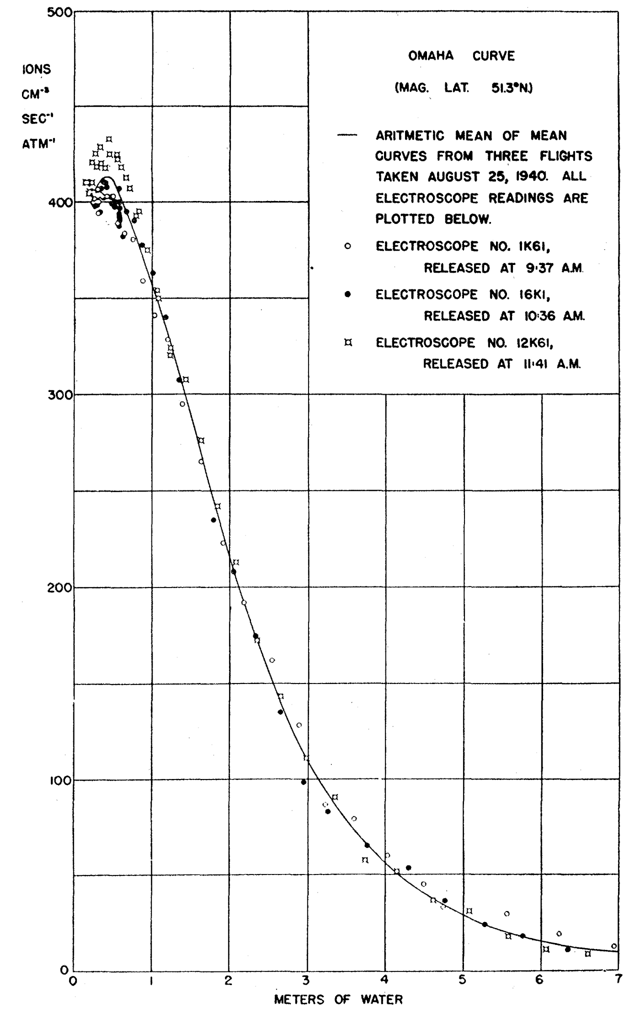

FIG. 1. Details showing consistency obtained in three successive flights in one day. Similar consistency was shown between the component curves making up each of the five mean curves of Fig. 2. All of these flights were made within a period of 13 days from August 21 to September 3, 1940.

cause of its weakness and closeness to O (atomic numbers 10 and 8, respectively) we had no hope of separating the two effects and decided to designate the whole effect as the "O annihilation band," as we had done with the similarly complex silicon band.

But because of the equality in the interstellar abundance of O and N at a latitude a very small number of degrees north of the first appearance of the O band, the N band should add its equal intensity to the total vertically incoming energy. We of course had little hope of separating these O and N bands, but we had confidence that the vertically incoming cosmic-ray intensity at Pasadena (Mag. Lat. 40.7° N) would represent the addition of the intensity of the O and N bands and would therefore be notably larger than that to be found at the first latitude at which the O band came in in its full strength.

Again, we predicted from our hypothesis that in going a *very* few degrees north of Pasadena there would be another increase in vertically incoming cosmic-ray intensity roughly equal to half the increase between Mag. Lat. 33° N and 40.7° N, which latter increase should be due to the joint effect of the O and N bands, for according to Bowen O, N, and C all had equal abundance.

We expected the most crucial test of this whole hypothesis to come in the testing of the prediction that between say Mag. Lat. 44° N and 54° N there should be found another flat plateau of constant vertically incoming cosmic-ray energy since there are no abundant atoms at all of weights between *those of C and He and we computed from Lemaitre and Vallarta's curves that the He atom-annihilation ray could not break vertically through the earth's magnetic field farther south than at about Mag. Lat. 54° N.*

IV. REASONS FOR MAKING THE 1940 SERIES OF INCOMING COSMIC-RAY ENERGY MEASUREMENTS WITH ELECTROSCOPES AT A SERIES OF DIFFERENT LATITUDES

All the foregoing predictions had been made upon our return from India in the early spring of 1940. We knew, however, that we could not develop the necessary new equipment for putting these predictions to careful experimental test in less than a year's time. But we were already well supplied with excellent recording electroscopes such as we had used, in part, in India. Since, then, the total cosmic-ray energy coming into the atmosphere at any latitude, no matter from what direction, is an important measurement which could be made most accurately with electroscopes, since, further, a series of such measurements taken in a series of latitudes in such rapid succession as to reduce the likelihood of time-changes was a matter of considerable importance, and since this was the only accurate way of getting the ratio of the field sensitive to the non-field sensitive incoming rays, we decided to interrupt, in the summer of 1940 for a period of two weeks, the preparations for the above-mentioned vertical counter tests planned for the year 1941–1942, in order to make a series of flights with electroscopes in September, 1940.

The absorption of all three of us in war projects has delayed the working up and presentation of these data until the present.

Meantime, the crucial vertical counter tests of the above-mentioned series of predictions as to cosmic-ray bands and plateaus were carried out in December, 1941 and March and April, 1942, and have already been published[6] *with results which appear to confirm all of the predictions made.* However, we have not yet located with the aid of vertical counters the latitude of entrance of the vertically incident He annihilation rays. We hope to do this at the close of the war, but the present data, taken with electroscopes, throw some light on this point and on other points of importance, as indicated below.

This expedition, then, was undertaken in the summer of 1940 because (1) we had the equipment ready and could carry out these high flights with electroscopes to best advantage in the more stable weather conditions usually prevailing in the Middle West in late summer and early fall; (2) because we had already found in very high electroscopes flights in high magnetic latitudes (Omaha, Bismarck, Saskatoon) as yet unexplained variabilities with time[7] which were conspicuously absent in India[8] and which we thought might disappear in part, at least, if a series of flights at different latitudes was taken all in one season and in as rapid succession as possible; (3) because such a series of more accurate electroscope flights would establish with greater certainty the relation between the field sensitive and the non-field sensitive portions of the incoming cosmic rays such as had been thus far defined only in terms of the total integrated value of the ionization found at a given latitude through ionization measurements made with electroscopes or single counters as they are carried from the lowest point at which cosmic-ray ionization can be detected up to the top of the atmosphere; (4) because on account of the shape of the Lemaitre-Vallarta curves in high magnetic latitudes there was a possibility of throwing some light, even with electroscope flights, on the reality, and latitude of entrance, of the helium annihilation band.

V. THE NEW MEASUREMENTS

At Bismarck, North Dakota three flights were made on the same day, August 21, 1940, the first pair of balloons used for carrying up the recording electroscope being released at 8:45 A.M., the second at 9:50 A.M., the third at 1:46 P.M. Four days later, namely, on August 25, 1940, three more similar flights were made at Omaha, the first ascent starting at 9:37 A.M., the second at 10:36 A.M., the third at 11:41 A.M. This process was repeated at Oklahoma City on

[6] R. A. Millikan, H. V. Neher, and W. H. Pickering, Phys. Rev. **63**, 234–245 (1943).

[7] See R. A. Millikan, H. V. Neher, and W. H. Pickering, Phys. Rev. **61**, 405–406 (1942).

[8] See R. A. Millikan, H. V. Neher, and W. H. Pickering, Phys. Rev. **61**, 408 (1942), and especially Fig. 1 on page 408.

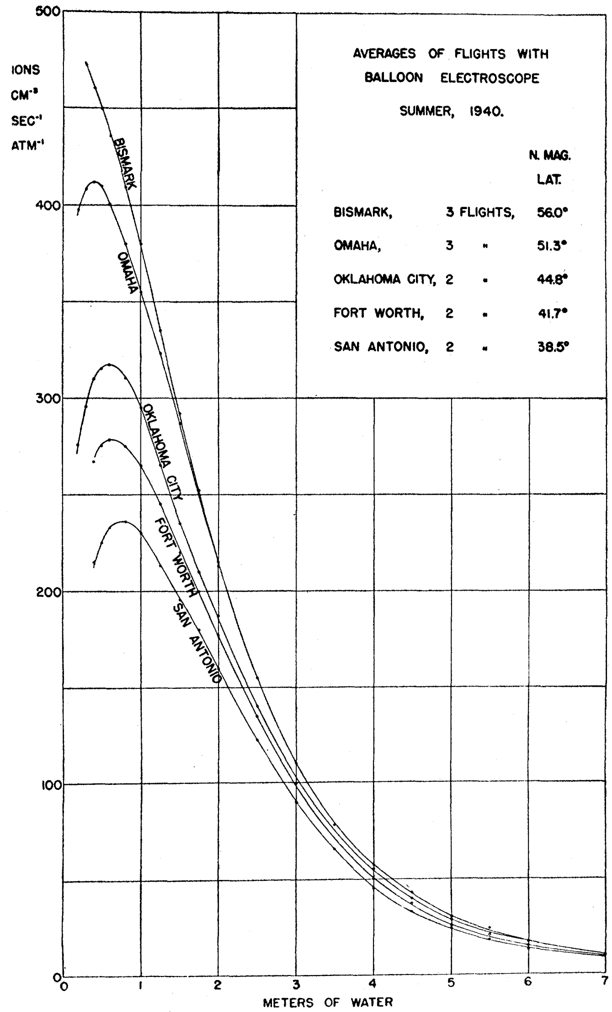

FIG. 2. Soft He rays make Bismarck-Omaha difference only at high altitudes; penetrating carbon rays cause difference, extending down to low altitudes, between Oklahoma City and Omaha curves.

August 29, at Fort Worth on September 2 with two flights, and at San Antonio on September 3 with two flights. All of these flights, then, were taken within a period of thirteen days, during which the atmospheric conditions were particularly stable. The set of resulting curves, therefore, has a much better chance of being free from errors caused by possible changes with time in the incoming cosmic-ray intensities than have those we have previously published, some of which were taken some years apart. Furthermore, we have never taken a series of flights with different electroscopes that showed throughout as great a self-consistency in each given locality as do these.

Figure 1 shows just how we treated the data at Omaha, for example. It is altogether typical of the data taken in all five of the different latitudes.

In this figure every one of the readings of the photographic film for each of the three flights made at Omaha is recorded. Our procedure for getting the final mean curve for this locality from these readings was to draw separately the full curve of each flight; then to read off on each of these three curves the ionization at a given altitude, say the ionization corresponding to 2 meters of water beneath the top of the atmosphere; then to average these three readings at 2 meters; and having obtained this sort of an average at a whole series of chosen altitudes up to as near the top of the atmosphere as the readings extend, to draw through these averaged points the final curve. This is the full line shown in Fig. 1. It is reproduced again in Fig. 2 along with the mean curves obtained in this way at all of the five indicated latitudes. The area underneath each of these curves is the total integral of the cosmic-ray energy entering the atmosphere at the latitude in question. It can be reduced to electron volts by multiplying by 31, the average number of electron-volts required to produce an ion in air.

VI. THE BEARING OF THESE CURVES ON THE REALITY, AND LATITUDE OF ENTRANCE, OF HELIUM ANNIHILATION RAYS

From as careful a study as we have been able to make of the Lemaitre-Vallarta curves reproduced in Fig. 2 of our earlier paper,[9] supplemented by a little adjustment, especially in the case of the O band because of our own observations shown in Fig. 4, page 403 of reference 9(a), and because of the evidence presented in pages 239 and 242 of reference 6, that off the west coast of South America the electronic energy necessary to get vertically through the resistance of the earth's magnetic field at the equator is 13 Bev instead of the 15 Bev assumed by Lemaitre and Vallarta (probably a correct assumption for computing incoming energies north of Pasadena), we have estimated in Table I the critical magnetic latitudes for the entry of the cosmic annihilation rays as measured by electroscopes.

The table brings to light the bad overlapping of the various bands and the impossibility of unscrambling them, *save in the case of the helium band.* But because of the wide difference in energy between helium and carbon, the former is able to appropriate to itself entirely all latitudes above 51°, so that on the basis of the atom-annihilation hypothesis, if in going north new cosmic rays come in between 51° and 59°, they can only be helium annihilation rays. In fact, as shown in Fig. 2 there can be no uncertainty about the appearance of a new band of rays between Omaha and Bismarck and a band so little penetrating, as helium rays should be, that it cannot throw its influence down even to the 2-meter level, after which depth the Bismarck curve is seen in Fig. 2 to coincide completely with the Omaha curve.

This is not at all so with the difference between the Omaha and the Oklahoma City curve because this difference is primarily due to the carbon band, for carbon rays have an energy of 5.6 Bev and this is sufficient to throw their influence down nearly to sea level. If, on the other hand, one assumed incoming cosmic-ray energy lying between the C and He bands, then the Oklahoma City curve would certainly be expected to run together with the Omaha curve at a much higher altitude than Fig. 2 shows to be the case.

We expect as soon as possible after the war to locate the latitude of entrance of the vertically incoming helium rays by the vertical counter technique, but the evidence here as to its existence and location is so good that we feel confident that this prediction will be verified as well as this has been done with the whole series of predictions discussed above.

VII. THE VARIABILITY IN INCOMING COSMIC-RAY INTENSITIES

We have published on several occasions in recent years the discovery of large and thus far wholly inexplicable changes in incoming cosmic-ray intensities as measured by the electroscope technique in the northern part of the United States.[10]

TABLE I. Critical magnetic latitudes in North America for entry of cosmic annihilation rays.

Atom	Energy	Latitude of first entry on western horizon	Latitude of full entry on eastern horizon	Latitude of vertical entry
He	1.88 Bev	~51°	~59°	~54°
C	5.6 Bev	~32°	~49°	~42°
N	6.6 Bev	~27°	~48°	~39.5°
O	7.5 Bev	~22°	~46°	~33.5°

Thus, between August 21 and September 3 of 1937 we made good flights with electroscopes at Omaha to such heights as to get definitely over the maximum of ionization in our electroscopes. The value of this ionization at the top when reduced to standard air density was 338 ions

[9] (a) Millikan, Neher, and Pickering, Phys. Rev. **61**, 397 (1942). (b) See also Lemaitre and Vallarta's original papers in Phys. Rev. **49**, 719 and **50**, 503 (1936); also **43**, 87 (1933).

[10] See R. A. Millikan and H. V. Neher, Phys. Rev. **56**, 491 (1939).

per cm^3. In 1938 we went purposely to Omaha in winter (December 22 and 23) and repeated these experiments and found a maximum ionization at the top of 364 ions, an increase of nearly 8 percent. Again, the maximum ionization shown

TABLE II. Time changes in the earth's magnetic field. H in c.g.s. units.

	1915	Annual changes	1940
Bismarck	0.161	−0.00027	0.154
Omaha	.197	−.00049	.185
Oklahoma City	.241	−.00063	.225
St. George	.247	−.00037	.238
Mt. Wilson	.265	−.00035	.256

Magnetic records from Mount Wilson, California; Tucson, Arizona; Cheltenham, Maryland

1937	September	3	No storm
1938	July	10	No storm
	July	15	Small storm July 13 to 17
	December	22	*H* about 0.00030 lower than normal
1940	August, September		No storms.

at the top in the September 25, 1940 flights herewith reported (see Figs. 1 and 2) is 413 ions per cm^3, an increase over December, 1938 of 13.4 percent.

Again, the maximum which we obtained in Bismarck in the summer of 1938 (July 5) was 374 ions/cm^3, while the 1940 maximum found at Bismarck August 21 and reported in Fig. 2 was 485 ions/cm^3, or an increase over 1938 of 29.7 percent.

Still further, our maximum obtained in Oklahoma City in mid July (11, 12, 13) of 1938 was 280 ions/cm^3, while the maximum shown in summer of 1940, August 29 (see Fig. 2), is 319 ions/cm^3, an increase over 1938 of 14 percent.

Contrast the foregoing findings with the following facts. We made electroscope flights at San Antonio in the summer of 1936 and got a maximum of 234 ions/cm^3 and repeated these flights on September 3, 1940 and found (see Fig. 2) precisely the same maximum as in 1936, *viz.*, 234 ions/cm^3. Also, not only was our maximum at Madras, India in 1936 the same as in February in 1940, but as shown in the report of our India work[11] the two curves taken four years apart are everywhere indistinguishable.

In a word, while the cosmic rays coming in at Madras and at San Antonio seem to remain amazingly constant, yet in the northern part of the United States they show amazing fluctuations.

We seek the explanation of this strange behavior in the atom-annihilation hypothesis as follows: We assume for simplicity that the cosmic processes creating the cosmic rays cause an essentially constant incidence of all the five cosmic-ray bands on the earth. According to Bowen, He is ten times more abundant than C, N, O, or Si, and even from the energy standpoint, if the probability of the transformation of the whole rest mass into an electron pair is the same for all atoms, the energy in the helium band is some 3.5 times that in the C, N, and O bands, and some $1\frac{1}{2}$ times the energy of the Si band.

Bismarck and Omaha both lie close to the southern edge of the polar cap of He annihilation rays, which we have computed as located for vertical-counter measurements at 54° N Mag. Lat. and for electroscope measurements at between 51° N Mag. Lat. and 59° N Mag. Lat. With even slight changes in the earth's magnetism this edge of the He polar cap will move north or south and, since the resistance to incoming electrons is very small in northern latitudes, the relative effect of changes in the earth's magnetic field may be correspondingly large depending upon the kind of cause that makes the change. In equatorial latitudes, where the resistance to incoming electrons is say 8 times larger than at Bismarck or Omaha, the fluctuations in the earth's magnetic field will have smaller influence and *no influence at all unless the point of observation is near the edge of one of the five polar caps corresponding to the latitude of entrance of one of the cosmic-ray bands.*

TABLE III. Tucson values of H corresponding to the year and month at which the foregoing cosmic-ray measurements were made.

Omaha	Bismarck	Oklahoma City
Sept. 1–30, 1937, 26189γ	July 1–31, 1938, 26182γ	July 1–31, 1938, 26182γ
Dec. 1–31, 1938, 26162γ	Aug. 1–31, 1940, 26141γ	Aug. 1–31, 1940, 26141γ
Aug. 1–31, 1940, 26140γ		

In the present experiments according to Table I the helium rays from the western horizon are already passing through the electroscope in the upper levels of the atmosphere and a very little change in the earth's magnetic field that would make the band move say 2° southward, according to Lemaitre-Vallarta curves, would throw the

[11] See H. V. Neher and W. H. Pickering, Fig. 1, Phys. Rev. 61, 408 (1942).

greater part of the helium band's energy into the top of the Omaha curve. *It is presumably actually there in these observations* in which the Omaha curve runs higher than we have observed it to do in any of our other trials there.

Oklahoma City is the only place, save Bismarck and Omaha, at which these changes have been observed, and here they were in smaller amount. Oklahoma City is actually quite close to the computed latitude of entrance of the carbon band. The reason for the high values of the Bismarck, Omaha, and Oklahoma City curves in 1940 is that then the edge of the He and C polar caps were farther south than at the time of previous readings. Furthermore, as Table I shows, the width of the slope of the rise to a new plateau increases greatly for the more southerly bands. This is another reason why a small change in the earth's magnetic field has little effect in the more southerly latitudes, but a large effect in the northerly ones.

According to this hypothesis, an increase in the intensity of the cosmic rays at Bismarck and Omaha means a southward movement of the edge of the helium polar cap and such a movement in turn means a weakening in the strength of the earth's magnetic field. Such a weakening is actually shown in Table II which has just been prepared by Dr. S. B. Nicholson of the Mount Wilson Observatory who is looking for correlations between sun-spot activity and magnetic storms, and who also tells us that magnetic storms in general correspond to a weakening rather than a strengthening in the earth's field. Further evidence of such weakening of the earth's field between May, 1936 and May, 1938 can be seen in Fig. 1,[12] which we took from the paper of S. E. Forbush,[13] of the Carnegie Institution of Washington.

[12] See R. A. Millikan and H. V. Neher, Phys. Rev. 56, 492 (1939).

[13] S. E. Forbush, Phys. Rev. 54, 975 (1938).

In further check on the foregoing apparent correspondence between cosmic-ray intensity and the strength of the earth's magnetic field in the western part of the United States, Director L. O. Colbert of the Coast Geodetic Survey has kindly furnished us with the data which we have put together in the form shown in Table III. This is more definite than Table II in indicating that at every one of the above observed times (year and month) in which we observed an increase in cosmic-ray intensity over its value at the same station at some other year and month there had also occurred a corresponding small decrease in the horizontal component H of the earth's magnetic field at the Tucson magnetic observatory.

VIII. THE FIELD SENSITIVE COMPONENT OF THE COSMIC RAYS

In view of the greater accuracy in these electroscope measurements than of our preceding ones, we have computed from the new curves of Fig. 2 combined with the unchanged measurements at Madras,[14] that *the field sensitive component defined through these electroscope readings is 65 percent of all incoming cosmic-ray energy.* All this is due to incoming charged particles, certainly electrons or protons. There is at present no way of finding what fraction of the non-field sensitive component is due to charged particles and what to photons, though part of it is unquestionably due to the former.

This investigation, like many preceding ones, has had support from the Carnegie Corporation of New York administered by the Carnegie Institution of Washington. For this aid, as well as for the aid given us by Dr. Seth B. Nicholson and Director Colbert, the authors wish to express their keen appreciation.

[14] See Figs. 1, 7, and 8 of Millikan, Neher, and Pickering, Phys. Rev. 61, 397 (1942).