# Letters to the Editor

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## On a Recent Paper by Jaffé

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I HAVE been interested in comparing a paper, "On the currents carried by electrons of uniform velocity," by George Jaffé,[1] with an earlier paper on the subject by C. E. Fay, A. L. Samuel, and W. Shockley.[2] These treatments agree in all cases in which the current is not "space-charge limited." The difference may be illustrated by the following special case.

Suppose we have a thermionic cathode, a positive grid close to it, and an anode at the same potential as the grid, spaced many times as far from the grid as the grid is from the cathode. Imagine that we gradually increase the temperature of the cathode, and hence the current from it, until the cathode current finally becomes space-charge limited.

According to Fay, Samuel, and Shockley, the entire cathode current can be transmitted through the grid-anode region until a "limiting current" is reached. For cathode currents greater than this, and also as an alternative possibility for currents somewhat smaller, they would regard as valid an expression which involves a potential minimum at cathode potential between grid and anode, with part of the electron flow turned back toward the cathode at this potential minimum. Under this condition the anode current is considerably less than the "limiting current." Clearly, Fay, Samuel, and Shockley regard this condition only as representing approximately the actual solution with thermal velocities, for they say, "The assumption of equal energy electrons leads to indeterminacy at planes where the potential and its gradient are both zero. At these planes the existence of non-uniform electron velocities will be recognized insofar as it provides a selective mechanism to resolve the mathematical indeterminacy."

On the other hand, Jaffé rejects completely any expressions implying zero electron velocity and infinite charge density at a plane, and according to his treatment the final value of the anode current is the "limiting current" of Fay, Samuel, and Shockley, mentioned above.

Jaffé says, "Now, the velocity of the electrons naturally becomes zero at a point where $\eta = -1$ [see (3)], and therefore the density $n$ would have to be infinite. We are of the opinion that no solution should be admitted which makes a physically measurable quantity infinite. We, therefore, conclude that the limiting case $X_1=0$ (on branch II) should be excluded even as a possible form of unstable discharge." This, of course, excludes all solutions for which a part of the current is turned back, and indeed, Jaffé says in his introduction, "In general, the minimum will not be low enough to stop electrons which have entered the discharge space. In spite of this fact the current will be space-charge limited (for sufficiently low values of the potential) because the solution of the problem does not exist for any given value of the plate distance unless the current is below a critical value. Thus the space-charge limitation is caused, in this case, not by the fact that some electrons return to the cathode, but by the fact that only a limited number of them can get into the discharge space."

It is pretty clear experimentally that electrons can be turned back in a space between approximately plane electrodes. The writer believes that expressions given by Fay, Samuel, and Shockley which involve a plane of zero velocity and infinite charge density (the charge is not infinite) are physically acceptable as representing a limit which the correct solutions involving thermal velocities must approach as the parameter expressing the ratio of the thermal energy to the energy at the entrance or exit plane is made very small.

In regard to stability, Jaffé's argument that in cases in which the velocity does not go to zero, the solution with the lower potential minimum (Fay, Samuel, and Shockley $C$ overlap solution) is unstable because "it would require work to change the discharge with the higher potential curve to that with the lower potential curve" is not decisive, as by this criterion a bottle is unstable when standing upright. Fay, Samuel, and Shockley also regard the $C$ overlap solution as unstable, and there are reasons to suspect that it is completely unstable.

In a paper, "Vacuum-tube networks," by F. B. Llewellyn and L. C. Peterson,[3] the a.c. problem is treated. For an undisturbed current injected into a diode, the relation between a.c. (or perturbation) voltage $V$ across the diode and a.c. (or perturbation) current $I$ in the diode is given by an expression which for very low frequencies reduces to

$$V = I(u_a+u_b)(T/\epsilon)(1-\zeta+\zeta\beta/3)/\beta.$$

Here $u_a$ and $u_b$ are velocities at the entrance and exit planes, $\epsilon$ is the dielectric constant of vacuum, $T$ is transit time across the diode, $\zeta$ is a "space-charge factor" which for the $C$ overlap solution is always greater than unity, and $\beta = i\omega T$. We note that for certain small values of $\beta$ such that

$$\beta = 3(\zeta-1)/\zeta,$$

current oscillations can occur in a short-circuited diode ($V=0$). We also note that for $\zeta$ a little less than unity the oscillation starts as an exponential decay of perturbation current with time, while for $\zeta$ a little larger than unity the oscillation starts as an exponential increase of perturbation current with time, indicating an instability.

[1] George Jaffé, Phys. Rev. **65**, 91–98 (1944).
[2] C. E. Fay, A. L. Samuel, and W. Shockley, Bell Sys. Tech. J. **17**, 49–79 (1938). The same matter was treated by B. Salzberg and A. V. Haeff in the RCA Review (Jan. 1938).
[3] F. B. Llewellyn and L. C. Peterson, Proc. I.R.E. **32**, 144–166 (1944).